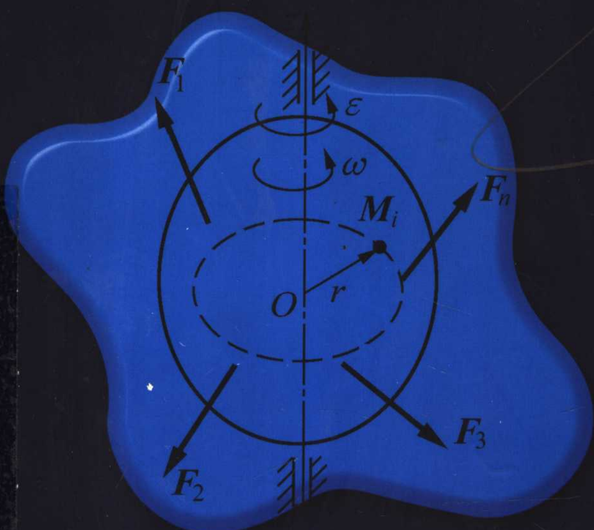
Textbook Series for 21st Century

THEORETICAL MECHANICS

理论力学

Chief Editors
Luan Xifu Zhang Tao Zhao Chunxiang



哈爾濱工業大學出版社

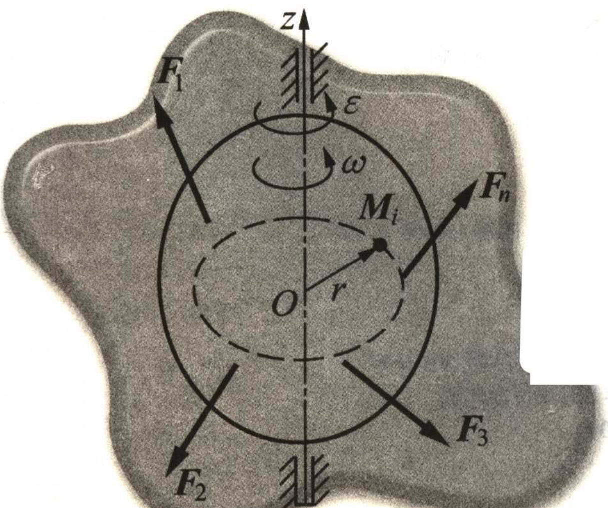
extbook Series for 21st Century

THEORETICAL MECHANICS

理论力学

Chief Editors

Luan Xifu Zhang Tao Zhao Chunxiang



哈爾濱工業大學出版社

Synopsis

This book is based on the basic teaching requirement for the course of theoretical mechanics in advanced industrial colleges in the People's Republic of China.

The book includes 16 chapters: principles of statics and the free-body diagrams, basic operations with force systems, reductions and resultants of force systems, equilibrium of coplanar force systems, equilibrium of noncoplanar force systems, friction, kinematics of a point, translation and rotation of rigid bodies, composite motion of a point, plane motion of rigid bodies, kinetics of a particle, principle of linear impulse and momentum, principle of angular impulse and momentum, principle of work and kinetic energy, D' Alembert's principle, and principle of virtual displacements.

This book can be used as a bilingual teaching textbook of the course of theoretical mechanics for undergraduates in advanced engineering universities. It can also be used as a reference for related teachers, students, and engineering technicians.

图书在版编目(CIP)数据

理论力学 = Theoretical Mechanics:英文/栾锡富,张涛,赵春香主编.—哈尔滨:哈尔滨工业大学出版社,2007.8

ISBN 978-7-5603-2533-0

Ⅰ.理… Ⅱ.①栾…②张…③赵… Ⅲ.理论力学-英文 Ⅳ.031

中国版本图书馆 CIP 数据核字(2007)第 065658 号

责任编辑　杜　燕
封面设计　卞秉利
出版发行　哈尔滨工业大学出版社
社　　址　哈尔滨市南岗区复华四道街 10 号　邮编 150006
传　　真　0451-86414749
网　　址　http://hitpress.hit.edu.cn
印　　刷　肇东粮食印刷厂
开　　本　787mm×960mm　1/16　印张 16.75　字数 305 千字
版　　次　2007 年 8 月第 1 版　2007 年 8 月第 1 次印刷
书　　号　ISBN 978-7-5603-2533-0
印　　数　1~3 000 册
定　　价　28.00 元

Preface

For the convenience of the bilingual teaching in the course of theoretical mechanics, we wrote and edited this book. The salient features of the book are as follows:

1. This book is written according to "The basic teaching requirement for the course of theoretical mechanics in Advanced Industrial Colleges" drawn up by the Ministry of Education of China.

