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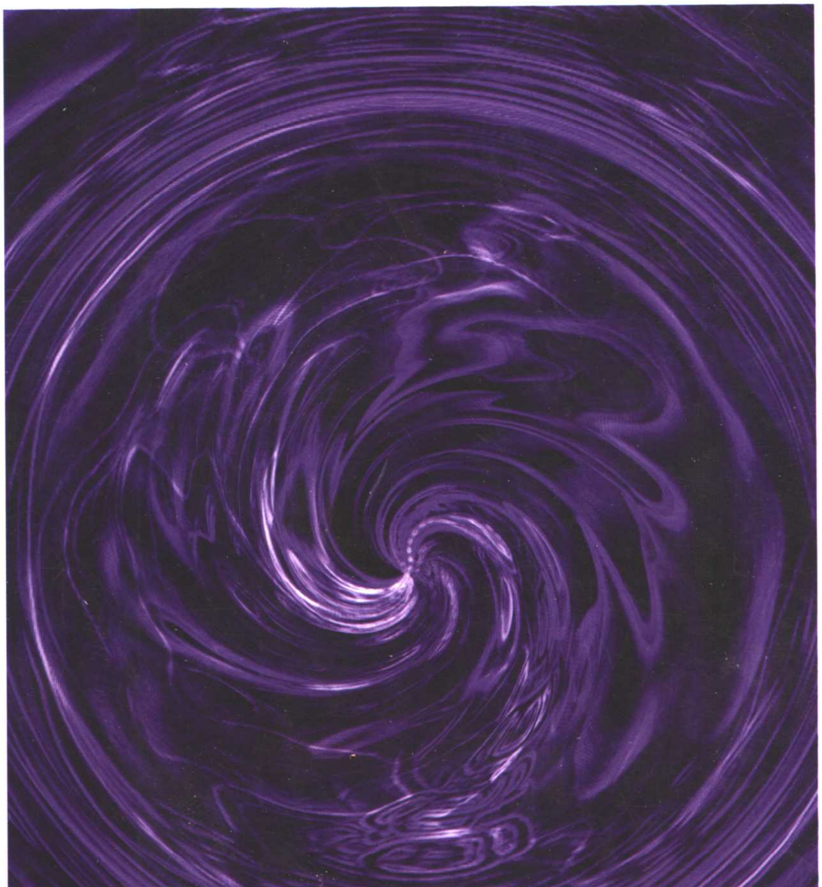
(英文版)



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西 北 大 学

机 械 工 业 出 版 社
China Machine Press

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本书版权登记号：图字：01-2005-1181

图书在版编目（CIP）数据

动力系统导论（英文版）/（美）罗宾逊（Robinson, R. C.）著. –北京：机械工业出版社，2005.5

（经典原版书库）
书名原文：An Introduction to Dynamical Systems: Continuous and Discrete
ISBN 7-111-15913-6

I. 动 … II. 罗 … III. 动力系统（数学）– 英文 IV. O175

中国版本图书馆CIP数据核字（2004）第141023号

机械工业出版社（北京市西城区百万庄大街22号 邮政编码 100037）
责任编辑：迟振春
北京昌平奔腾印刷厂印刷 · 新华书店北京发行所发行
2005年5月第1版第1次印刷
787mm × 1092mm 1/16 · 41.5印张
印数：0 001-3 000册
定价：65.00元

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前　言

本书可作为高年级本科生非线性常微分方程或动力系统课程的教材，书中部分内容也可在低年级硕士生课程中讲授。本书旨在提供计算的范例和方法，同时介绍相关数学概念。无论是介绍动力系统有关应用的概念还是更带理论性的数学引论的课程，主讲教师均可从本书选材。进一步的使用说明可参考下列“组织结构”中的有关讨论。

