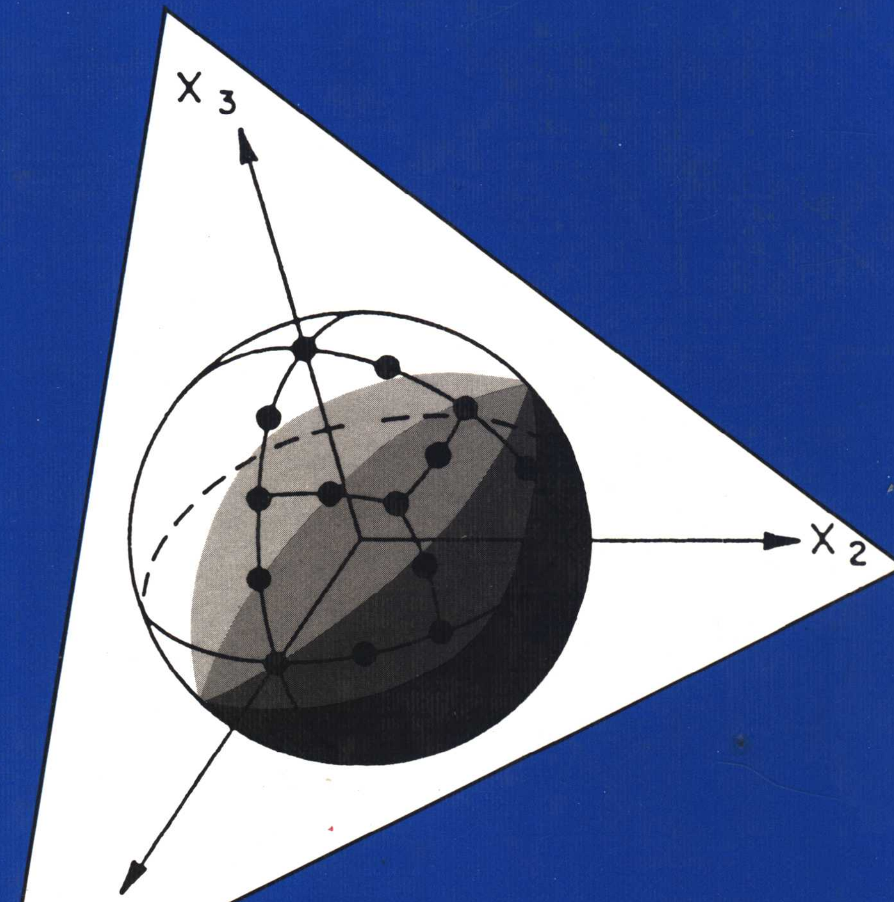
BOUNDARY ELEMENT TECHNOLOGY VI

EDITOR: C.A. BREBBIA



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Boundary Element Technology VI

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PREFACE

This book presents a series of papers dealing mainly with the applications of the boundary element method rather than its more theoretical aspects. The contributions are all written by scientists and engineers working on the latest developments of the technique and its applications to solve engineering problems.

The volume presents the state of the art in boundary elements in fields such as fluid flow, electromagnetics and electrical engineering, hydrodynamics, heat conduction, different potential type problems, stress analysis, fracture mechanics and some of the latest developments in numerical and mathematical techniques which are essential to write efficient boundary element systems.

The book is the edited version of the Proceedings of the 6th International Boundary Element Technology Conference held at Southampton in June 1991 and organized by the Wessex Institute of Technology.

The Boundary Element Technology series of conferences (BETECH) is now well established as the pre-eminent forum for the presentation of application oriented work in Boundary Elements and associated computational techniques. From its inception in 1985 in Australia the BETECH conferences have documented the increasing application of the Method and the Proceedings have become a standard reference for the engineering community. The BETECH conferences have always had an international emphasis with venues in Adelaide, Australia 1985, M.I.T., USA 1986, Rio de Janeiro, Brazil 1987, Windsor, Canada 1989, Delaware, USA 1990, and Southampton, UK 1991.

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Carlos A Brebbia
Southampton, June 1991

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SECTION 1: FLUID FLOW

Numerical Investigation of Fluid Flow with Natural Convection due to Non-linear Material Behavior and Double-Diffusion Effects

I. Zagar, P. Skerget, A. Alujevic

Faculty of Engineering, University of Maribor, Slovenia

ABSTRACT

The paper reports numerical simulation results of natural convection when the flow is driven by density variations due to combined heat and mass transfer. The Boussinesq approximation is used to take into account the temperature and concentration dependent density. Non-steady state solutions are performed for various combinations of boundary conditions and for different values of the thermal and mass transfer Grasshoff numbers. The influence of the fluid density inversion on the flow structure is also studied. Boundary-domain integral method has been applied.

