W.G.V. Rosser

An Introduction to the Theory of Relativity

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W. G. V. ROSSER, M.Sc., Ph.D.
Lecturer in Physics, Exeter University

LONDON
BUTTERWORTHS
1964

PREFACE

There are a large number of textbooks on the theory of relativity, but they are generally either advanced treatises or semi-popular accounts. The aim of the present book is to fill the gap between the two, a gap which is at present filled mainly by short monographs or by the odd chapter in textbooks on modern physics. The author has found that students are interested in a more comprehensive discussion of the theory of relativity than can be given in a short monograph. The book is written at a level which the *average* honours *physics* or pass degree student should be able to understand in his second or third year at the University, and it is written primarily with these in mind, though it is hoped that it will prove useful for mathematicians who are interested in the theory of relativity as something deeper than a mathematical exercise. The book should also be suitable for electrical engineers, who seem to be showing an increasing interest in the theory of relativity, particularly in relation to electromagnetism. It is also hoped that it will prove useful for those people who may wish to read about the theory of relativity, after a lapse of a few years. For the most part only simple algebra and calculus are used and the 4-vector methods are only introduced in Chapters 6 and 10 after the ground has already been covered using the simpler mathematical methods. Having understood the physical principles first, the average experimental physicist can then appreciate the elegance, and conciseness of the 4-vector method realizing, of course, that it adds nothing new to the physics of the special theory. Algebraic proofs are generally given in full, as many students are worried until they have followed them through. The better students may prefer to go on quickly to the 4-vector methods. The author has given a more comprehensive treatment than is normal in undergraduate physics courses, as he has found that students are generally anxious to read around the subject. A suggested reading list is given at the end of each chapter and references to the original literature are given whenever appropriate, these references being listed at the end of each chapter.

Professor Holton wrote recently in the *American Journal of Physics**: 'Over the next few years a good deal more SRT (special relativity theory) will find its way into introductory and intermediate college

* *Amer. J. Phys.* **30** (1962) 462.

physics courses'. The author's experiences as a Visiting Associate Professor at the State University of Iowa showed that most of the present book is suitable for an intermediate course at American Colleges, given during the junior or senior year.

Since the book is written by an experimental physicist, the author has been at great pains to relate the theory to experiment at all stages. The postulates of the theory are discussed critically in terms of the experimental evidence available. An attempt has been made to state clearly the assumptions made at each stage, and the conclusions of the theory are always compared with the experimental results available.

The undergraduate period, during which the students attend many courses, is a formative period and the *average* student has not had time to assimilate everything completely and to put everything into the correct perspective. The students may have understood the courses when they were given, but they still like to be reminded briefly of standard topics, when they are introduced, with perhaps a slightly different emphasis, into a course on relativity. For this reason, short reviews of physical principles are often given in the text, for example, Maxwell's equations are discussed in Section 2.2 and the definition of the electric and magnetic vectors is given in Appendix 2. Nowadays, Maxwell's equations are almost invariably taught in undergraduate courses in physics, and generally vector analysis is used. The author has not hesitated to introduce Maxwell's equations and vector analysis whenever appropriate, but such sections are rare before Chapter 8 and can be omitted.

It has often been said that most physicists are merely technicians, in the sense that they can use the equations of physics, but they never question the validity of the laws they use, how and why the laws were introduced and how the various scientific laws are related to each other. Very few courses in physics include a historical survey and a review of the scientific methods involved. A course on the theory of special relativity affords the opportunity of discussing, briefly, the history and methods of several branches of physics, such as mechanics, optics and electromagnetism, and illustrating the relationships between them, before going on to discuss the changes in outlook that resulted from the theory of special relativity. This approach should, of course, be extended in a course on quantum mechanics. The short accounts of scientific method scattered here and there in the present text are not meant to be a complete survey, but merely a short discussion of those parts which relate to the theory of relativity. An attempt was made to try to omit all controversial 'doctrines' from these short discussions apart from the clock paradox. In the latter case an attempt is made to present, as objectively as possible, the views of both sides in the controversy.