2. The book provides the students with a clear and brief presentation of the theory and application of theoretical mechanics.

3. This book is written fluently and easily to read. Its words are the most commonly used in science and technology.

The book includes three parts: statics, kinematics, and kinetics. It is divided into sixteen chapters. The first part, statics, introduces basic concepts and principles of statics, the free-body diagrams, resultants and equilibrium of force system, friction, and the principle of virtual displacement, etc.. The second part, kinematics, covers kinematics of a point, translation and rotation of rigid bodies, composite motion of a point, and plane motion of rigid bodies. The third part, kinetics, covers fundamental laws of kinetics and differential equations of a particle, the principle of linear impulse and momentum, the principle of angular impulse and momentum, the principle of work and kinetic energy, and D'Alembert's principle. For most specialties with moderate period of theoretical mechanics, the use of this book should be enough.

The definition, theory and expression in this book are not exclusive in some situation, even not the best. What we should do is just to offer a few commonplace remarks which may come up with some valuable views. We hope that teachers and students who use this book can attach more concern, thinking, love and action on bilingual education of the theoretical mechanics; we really appreciate your precious opinions.

We wrote this book under two motivations:

1. With the rapid development of science and technology, the open model of mechanics education appears which is bound to cause the necessity of bilingual education and needs our

mechanical teachers to make contribution for it.

2. The second motivation is the authors' long-term enthusiasm on bilingual education of the theoretical mechanics. The clear English expression of the definition, theory and application in the theoretical mechanics will help students to establish the ability of thinking and solving the mechanical problems in English and improve their development.

For the structure of this book, our objective is to try to set up a scientific and reasonable design. We hope that the content of the book can be close to the engineering application as far as possible. We will try our best to make this textbook of bilingual education more deeply in the content, more prominent in the features.

This book is intended to serve as a bilingual teaching textbook of the course of theoretical mechanics for undergraduate students majored in mechanical engineering, aerospace engineering and civil engineering. It can also serve as a reference book for students, teachers, and the technicians in related areas.

In this book chapter 1 to 3,15 and 16 are edited and written by Luan Xifu of Jiamusi University, chapter 7 to 10 by Zhang Tao of Qiqihar University, chapter 11 to 14 by Zhao Chunxiang of Heilongjiang College of Science and Technology, and chapter 4 to 6 by Zhao Hong of Qiqihar University. All the figures are plotted by Zhou Jun of Jiamusi University. In this book Luan Xifu, Zhang Tao, and Zhao Chunxiang hold the chief editors.

Although our colleagues and we have done our best, we are afraid that errors are unavoidable. We would greatly appreciate hearing from you whatever you have any comments, suggestions, or problems related to this book.

Finally, we express our thanks to those teachers and students who offer their help to this book.