本书假定学生选修过单变量和多变量微积分、线性代数和微分方程导论的课程。多变量微积分中有关偏导数的素材在本书中广泛使用，少数地方还用到多重积分和面积分(见附录A)。特征值和特征向量是书中用到的线性代数的主要概念，有关线性代数的其他主题请见附录C。微分方程基础知识只在本书第一部分用到；我们假定学生能用变量分离法解一阶方程，并了解二阶标量方程解的形式。选修过微分方程基础课程的学生通常对常系数线性系统（至少有实特征根的情形）比较熟悉，但本书第2章重述了部分材料，其中对相图也作了介绍。有些学生选修过涵盖本书微分方程第一部分的课程而先前却未学过有关微分方程的入门知识；他们能够理解一些领域的新材料，在这些领域他们乐于用额外的努力弥补所缺少的背景知识。最后要提到的是，阅读本书并不要求学生学过实分析或高等微积分的课程。然而，使用这些课程中的一些术语会带来方便，为此，我们提供一个有关连续性和拓扑学术语的附录。

组织结构

本书分两部分介绍动力系统的概念，两部分无先后之分：第一部分讨论非线性常微分方程组的各个方面，第二部分讨论叠函数的相关方面。两部分中任一部分均可用于半学期、两个半学期甚至一学年的课程。在西北大学我们开设了两门课程，一门课程用半学期讲授第一部分，另一门课程用两个半学期讲授第二部分。在半学期的微分方程课程中，很难讲授混沌吸引，甚至不得不略去各章末尾的许多应用实例和证明。一学期的微分方程也可能从第9章到第11章选用有关叠函数的题材。在用本书第二部分的离散动力系统课程中，我们用半学期讲授一维叠函数(第9~11章）的大部分材料。有关高维叠函数（第12~13章）的材料当然依赖于一维函数的材料，但一学期的课程则可以在讲授第9~11章时融入一些高维函数的例子。最后，第14章分形可放在几章之前讲授。分形维可在微分方程课程结束时结合到混沌吸引子的材料中讲授；分形维或叠函数系统的材料可在一维叠函数的课程中讲授。

各章前面几节主要讲解概念，其后一节介绍某些应用，再后面一节是对较难结果的证明和更具理论性的材料。这种节之间的材料划分带有某种随意性。例如，有关竞争种群和捕食与被捕食体系的材料就安排在有关章的前面几节之一中，而不是放在各章末的应用部分，因为这些主题是为展现主要方法服务的。另外，把某些含有较复杂计算和有助于使概念更加清晰的证明放在一些主要节中。对较长且技巧性较强的证明和进一步的理论探讨分别在每一章的结尾给出。

对于着重从应用出发来讲授动力系统概念的课程，可从本书主要章节选材，不用各章末尾有关应用和包含更多理论材料的几节。

各章应用部分提供动力系统的诱导因素，并说明概念的用处。这一节的材料并不是后面主要章节的论述所必需的。这部分材料越多越能加强应用性。

用本书作为教材，教师可以通过舍弃较繁难的证明界定课程的理论水平。具有较高理论水准的课程可以考虑采用各章后面的大部分证明。

计算机程序

本书并未明确介绍计算机编程问题。但是，选用的一些习题需要用计算机模拟产生微分方程的相图或叠函数。Sample Maple电子表格可从网页http://www.math.northwestern.edu/~clark/dyn-sys获得，学生对其加以修改可以用来解决一些其他计算问题（有关本书的订正及更新也可通过该网址查到）。

有几本用Maple 和Mathematica 来处理动力系统的书，其中两本是：M. Kulenović [58] 和S. Lynch [70]。J. Polking 和 D. Arnold的书[85] 中讨论了用Matlab求解微分方程，所用软件包可从http://www.math.rice.edu/~dfield获得。H. Nusse和J. Yorke 的书[80]中有其专门的动力系统软件包。

致谢

我谨对其他几本书的作者深表谢意，我在讲授这一题材时曾用过他们的书，这些书影响了我对题材的理解，特别是在有效地介绍题材方式方面。我难以一一列出那些同样对我产生影响的更高级的书籍。关于微分方程部分我用过的参考书有：F. Brauer and J. Nohel [19], M. Hirsch and S. Smale [51], M. Braun [21], I. Percival and D. Richards [84], D. W. Jordan and P. Smith [55], J. Hale and H.Koçak [48], S. Strogatz [104]。有关叠函数部分我用过的参考书包括：R. Devaney 的两本书 [31]和[32]，D. Gulick[45]，K. Alligood, T. Sauer, and J. Yorke [7]。