In Chapter 1 a brief review is given of the scope and limits of Newtonian mechanics. The historical introduction is continued in Chapter 2, where an attempt is made to illustrate how the theory of special relativity arose out of classical electromagnetism. The usual course on relativity generally starts with accounts of experiments relating to the properties of the ether, such as the aberration of light from stars, Airy's experiment, Hoek's experiment, Fizeau's experiment, etc. These experiments are then interpreted in terms of the transformations of the theory of special relativity later in the course. The various ether theories must have led to great confusion in the nineteenth century, but it is fairly safe to say that they lead to even more confusion in the twentieth century, at least among students. For these reasons a fairly full account of these experiments and theories is given. The amount of space devoted to them tends to over-emphasize their significance in the twentieth century. In Chapter 3 the Lorentz transformations are derived. After discussing the postulates, the theory is developed axiomatically and then the predictions of the theory are compared with experiments. The velocity transformations are derived and applied to relativity optics in Chapter 4. Relativistic mechanics is considered in Chapter 5. In Chapter 6 the two methods of representing the Lorentz transformation geometrically are discussed. Even though it is rather complicated, the method based on the use of real variables only is discussed, since it is applied extensively in the various textbooks such as Eddington's *Space, Time and Gravitation.*

In Chapter 7, electromagnetism is developed in an axiomatic way from the transformations of the theory of special relativity, the principle of constant charge and Coulomb's law. The main object of this approach is to illustrate the essential unity of electromagnetism. It is also a good way of illustrating the concept of action at a distance. The electromagnetism section of the book, namely Chapters 7, 8, 9 and 10, is meant to complement rather than compete with the normal courses given on electromagnetism. In Chapter 8 it is shown that Maxwell's equations are Lorentz covariant. A brief description of motional e.m.f's is included, since this is one of the few cases of e.m.f's when it is possible to form a reasonably clear picture of what is going on. Chapter 9 on the scalar and vector potentials is expanded to include a brief account of the Liénard–Wiechert potentials and their application to the calculation of the radiation from a moving charge. In Chapter 10 the equations of classical electromagnetism are expressed in matrix and tensor form. In this form their covariance becomes more apparent. In Chapter 11 a fairly full account is given of the clock paradox. This subject always provokes tremendous interest, but the accounts in textbooks are generally rather short and often rather biased. In Chapter 12 a brief account of some of the features of the theory of general relativity is given in terms of the Principle of Equivalence. This chapter illustrates some of the limitations of the special (or restricted) theory.

The author has benefited from the reading of many textbooks, particularly those by Einstein, Møller, Synge and Tolman, and the author hopes that this book will serve as an introduction to these and other more advanced textbooks. The influence of Cullwick's *Electromagnetism and Relativity* will be apparent in Chapter 8. The author was greatly influenced by Born's *Einstein's Theory of Relativity* and stimulated, to say the least, by O'Rahilly's *Electromagnetics*. He has also been influenced extensively by *Classical Electricity and Magnetism* by Panofsky and Phillips, which he has used as a course textbook at various times. Whittaker's books have proved invaluable on the historical side.

Problems are given at the end of most chapters. These problems are designed to help the student, not to baffle him. Some of the problems are of the descriptive type designed to test whether or not the reader has grasped the subject matter of the previous chapter. Some of the problems require substitution in standard formulae. These are designed to familiarize the reader with the equations and to give him an idea of the orders of magnitude of various phenomena. Some examples are designed as extensions to the text, and references are given in these cases to encourage the reader to read widely around the subject.

The author would like to thank Professor G. K. T. Conn for his encouragement throughout the preparation of the manuscript. He would also like to thank the University of Exeter for permission to reproduce examination questions.

W. G. V. Rosser

Exeter,
February, 1964

Suggested U.D.C. Number: 530.12

Printed in Northern Ireland at The Universities Press, Belfast

CONTENTS

1

HISTORICAL INTRODUCTION I—NEWTONIAN MECHANICS

1.1. INTRODUCTION

The enormous success of Newtonian mechanics in interpreting many of the phenomena familiar to them in their daily lives gives students the impression that it is infallible and correct in every detail. It is surprising how many students are hazy about the fundamentals of Newtonian mechanics, even about such things as the definition of inertial mass and force. At school level the understanding of physics is largely based on familiarity with the equations of physics. When problems are set in mechanics, if they can use the appropriate equation to obtain the correct solution to the problem, students feel that they understand the subject. In this chapter some features of Newtonian mechanics will be discussed a little more critically than is normally done at school level, in preparation for the changes in interpretation necessary when one comes to discuss the theory of special relativity. A few features of scientific theories in general will be pointed out within the context of Newtonian mechanics. This may help to make it a little easier for those readers meeting relativity for the first time to accept that Newtonian mechanics is not infallible, and that, when the velocities of the particles are comparable with the velocity of light, then Newtonian mechanics must be replaced by an entirely new theory.