Editors

May, 2007

List of symbols

$\boldsymbol{a}$ acceleration

$\boldsymbol{a}_n$ normal acceleration

$\boldsymbol{a}_\tau$ tangential acceleration

$\boldsymbol{a}_e$ transport acceleration

$\boldsymbol{a}_C$ Coriolis acceleration

A area

A_O angular impulse with respect to point O

f_k kinetic friction factor

f_s static friction factor

$\boldsymbol{F}$ force

$\boldsymbol{F}_I$ inertia force

$\boldsymbol{F}_s$ static friction force

$\boldsymbol{F}_N$ normal reaction

$\boldsymbol{g}$ acceleration due to gravity

h height

H_O angular momentum with respect to O point of a rigid body

H_C angular momentum with respect to center of mass

H_z angular momentum with respect to z-axis

$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$ base vectors

$\boldsymbol{I}$ linear impulse

I_z moment of inertia with respect to z-axis

I_C moment of inertia with respect to center of mass

L length

m mass

$M_z(\boldsymbol{F})$ moment with respect to z-axis

$\boldsymbol{M}_O(\boldsymbol{F})$ moment with respect to point O

P weight, power, linear momentum

q intensity of loads

r radius

$\boldsymbol{r}$ position vector

R radius

s arc coordinates

t time

T kinetic energy

U work of force

$\boldsymbol{v}$ velocity

$\boldsymbol{v}_a$ absolute velocity

$\boldsymbol{v}_r$ relative velocity

$\boldsymbol{v}_e$ transport velocity

V potential energy, volume

x, y, z rectangular coordinates

δ variation

α, β, γ angles

ε angular acceleration

ρ radius of curvature

ω angular velocity

ω_a absolute angular velocity

ω_r relative angular velocity

ω_e transport angular velocity

Contents

Part 1 Statics

Part 2 Kinematics

Part 3 Kinetics

Part 1 Statics

Introduction

Theoretical Mechanics is the science that studies the general laws of mechanical motions of bodies. **Mechanical motion** refers to that relative positions of bodies in space change with time.

Theoretical mechanics is of great use for the design and analysis of many types of structural or mechanical devices encountered in engineering. Also, theoretical mechanics forms the foundation of many engineering disciplines.

Theoretical mechanics is commonly divided into three areas: **statics**, **kinematics**, and **kinetics**. Statics deals with the conditions of equilibrium of bodies that are acted on by forces. Kinematics is the study of the geometry of motion. It is not concerned with the causes of motion. In kinetics, we study the relationships between the forces acting on the body and the resulting motion.

The **system of forces** means a few forces that act on bodies.

The force systems may be classified as follows:

- force systems
  - coplanar
    - concurrent
    - parallel
    - general case
  - noncoplanar
    - concurrent
    - parallel
    - general case

Two force systems that produce the same external effects on a rigid body are said to be **equivalent**.

In mechanics, the term **equilibrium** implies a body is at rest or moving with constant velocity in a straight line with respect to the surface of the earth. The equilibrium can be considered as a special case of mechanical motion.

In statics we study three fundamental problems as follows:

(1) The analysis of forces acting on the bodies and the free-body diagram.

(2) The reduction of a force system, namely, which reduces a system of forces to a simpler, equivalent system of forces.

(3) The conditions of equilibrium of bodies acted by various force system and the equations of equilibrium.

The knowledge obtained in statics can be applied directly to kinetics, mechanics of materials, and many fields of engineering. For example, mechanical, aerospace, or architectural engineer who designs structures uses the equilibrium equations derived in statics.

Chapter 1 Principles of Statics and the Free-Body Diagrams

In this chapter we introduce fundamental concepts and principles of statics, and show how to draw the free-body diagram.

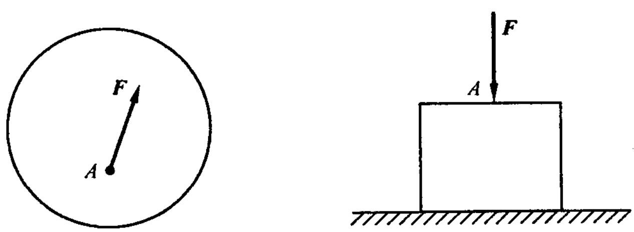
1.1 Fundamental Concepts of Statics

1. Force

Force is the mechanical interaction between bodies that changes or tends to change the state of motion of the body or changes shape which it acts on.

A force can affect both the motion (external effect of forces) and the deformation (internal effect of forces) of the body on which it acts. Forces may arise from direct contact between bodies, or there may be applied at a distance (such as gravitational attraction).

Sometimes the area over which a contact force is applied is so small that it may be approximated a point, in which case, the force is said to be concentrated at the point of contact. The contact point is also called the **point of application** of the force. The **line of action** of a concentrated force is the line that passes through the point of application and is parallel to the force. Force is a vector, which possesses direction as well as magnitude and can be represented graphically by a vector shown in Fig. 1.1.