INTRODUCTION

The transport phenomenon of natural convection in general fluid flow is included into governing equations through the Boussinesq approximation. In this paper two different problems of natural convection in fluids are studied.

In the first problem, natural convection motion due to simultaneous existence of temperature and concentration gradients in fluids is considered. A practical example of such complex phenomenon is a storage tank of liquified natural gas. Fluid properties of liquified natural gas ($Pr = 2.2$; $Sc = 130$; $Gr_t = 2500$; $Gr_m = -1000$) are taken into account in the numerical example of a closed cavity subject to different kinds of boundary condition.

In the second problem, the nonlinear Boussinesq approximation was considered. Neglecting the nonlinear term in the Boussinesq approximation for the same fluids may lead to a completely wrong picture of the real situation. Example of such fluid is water in the range of 0^oC to 8^oC. Density of water is increasing to 4^oC and

then decreasing. Effect of such anomaly of water density is evident from the development of the temperature and velocity field.
The boundary-domain integral method is used for the numerical simulation of the time-dependent set of governing equations in velocity-vorticity formulations. The numerical results agree with those obtained from experiments [1] and calculations of other authors[2],[5].

GOVERNING EQUATIONS

The partial differential equations set, governing the motion of viscous incompressible fluid is known as nonlinear Navier-Stokes equations expressing the basic conservation balances of mass, momentum, energy and mass concentration. Introducing vorticity ω and stream function ψ of the solenoidal velocity field, the computation of the flow is divided into kinematics given by the Poisson's elliptic equation, written in plane $x-y$ by

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \omega = 0 \tag{1}$$

and into the kinetics described by the vorticity equation

$$\frac{\partial\omega}{\partial t} + v_x \frac{\partial\omega}{\partial x} + v_y \frac{\partial\omega}{\partial y} = \nu \left(\frac{\partial^2\omega}{\partial x^2} + \frac{\partial^2\omega}{\partial y^2}\right) + g_y \frac{\partial F}{\partial x} - g_x \frac{\partial F}{\partial y} \tag{2}$$

The buoyancy effect is included by energy and molar fraction equations

$$\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = a \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

$$\frac{\partial C}{\partial t} + v_x \frac{\partial C}{\partial x} + v_y \frac{\partial C}{\partial y} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) \tag{4}$$

and Boussinesq approximation, given for linear normalised difference of density

$$\frac{\rho - \rho_o}{\rho_o} = F = -\beta_t \,(T - T_o) - \beta_m \,(C - C_o) \tag{5}$$

For water in the region of anomaly from $0^o\ C$ to $8^o\ C$ the normalized difference of density is given [5]

$$\frac{\rho - \rho_o}{\rho_o} = F = (0.066576\, T - 0.008322\, T^2)/\rho_o \tag{6}$$

BOUDARY-DOMAIN INTEGRAL EQUATIONS

The boundary domain integral statement for the flow kinematics can be derived from the vector elliptic equation for vector potential ψ_i [3] applying Green's theorem for the vector functions and the elliptic fundamental solution u^*, resulting in the following statement

$$c(\xi)v_x(\xi) + \int_\Gamma v_x \frac{\partial u^*}{\partial n}\, d\Gamma = \int_\Gamma v_y \frac{\partial u^*}{\partial t}\, d\Gamma - \int_\Omega \omega \frac{\partial u^*}{\partial y} d\Omega \tag{7}$$

$$c(\xi)v_y(\xi) + \int_\Gamma v_y \frac{\partial u^*}{\partial n}\, d\Gamma = -\int_\Gamma v_x \frac{\partial u^*}{\partial t}\, d\Gamma + \int_\Omega \omega \frac{\partial u^*}{\partial x} d\Omega \tag{8}$$