1.2. THE STANDARDS OF LENGTH, TIME AND MASS

The feature which marked the rise of physical science in the sixteenth and seventeenth centuries was that scientists began to make quantitative observations and to carry out experiments under controlled conditions. In order to make quantitative measurements, standards have to be chosen. This book begins with a review of the primary standards of the so-called fundamental units of length, time and mass.

The international standard of length is the metre. Until recently it was defined as the distance between two defining marks engraved on a platinum–iridium bar, when the temperature of the bar was that of melting ice. The standard metre was kept at the International Bureau of Weights and Measures at Sèvres near Paris. Accurate copies of this standard metre were made and these secondary standards were then used to calibrate other secondary devices such as rulers, which were then used for measuring lengths. The standard metre was calibrated in terms of the wavelength of the red cadmium line and other spectral lines. For example, using a Fabry and Perot interferometer, it was shown by Benoit, Fabry and Perot in 1913 that there were 1,553,164·13 wavelengths of the red cadmium line in a metre of dry air at 15°C and 760 mm pressure. The wavelength of the cadmium line under these conditions was 6438·4696 Å. This wavelength was used for many years as a secondary standard of length and could have been used to calibrate a new standard metre to an accuracy of at least one part in a million. At the Eleventh General Conference on Weights and Measures meeting in Paris in October 1960 it was decided to replace the old definition of the metre in terms of the platinum–iridium prototype by the statement that the metre *is* 1,650,763·73 wavelengths of the orange–red line of krypton–86. The new metre agrees with the old to within about one part in 10^7. If one were in a rocket, moving with uniform velocity relative to the earth, one could still use the new definition of the metre as the unit of length, and one could build an optical apparatus from materials in the rocket and use it to calibrate secondary standards such as rulers, which were at rest relative to the rocket.

In order to measure time one must choose a repetitive process and assume that it recurs at constant time intervals. Any repetitive process could be chosen as a primary standard; the one actually chosen was the mean solar day, which was defined in terms of the average time taken by the earth to make one complete revolution on its axis with respect to the sun. On account of the orbital motion cf the earth around the sun, the solar day varies during the year; hence the average value was taken as the standard. Astronomers prefer to take the period of rotation of the earth relative to the 'fixed' stars, and they use sidereal time. In the laboratory, clocks such as an oscillating pendulum can be calibrated in terms of the mean solar day. The frequency of electrical signals can be measured very accurately and quartz crystal clocks can be used as accurate secondary standards. The frequencies associated with some atomic and nuclear processes have also been used as secondary standards of frequency, for example in ammonia vapour and caesium clocks. There may be secular changes in the angular velocity of the earth's rotation so that the mean solar day may vary from year to year. Pending a possible re-definition of the second in terms of an atomic or molecular frequency, the Eleventh General Conference on Weights and Measures decided to define *the* second as 1/31,556,925·9749 of the tropical year 1900. This fixes the unit of time. If one were in a rocket moving relative to the earth one could set up a secondary atomic clock and calibrate other secondary clocks at rest in the rocket in terms of the frequency of an atomic process, the atomic frequency having previously been determined in terms of the solar day of 1900 using apparatus at rest on the earth.

The international primary standard of mass is the kilogram; this is defined as the mass of a cylinder of platinum–iridium kept at Sèvres. The comparison of the gravitational masses of other bodies with the standard can be carried out by weighing. The determination of inertial mass involves the application of the theory of mechanics and a discussion of this is deferred until after the consideration of Newton's laws of motion. In principle, the rest mass of the proton could be used as a secondary standard of mass suitable for calibrating masses at rest on a rocket moving with uniform velocity relative to the earth.

Other mechanical quantities are expressed in terms of mass, length and time. For example, the velocity of a body is defined as the distance the body moves in unit time. The velocity of light *in vacuo* can be measured accurately, and it has been suggested that one could adopt the velocity of light *in vacuo* as a primary standard, assigning it an arbitrary value. A time interval could then be measured in terms of the distance travelled by light *in vacuo* in that interval. Such a procedure would be especially convenient within the context of the theory of special relativity, since in that theory it is assumed that the velocity of light *in vacuo* has the same numerical value in all inertial reference frames.