(a) (b)

Fig. 1.1 Representing the force vector F acts on a body

For the complete definition of a force, we must know: (1) its magnitude; (2) its point of application, and (3) its direction. These three quantities that completely define the force are called its characteristics, or three elements.

In the SI system, the unit of force is Newton ($1\ N = 1\ kg \cdot m/s^2$).

2. Rigid Body

Physical bodies, such as engineering structures and machine parts, are never absolutely rigid and deform slightly under the action of the loads that they carry. Such deformation, however, is usually very small and can be completely ignored in the investigation of conditions of equilibrium. Thus, in statics, we made the assumption that we are dealing with rigid bodies. A **rigid body** is defined as one, which does not deform at all under the action of applied loads. Obviously, a rigid body is an abstracted mechanical model. For a deformable body, in which the effect of small deformations in physical bodies must be taken into account are generally treated in books on mechanics of materials.

1.2 Principles of Statics

The study of statics rests on five fundamental principles based on experimental evidence. These principles may be stated as follows:

First Principle (Two-Force Equilibrium Principle): If a rigid body is held in equilibrium by two forces only, the two forces must have the same magnitude, the same line of action, and opposite sense. For example, a rigid straight bar is subjected to a pair tensile or compressive forces that have the same magnitude, the same line of action, and opposite sense in equilibrium as shown in Fig. 1.2.

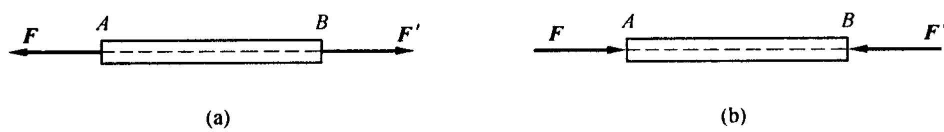


Fig. 1.2 The rigid bar *AB* is subjected to a pair tensile or compressive forces in equilibrium

When a body (or member) is subjected to no couple moments and forces are applied at only two points on a body (or member) in equilibrium, the body (or member) is called a **two-force body** (or member). An example is shown in Fig. 1.3. The forces at *A* and *B* are summed to obtain their respective resultants F_A and F_B. Using the two-force equilibrium principle, we know, without writing any equilibrium equations, that the resultant forces at *A* and *B* are equal in magnitude and oppositely directed along the line joining *A* and *B*.

Second Principle (Plus or Minus a Force System of Equilibrium Principle): The action of a given system of forces acting on a rigid body will in no way be changed if we add to, or subtract from, these forces any other system of forces in equilibrium. For example, two given systems of forces as shown in Fig. 1.4 are equivalent.

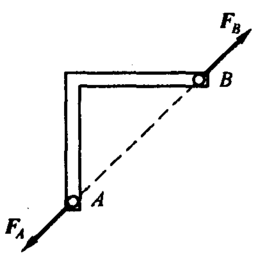


Fig. 1. 3 The rigid body *AB* is subjected to two forces in equilibrium

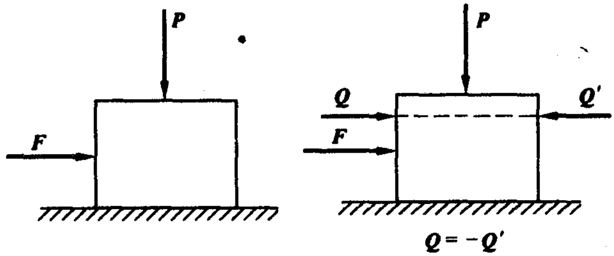


Fig. 1.4 Two figures are equivalent

Using the above principle, it can be proved that a force may be moved anywhere along its line of action without changing its external affects on a rigid body. This conclusion is summarized by **the principle of transmissibility**. Thus, in the case of forces acting on a rigid body, however, the point of application of the force does not matter, as long as the line of action remains unchanged. In other words, forces acting on a rigid body are vectors which may be allowed to slide along their line of action; such vectors are called sliding vectors.