Describing the laminar transport of the vorticity, temperature and molar fraction in the integral statement, one has to take into account that the vorticity, temperature and mass concentration obey a nonhomogeneous parabolic equation[3],[6]. So the following boundary-domain integral formulations can be derived for the plane

$$c(\xi)\omega(\xi, t_F) + \nu \int_\Gamma \int_{t_{F-1}}^{t_F} \omega \frac{\partial u^*}{\partial n}\, dt\, d\Gamma = \nu \int_\Gamma \int_{t_{F-1}}^{t_F} \frac{\partial \omega}{\partial n} u^*\, dt\, d\Gamma$$

$$- \int_\Gamma \int_{t_{F-1}}^{t_F} (\omega v_n + g_t\, F) u^*\, dt d\Gamma$$

$$+ \int_\Omega \int_{t_{F-1}}^{t_F} (\omega\, v_x + g_y\, F,\ \omega\, v_y - g_x\, F)\, (\frac{\partial u^*}{\partial x}, \frac{\partial u^*}{\partial y})\, dt d\Omega + \int_\Omega \omega_{F-1}\, u^*{}_{F-1}\, d\Omega \tag{9}$$

$$c(\xi)T(\xi, t_F) + a \int_\Gamma \int_{t_{F-1}}^{t_F} T \frac{\partial u^*}{\partial n}\, dt\, d\Gamma = a \int_\Gamma \int_{t_{F-1}}^{t_F} \frac{\partial T}{\partial n} u^*\, dt\, d\Gamma$$

$$- \int_\Gamma \int_{t_{F-1}}^{t_F} T\, v_n u^*\, dt\, d\Gamma$$

$$- \int_\Omega \int_{t_{F-1}}^{t_F} (T\, v_x,\ T\, v_y)\, (\frac{\partial u^*}{\partial x}, \frac{\partial u^*}{\partial y})\, dt\, d\Omega + \int_\Omega T_{F-1}\, u^*{}_{F-1}\, d\Omega \tag{10}$$

$$c(\xi)C(\xi, t_F) + D \int_\Gamma \int_{t_{F-1}}^{t_F} C \frac{\partial u^*}{\partial n}\, dt\, d\Gamma = D \int_\Gamma \int_{t_{F-1}}^{t_F} \frac{\partial C}{\partial n} u^*\, dt\, d\Gamma$$

$$- \int_\Gamma \int_{t_{F-1}}^{t_F} C\, v_n u^*\, dt\, d\Gamma$$

$$- \int_\Omega \int_{t_{F-1}}^{t_F} (C\, v_x,\ C\, v_y)\, (\frac{\partial u^*}{\partial x}, \frac{\partial u^*}{\partial y})\, dt\, d\Omega + \int_\Omega C_{F-1}\, u^*{}_{F-1}\, d\Omega \tag{11}$$

NUMERICAL RESULTS

Water anomaly in a closed cavity

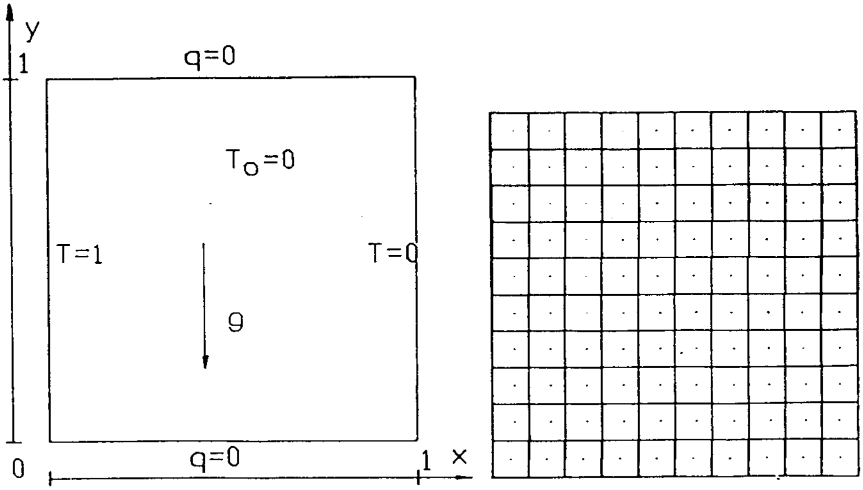

Fig. 1: Problem geometry and boundary conditions

Geometry and boundary condition for buoyancy driven laminar flow in a closed square cavity, filled by water exhibiting an anomaly of density is shown on Fig.1. A mesh of 40 elements (80 nodes) and 100 internal cells has been used.A model was taken as in [4] where linear Boussinesq approximation for air has been used. [1],[5].