还要感谢我读研究生期间指导过我的三位教授：Charles Pugh、Morris Hirsch和Stephen Smale。他们把我引领到动力系统这门学科，并教给我许多思想和方法，使我终身受益。还有我在西北大学的许多同事以不同方式深深影响着我，他们之中有John Franks、Donald Saari和Robert Williams。

下面的审阅人对本书初稿的改进提出了许多宝贵意见和建议，在此我也一并感谢。他们是：John Alongi（波莫纳学院），Pau Atela（史密斯学院），Peter Bates（伯明翰扬大学），Philip Bayly（华盛顿大学），Michael Brin（马里兰大学），Roman Grigoriev（佐治亚理工学院），Palle Jorgensen（艾奥瓦大学），Randall Pyke（雷尔森大学），Joel Robbin（威斯康星大学），Bjorn Sandstede（俄亥俄州立大学），Douglas Shafer（北卡罗来纳大学夏洛特分校），Milena Stanislavova（堪萨斯大学），Franz Tanner（密歇根理工大学），Howard Weiss（宾夕法尼亚州立大学）。

我还要感谢组稿编辑George Lobell对这个项目的鼓励，制作编辑Lynn Savino Wendel为改进表达的清晰性所提出的建议，Adam Lewenberg对准备付印的最后电子文档给予的帮助，Julio Ottino 为本书提供封面照片，Miguel Lerma对解决LaTeX 和图形的各种问题提供的帮助，以及Marian Gidea对使用Adobe Illustrator和Kamlesh Parwani对使用Maple电子表格所给予的帮助。

特别要感谢我的妻子Peggie，她始终如一的宽容、耐心、理解和祝愿，使得本书最终得以完成。

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Contents

Historical Prologue

The theory of differential equations has a long history, beginning with Isaac Newton. From the early Greeks through Copernicus, Kepler, and Galileo, the motions of planets had been described directly in terms of their properties or characteristics, for example, that they moved on approximately elliptical paths (or in combinations of circular motions of different periods and amplitudes). Instead of this approach, Newton described the laws that determine the motion in terms of the forces acting on the planets. The effect of these forces can be expressed by differential equations. The basic law he discovered was that the motion is determined by the gravitational attraction between the bodies, which is proportional to the product of the two masses of the bodies and one over the square of the distance between the bodies. The motion of one planet around a sun obeying these laws can then be shown to lie on an ellipse. The attraction of the other planets could then explain the deviation of the motion of the planet from the elliptic orbit. This program was continued by Euler, Lagrange, Laplace, Legendre, Poisson, Hamilton, Jacobi, Liouville, and others.

By the end of the nineteenth century, researchers realized that many nonlinear equations did not have explicit solutions. Even the case of three masses moving under the laws of Newtonian attraction could exhibit very complicated behavior and an explicit solution was not possible (e.g., the motion of the sun, earth, and moon cannot be given explicitly in terms of known functions). Short term solutions could be given by power series, but these were not useful in determining long-term behavior. Poincaré, working from 1880 to 1910, shifted the focus from finding explicit solutions to discovering geometric properties of solutions. He introduced many of the ideas in specific examples, which we now group together under the heading of chaotic dynamical systems. In particular, he realized that a deterministic system (in which the outside forces are not varying and are not random) can exhibit behavior that is apparently random (i.e., it is chaotic).

In 1898, Hadamard produced a specific example of geodesics on a surface of constant negative curvature which had this property of chaos. G. D. Birkhoff continued the work of Poincaré and found many different types of long-term limiting behavior, including the α- and ω-limit sets introduced in Sections 5.4 and 11.1. His work resulted in the book **[16]** from which the term "dynamical systems" comes.