As an illustration of the principle of transmissibility, we consider the rigid block shown in Fig. 1.5. The block is subjected to a force $\boldsymbol{F}$ that acts at a point A. Using plus or minus a force system of equilibrium principle we can prove the two forces $\boldsymbol{F}$ and $\boldsymbol{F}'$, which acts at point B, are equivalent.

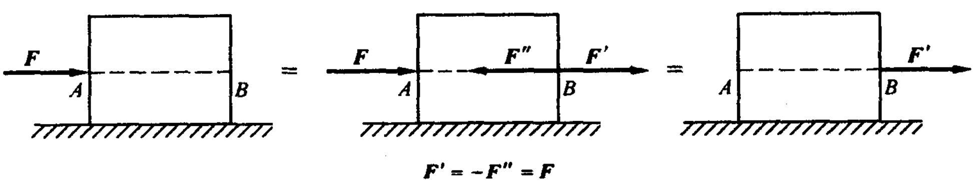


Fig. 1.5 Two forces $\boldsymbol{F}$ and $\boldsymbol{F}'$ are equivalent

Third Principle (Principle of The Parallelogram of Force): If two forces of $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$, represented by vector *AB* and *AC* are applied to a body at point A, their action is equivalent to the action of the one force of $\boldsymbol{R}$, represented by the vector *AD* obtained as the diagonal of the parallelogram constructed on the vectors *AB* and *AC* as shown in Fig. 1.6.

That is

$$R = F_1 + F_2.$$

The force $\boldsymbol{R}$ is called the resultant of the two forces $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$. The $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ are called components of the force $\boldsymbol{R}$. Thus, a force is equivalent to its components and vice versa. This principle is called **the principle of the parallelogram of force.**

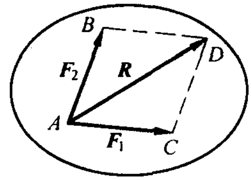
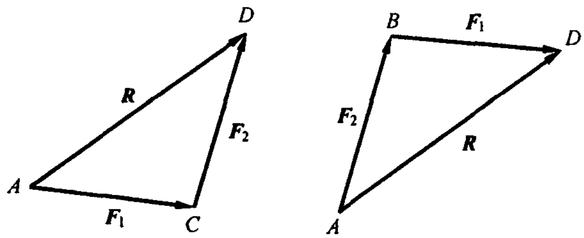


Fig.1.6 Parallelogram principle

An equivalent statement of the principle of the parallelogram of force is **the principle of the triangle of force**, which is shown in Fig. 1.7(a). Here the tail of $\boldsymbol{F}_2$ is placed at the tip of $\boldsymbol{F}_1$, and $\boldsymbol{R}$ (closed side) is the vector that completes the triangle drawn from the tail of $\boldsymbol{F}_1$ to the tip of $\boldsymbol{F}_2$. (Observe that the result is identical if the tail of $\boldsymbol{F}_1$ is placed at the tip of $\boldsymbol{F}_2$ and $\boldsymbol{R}$ is drown from the tail of $\boldsymbol{F}_2$ to the tip of $\boldsymbol{F}_1$ shown in Fig. 1.7(b)). The addition of two vectors $\boldsymbol{F}_1$ and $\boldsymbol{F}_2$ is defined to be the force vector $\boldsymbol{R}$ that is determined by the geometric construction (vector sum).



(a) (b)

Fig. 1.7 Triangle principle

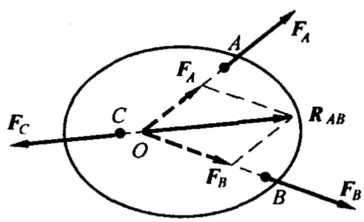


Fig. 1.8 Three forces intersect at a point O

Three-Force Principle: Three nonparallel, coplanar forces that hold a rigid body in equilibrium must be concurrent.