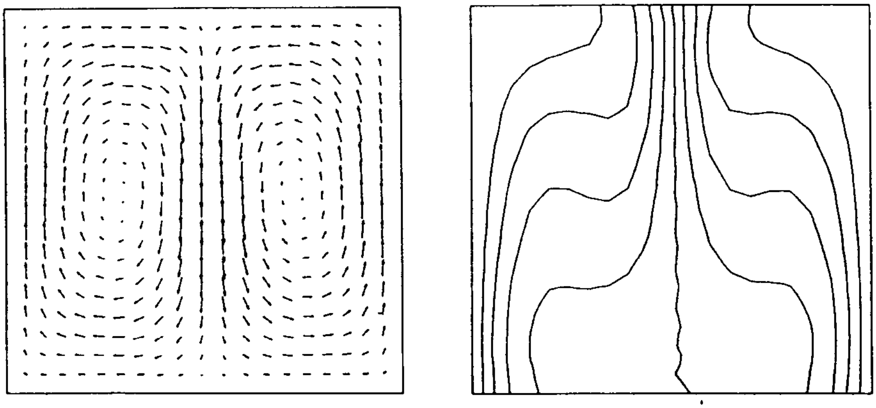

Fig. 2: Steady state results $Ra = 10^5$

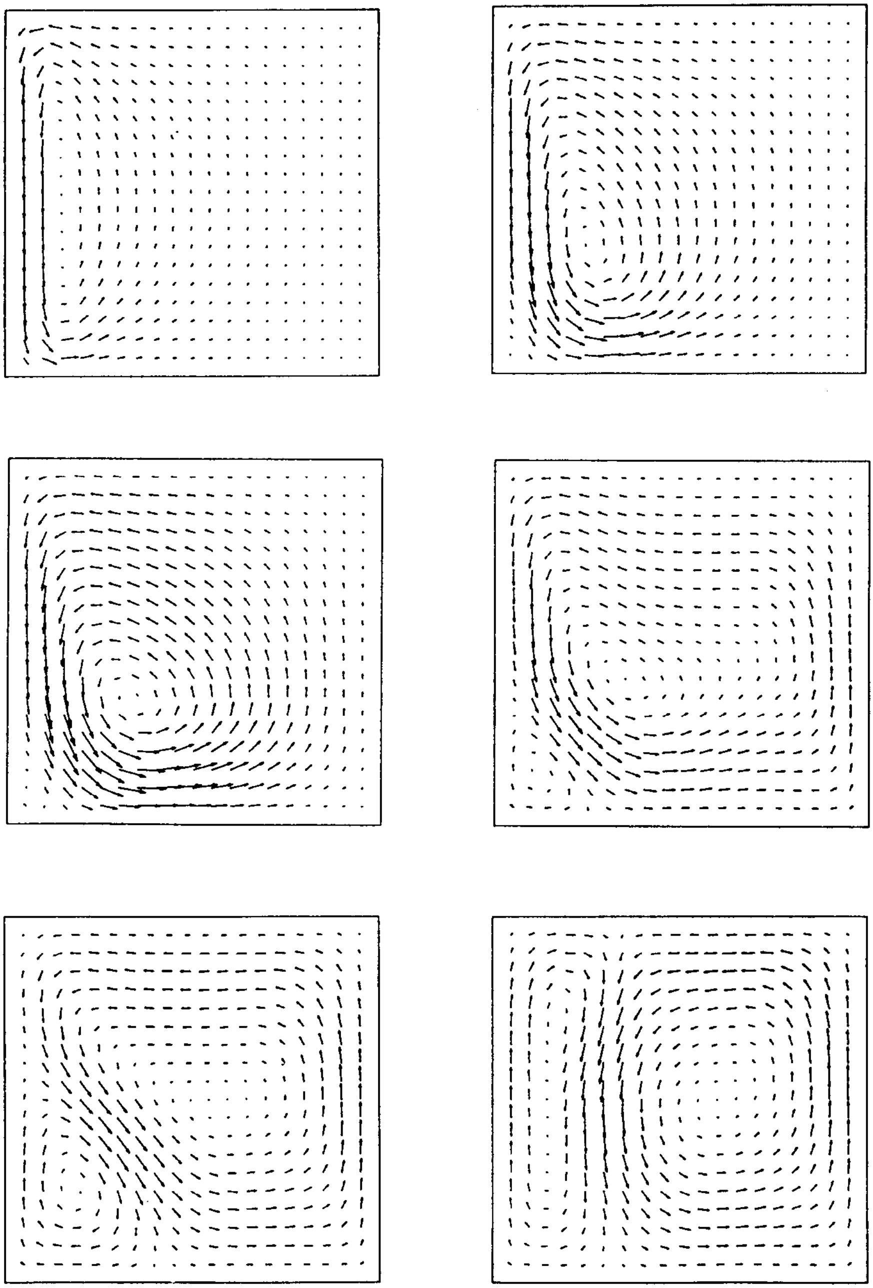

Fig. 3: Velocity field development, $Ra = 10^5$, $t = 0.2, 0.8, 1.2, 3.2, 6.4$ and $40s$

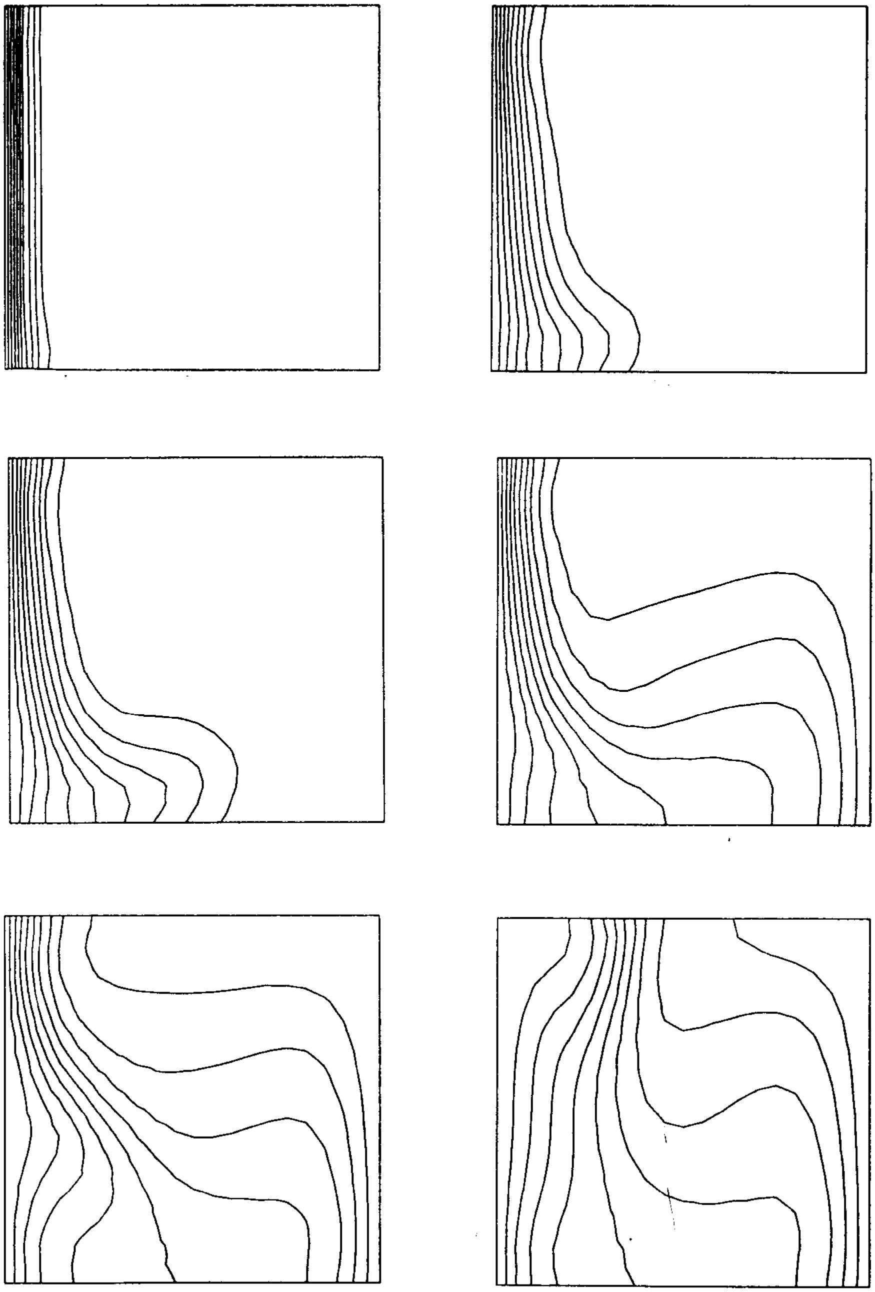

Fig. 4: Temperature field development $Ra = 10^5$, $t = 0.2$, 0.8, 1.2, 3.2, 6.4 and 40s

Comparison of velocity (Fig. 3) and temperature field (Fig. 4) shows evident difference in formation and development of water transients whith those obtained for air [4] in same range. Figure 4 gives the final steady state of both velocity and temperature distributions. There are two separated circulation zones observed, while in the middle a symmetry separation line developed.