During the first half of the twentieth century, much work was carried out on nonlinear oscillators, that is, equations modeling a collection of springs (or other physical forces such as electrical forces) for which the restoring force depends nonlinearly on the displacement from equilibrium. The stability of fixed points was studied by several people including Lyapunov.

(See Sections 4.4 and 5.3.) The existence of a periodic orbit for certain self-excited systems was discovered by Van der Pol. (See Section 6.3.) Andronov and Pontryagin showed that a system of differential equations was structurally stable near an attracting fixed point, **[8]** (i.e., the solutions for a small perturbation of the differential equation could be matched with the solutions for the original equations). Other people carried out research on nonlinear differential equations, including Bendixson, Cartwright, Bogoliubov, Krylov, Littlewood, Levinson, and Lefschetz. The types of solutions that could be analyzed were the ones which settled down to either (1) an equilibrium state (no motion), (2) periodic motion (such as the first approximations of the motion of the planets), or (3) quasiperiodic solutions which are combinations of several periodic terms with incommensurate frequencies. See Section 2.2.3. By 1950, Cartwright, Littlewood, and Levinson showed that a certain forced nonlinear oscillator had infinitely many different periods; that is, there were infinitely many different initial conditions for the same system of equations, each of which resulted in periodic motion in which the period was a multiple of the forcing frequency, but different initial conditions had different periods. This example contained a type of complexity not previously seen.

In the 1960s, Stephen Smale returned to using the topological and geometric perspective initiated by Poincaré to understand the properties of differential equations. He wrote a very influential survey article **[98]** in 1967. In particular, Smale's "horseshoe" put the results of Cartwright, Littlewood, and Levinson in a general framework and extended their results to show that they were what was later called chaotic. A group of mathematicians worked in the United States and Europe to flesh out his ideas. At the same time, there was a group of mathematicians in Moscow lead by Anosov and Sinai investigating similar ideas. (Anosov generalized the work of Hadamard to geodesics on negatively curved manifolds with variable curvature.) The word "chaos" itself was introduced by T.Y. Li and J. Yorke in 1975 to designate systems that have aperiodic behavior more complicated than equilibrium, periodic, or quasiperiodic motion. (See **[66]**.) A related concept introduced by Ruelle and Takens was a *strange attractor*. It emphasized more the complicated geometry or topology of the attractor in phase space, than the complicated nature of the motion itself. See **[90]**. The theoretical work by these mathematicians supplied many of the ideas and approaches that were later used in more applied situations in physics, celestial mechanics, chemistry, biology, and other fields.

The application of these ideas to physical systems really never stopped. One of these applications, which has been studied since earliest times, is the description and determination of the motion of the planets and stars. The study of the mathematical model for such motion is called *celestial mechanics*, and involves a finite number of bodies moving under the effects of gravitational attraction given by the Newtonian laws. Birkhoff, Siegel, Kolmogorov, Arnold, Moser, Herman, and many others investigated the ideas of stability and found complicated behavior for systems arising in celestial mechanics and other such physical systems, which could be described by what are called *Hamiltonian differential equations*. (These equations preserve energy and can be expressed in terms of partial derivatives of the energy function.) K. Sitnikov in **[97]** introduced a situation in which three masses interacting by Newtonian attraction can exhibit chaotic oscillations. Later, Alekseev showed that this could be understood in terms of a "Smale horseshoe", **[3]**, **[4]**, and **[5]**. The book by Moser, **[78]**, made this result available to many researchers and did much to further the applications of horseshoes to other physical situations. In the 1971 paper **[90]** introducing strange attractors, Ruelle and Takens indicated how the ideas in nonlinear dynamics could be used to explain how turbulence developed in fluid flow. Further connections were made to physics, including the periodic doubling route to chaos discovered by Feigenbaum, **[37]**, and independently by P. Coullet and C. Tresser, **[29]**.