Physical measurements on bodies are carried out by comparison with the primary standards (or with secondary standards which have themselves been calibrated by comparison with the primary or other secondary standards). For example, the length of a stationary body can be determined in practice by comparing the positions of the ends of the body relative to a secondary standard such as a ruler, whose length has been subdivided into equal intervals. In this way an *experimental* determination of the length of the stationary body is made, and the physicist is satisfied with a statement that the measured length of a body is a number between, say, 13·3 and 13·4 cm. Generally the physicist does not stop to worry about terms such as the correct or absolute length of a body, though, of course, he is always striving to improve the accuracy of his experimental measurements.

It is common practice to use symbols to denote the magnitudes of physical quantities relative to the primary standards. This enables the laws of physics to be written in a mathematical form. The mathematical expressions of the laws of mechanics do not depend on the actual sizes of the arbitrary units of mass, length and time. The sizes of the fundamental units chosen affect only the values of the numerical constants in the mathematical equations expressing the laws of physics.

1.3. A CRITIQUE OF NEWTONIAN MECHANICS

Prior to the time of Galileo, the prevailing views on motion were derived largely from Aristotle, and when they were not derived from pure reason without recourse to experiments, they were generally based on *qualitative* observations only. In the sixteenth and seventeenth centuries under the influence of people such as Copernicus, Tycho Brahe, Kepler, Galileo, etc., a new outlook arose based on quantitative observations and systematic experimentation under controlled conditions. From a limited number of observations laws were postulated. For example, from a study of the time taken by metal balls to roll down an inclined plane, Galileo postulated the law of falling bodies. At this stage in the development of science, physical laws were generally derived directly by 'induction' from a limited number of quantitative observations. The early investigations on mechanics culminated in Newton's laws of motion and Newton's theory of universal gravitation. A typical statement of Newton's laws of motion is as follows:

(*a*) All bodies continue in their state of rest or of uniform motion in a straight line unless they are compelled to change that state by external forces.

(*b*) The rate of change of momentum is proportional to the impressed force and takes place in the direction in which the force is acting.

(*c*) Action and reaction are equal and opposite.

If in a reference system a body not under the influence of any forces (i.e. a body far removed from all other bodies capable of exerting forces), moves in a straight line with constant speed, then Newton's first law is valid in this reference system. Such a reference system is called an inertial frame. The properties of inertial frames are elaborated in Sections 3.2 and 3.3.

According to Newton's law of universal gravitation, every particle of matter in the universe attracts every other particle with a force proportional to the product of the masses of the particles, inversely proportional to the square of the distance between them, and directed along the line joining the particles. Thus, if the gravitational masses of two 'point' particles are m_1 and m_2 respectively, and if r is their distance apart, then

$$f = G\frac{m_1 m_2}{r^2} \tag{1.1}$$

where G is the gravitational constant.

The object of a theory is to correlate laws so that a theory is more comprehensive than a single law. Newton was familiar with the law of falling bodies and with Kepler's laws of planetary motion. Newton suggested that, if it is assumed that every pair of particles in the universe attract each other with a force given by eqn (1.1), then on the basis of this assumption, plus Newton's laws of motion, the other individual laws can be derived and many other phenomena, such as the motion of the moon around the earth, can be interpreted. In addition to interpreting known laws, new theories are used to make new predictions, which should subsequently be tested by experiment. The degree of acceptance of a new theory depends largely on how well these new predictions agree with the experimental results.

In a theory it is postulated that nature behaves in a particular way. For this reason a theory is sometimes described as a model of nature; it is not necessarily a mechanical model but may be a functional relation such as eqn (1.1). A theory should enable one to predict the course of an experiment from given initial conditions. Mathematical reasoning is used in applying a theory to a particular case. The conclusions of the theory obtained in this way should be in agreement with the experimental results. If a theory were perfectly correct, then there would be a one to one correspondence between the predictions of the theory and the course of nature. When applying a theory one idealizes the system and considers only those quantities which produce effects of the order of magnitude of the accuracy required, for example, when calculating the acceleration of a ball near the surface of the earth, in the interests of simplicity, one would neglect the gravitational attraction of a distant star, though, in principle, it would always be present.

If a theory is to describe what happens in practice, then the quantities appearing in the mathematical expressions of the theory must relate directly or indirectly to quantities which can be measured in practice. The meaning of the quantities used in the statement of Newton's laws of motion will now be discussed. The measurement of length and time relative to arbitrary standards