Double-diffusion in closed cavity

On the same mesh as in previously case and similar boundary conditions exept for vertical walls, where linear variation of temperature and molar fraction is imposed

$$X = 0,1 \;\; T = 1 - Y \quad C = 1 - Y$$

problem of double-diffusion was studied. The linear variation of molar fraction can be interpreted as diffusion of the component through the wall. Fluid was considered for which body forces due to heat and mass transfer are opposed, $Ra_t = 5.5\,10^3$ ($Gr_t = 2500,\ Pr = 2.2$) and $Ra_m = -1.3\,10^5$ ($Gr_m = -1000,\ Sc = 130$) [2]. Figure 5,6,7 show development of velocity, temperature and molar fraction.

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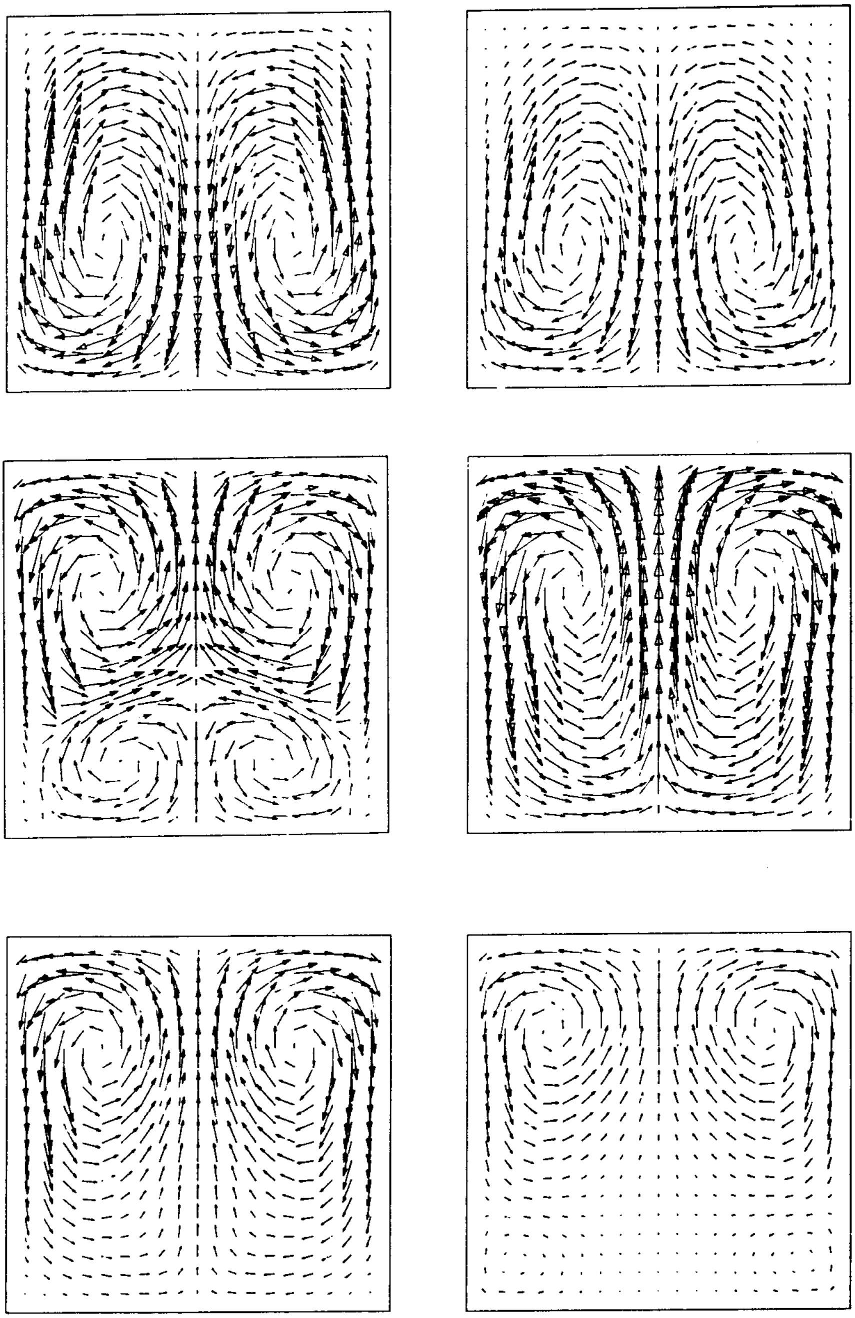