Relating to a completely different physical situation, starting with the work of Belousov and Zhabotinsky in the 1950s, certain mathematical models of chemical reactions that exhibit chaotic behavior were discovered. They discovered some systems of differential equations that not only did not tend to an equilibrium, but also did not even exhibit predictable oscillations. Eventually, this bizarre situation was understood in terms of chaos and strange attractors.

In the early 1920s, A.J. Lotka and V. Volterra independently showed how differential equations could be used to model the interaction of two populations of species, **[69]** and **[111]**. In the early 1970s, May showed how chaotic outcomes could arise in population dynamics. In the monograph **[75]**, he showed how simple nonlinear models could provide "mathematical metaphors for broad classes of phenomena." Starting in the 1970s, applications of nonlinear dynamics to mathematical models in biology have become widespread. The undergraduate books by Murray **[79]** and Taubes **[106]** afford good introductions to biological situations in which both oscillatory and chaotic differential equations arise. The books by Kaplan and Glass **[56]** and Strogatz **[104]** include a large number of other applications.

Another phenomenon that has had a great impact on the study of nonlinear differential equations is the use of computers to find numerical solutions. There has certainly been much work done on deriving the most efficient algorithms for carrying out this study. Although we do discuss some of the simplest of these, our focus is more on the use of computer simulations to find the properties of solutions. E. Lorenz made an important contribution in 1963 when he used a computer to study nonlinear equations motivated by the turbulence of motion of the atmosphere. He discovered that a small change in initial conditions leads to very different outcomes in a relatively short time; this property is called *sensitive dependence on initial conditions* or, in more common language, the *butterfly effect.* Lorenz used the latter term because he interpreted the phenomenon to mean that a butterfly flapping its wings in Australia today could affect the weather in the United States a month later. We describe more of his work in Chapter 7. It was not until the 1970s that Lorenz's work became known to the more theoretical mathematical community. Since that time, much effort has gone into showing that Lorenz's basic ideas about these equations were correct. Recently, Warwick Tucker has shown, using a computer-assisted proof, that this system not only has sensitive dependence on initial conditions, but also has what is called a "chaotic attractor." (See Chapter 7.) About the same time as Lorenz, Ueda discovered that a periodically forced Van der Pol system (or other nonlinear oscillator) has what is now called a chaotic attractor. Systems of this type are also discussed in Chapter 7. (For a later publication by Ueda, see also **[109]**.)

Starting about 1970 and still continuing, there have been many other numerical studies of nonlinear equations using computers. Some of these studies were introduced as simple examples of certain phenomena. (See the discussion of the Rössler Attractor given in Section 7.4.) Others were models for specific situations in science, engineering, or other fields in which nonlinear differential equations are used for modeling. The book **[36]** by Enns and McGuire presents many computer programs for investigation of nonlinear functions and differential equations that arise in physics and other scientific disciplines.

In sum, the last 40 years of the twentieth century saw the growing importance of nonlinearity in describing physical situations. Many of the ideas initiated by Poincaré a century ago are now much better understood in terms of the mathematics involved and the way in which they can be applied. One of the main contributions of the modern theory of dynamical systems to these applied fields has been the idea that erratic and complicated behavior can result from simple situations. Just because the outcome is chaotic, the basic environment does not need to contain stochastic or random perturbations. The simple forces themselves can cause chaotic outcomes.

There are three books of a nontechnical nature that discuss the history of the development of "chaos theory": the best seller *Chaos: Making a New Science* by James Gleick **[40]**, *Does God Play Dice?, The Mathematics of Chaos* by Ian Stewart **[102]**, and *Celestial Encounters* by Florin Diacu and Philip Holmes [**33**]. Stewart's book puts a greater emphasis on the role of mathematicians in the development of the subject, while Gleick's book stresses the the work of researchers making the connections with applications. Thus, the perspective of Stewart's book is closer to the one of this book, but Gleick's book is accessible to a broader audience and is more popular. The book by Diacu and Holmes has a good treatment of Poincaré's contribution and the developments in celestial mechanics up to today.

Part 1

Systems of Nonlinear Differential Equations