The proof of this principle can be obtained by referring to Fig. 1.8, which shown a rigid body subjected to the three nonparallel, coplanar forces $\boldsymbol{F}_A$, $\boldsymbol{F}_B$, and $\boldsymbol{F}_C$. Because the forces are not parallel, two of them—say, $\boldsymbol{F}_A$ and $\boldsymbol{F}_B$—must intersect at some point O, and have a resultant that is supposed as $\boldsymbol{R}_{AB}$ ($\boldsymbol{R}_{AB} = \boldsymbol{F}_A + \boldsymbol{F}_B$) passed through a point O. Therefore, according to two-force equilibrium principle we know, which force $\boldsymbol{F}_C$ must also pass through the same point O. This completes the proof of the principle.

Fourth Principle (Action and Reaction Principle): There are mutual actions between any two bodies such that the forces of action and reaction have the same magnitude, the same line of action, and opposite sense.

Fifth Principle (Principle of Solidification): If a freely deformable body subjected to the action of a force system is in equilibrium, the state of equilibrium will not be disturbed if the body solidifies. This principle is called the principle of solidification.

1.3 Reactions of Supports

Forces that act on a body can be divided into two general categories-**reactive forces** (or, simply, **reactions**) and **applied forces**. Reactions are those forces that are exerted on a body by the supports to which it is attached. Forces acting on a body that are not provided by the supports are called applied forces.

Before presenting a formal procedure as to how to draw a free-body diagram, we will first consider the various types of reactions that occur at supports and points of supports between bodies subjected to coplanar force system. As a general rule, if a support prevents the translation of a body in a given direction, then a force is developed on the body in that direction. Likewise, if rotation is prevented, a couple is exerted on the body.

Table 1.1 shows the reactions exerted by various coplanar supports. The reactions at each support are described below.

1. Flexible Cable

A flexible cable (such as a string, a chain or a strap that is negligible weight) exerts a tensile force in the direction of the cable that applies at the point of contact and the direction of it is away from the body.

2. Frictionless Surface

When a body is in contact with a frictionless surface, the reaction is a force that is perpendicular to the surface, acting at the point of contact and the direction of it is toward the body. This reaction is often referred to simply as the normal force.

3. Roller Support

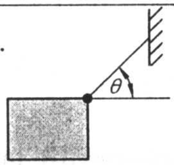
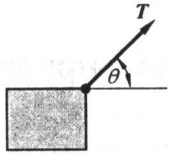
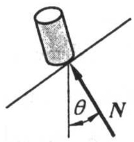
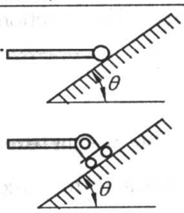
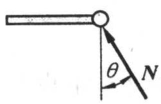
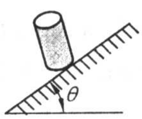
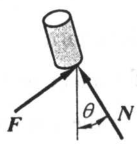
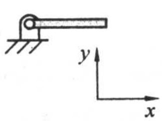
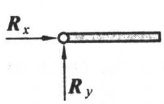
A roller support is equivalent to a frictionless surface: it can only exert a force that is perpendicular to the supporting surface.

4. Surface with Friction

A friction surface can exert a force that acts at an angle to the surface. The unknowns may be taken to be the magnitude and direction of the force; however, it is usually advantageous to represent the unknowns as N and F, the components that are perpendicular and parallel to the surface, respectively. The component N is called the normal force, and

F is known as the friction force.

Table 1.1 Reactions of Coplanar Supports

Supports	Reaction(s)	Description of reaction(s)	Number of unknowns
1.  Flexible cable of negligible weight		Tension of unknown magnitude ***T*** in the direction of the cord of cable	One
2.  Frictionless surface (single point of contact)		Force of unknown magnitude ***N*** directed normal to the surface	One
3.  Roller support		Force of unknown magnitude ***N*** normal to the surface supporting the roller	One
4.  Surface with friction (single point of contact)		Force of unknown magnitude ***N*** normal to the surface and a friction force of unknown magnitude ***F*** parallel to the surface	Two
5.  Pin support		Unknown two components $\boldsymbol{R}_x$ and $\boldsymbol{R}_y$ of the reaction ***R***	Two