Fig. 5: Velocity field development, $t = 4, 8$, 19.2, 57.6, 124.8., 240 and 480s

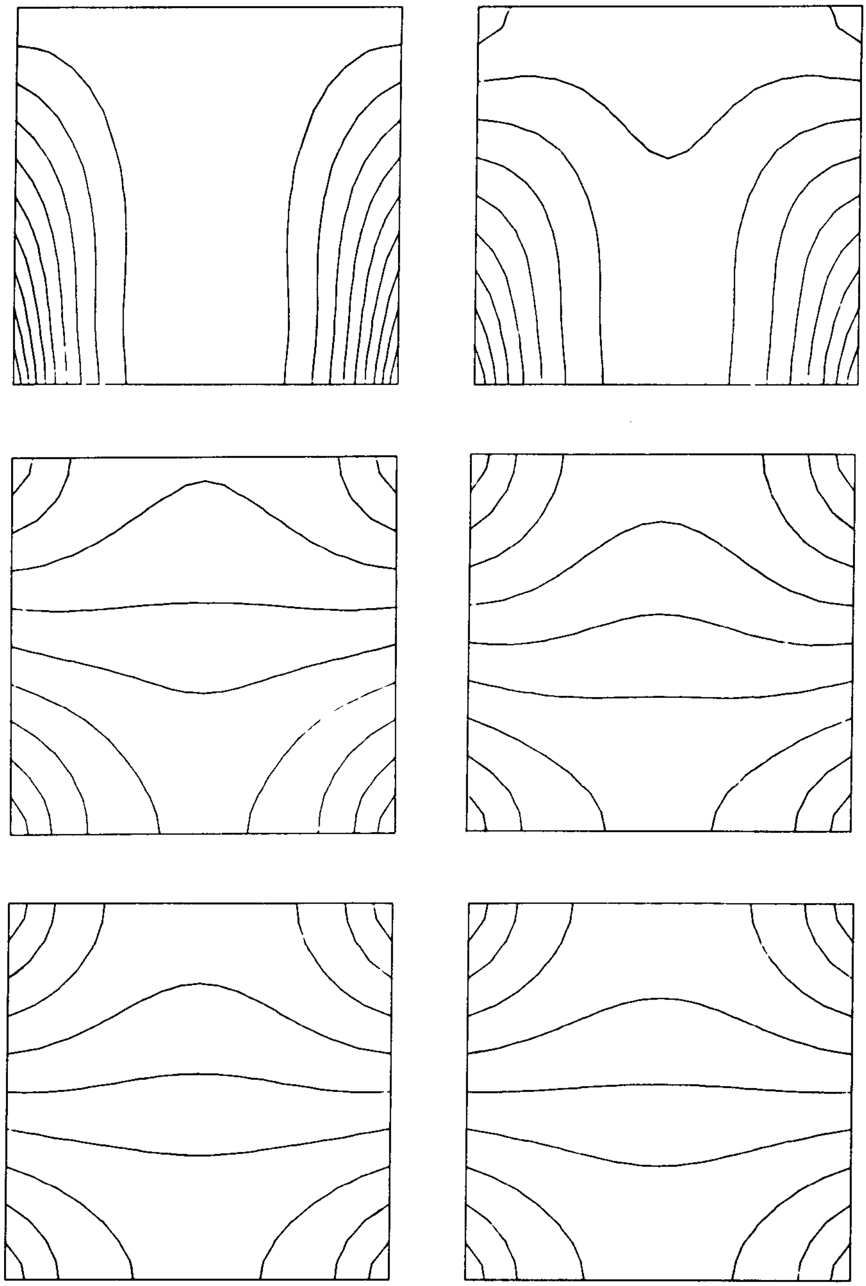

Fig. 6: Temperature field development, $t = 4, 8, 19.2, 57.6, 124.8., 240$ and $480s$