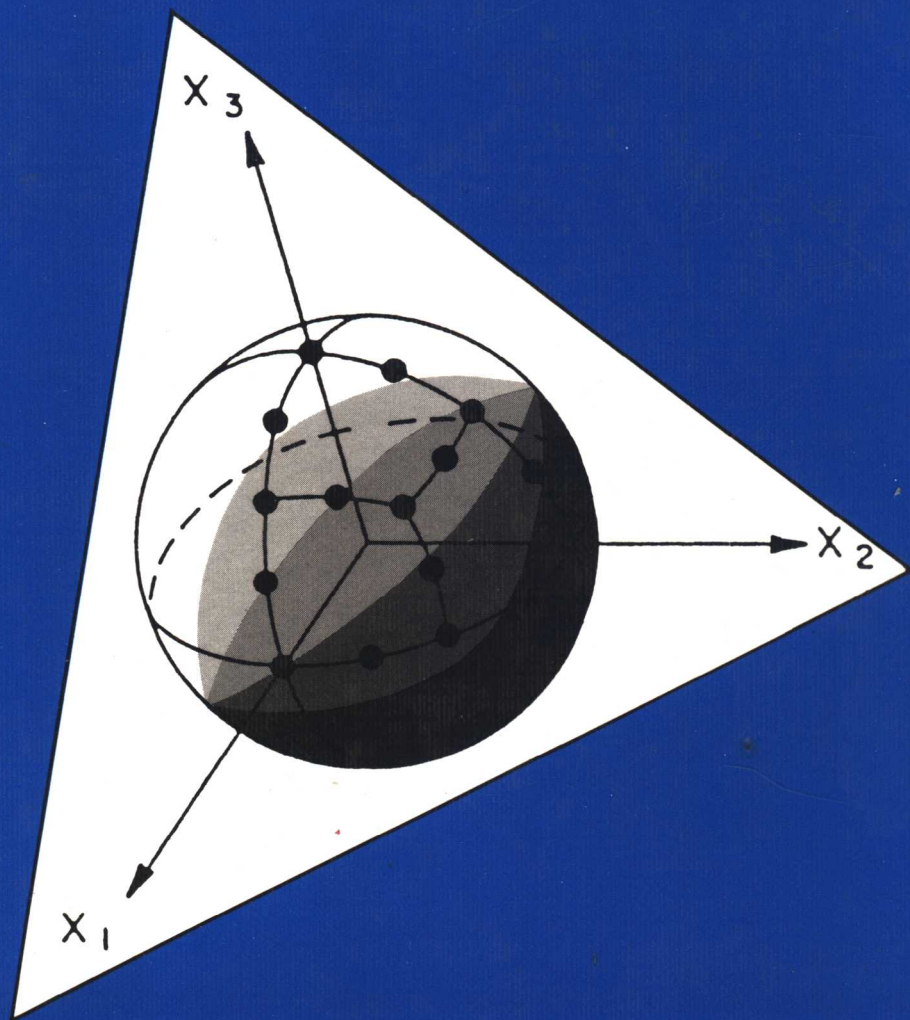


BOUNDARY ELEMENT TECHNOLOGY VI

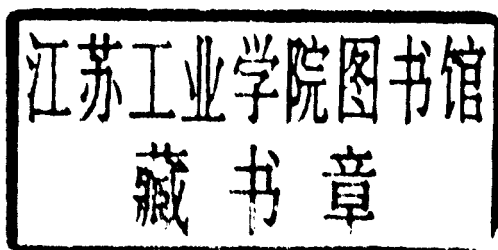
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Boundary Element Technology VI

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PREFACE

This book presents a series of papers dealing mainly with the applications of the boundary element method rather than its more theoretical aspects. The contributions are all written by scientists and engineers working on the latest developments of the technique and its applications to solve engineering problems.

The volume presents the state of the art in boundary elements in fields such as fluid flow, electromagnetics and electrical engineering, hydrodynamics, heat conduction, different potential type problems, stress analysis, fracture mechanics and some of the latest developments in numerical and mathematical techniques which are essential to write efficient boundary element systems.

The book is the edited version of the Proceedings of the 6th International Boundary Element Technology Conference held at Southampton in June 1991 and organized by the Wessex Institute of Technology.

The Boundary Element Technology series of conferences (BETECH) is now well established as the pre-eminent forum for the presentation of application oriented work in Boundary Elements and associated computational techniques. From its inception in 1985 in Australia the BETECH conferences have documented the increasing application of the Method and the Proceedings have become a standard reference for the engineering community. The BETECH conferences have always had an international emphasis with venues in Adelaide, Australia 1985, M.I.T., USA 1986, Rio de Janeiro, Brazil 1987, Windsor, Canada 1989, Delaware, USA 1990, and Southampton, UK 1991.

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Carlos A Brebbia
Southampton, June 1991

CONTENTS

SECTION 1: FLUID FLOW

- Numerical Investigation of Fluid Flow with Natural Convection due to Non-linear Material Behavior and Double-Diffusion Effects 3
I. Zagar, P. Skerget, A. Alujevic
- High Reynolds Number Flow about a Prolate Spheroid Moving near Ground 13
H. Schmitt
- Dual Reciprocity BIE in Lagrangian Form for Incompressible Unsteady Viscous Flow 29
X. Jin, D.K. Brown
- Forced Convection Cooling of Flat Electronic Substrates 39
G. De Mey, S. De Smet, M. Driscart
- An Analysis of the Kutta-Joukowski-Carafoli Condition at Axial Profile Cascades using the Boundary Element Method 51
I.N. Carte

SECTION 2: ELECTRICAL AND ELECTROMAGNETIC PROBLEMS

- Boundary Element Analysis of Electromagnetic Fields in Helically Symmetric System 65
H. Igarashi, T. Honma
- A Subdomain Method for FEM-BEM Coupling in Electromagnetism 77
F. Delincé, P. Dular, A. Genon, H. Hedia, W. Legros, A. Nicolet
- Modelization of Movement in Electromagnetic Devices with the Boundary Element Method 89
N. Bamps, F. Delincé, P. Dular, A. Genon, W. Legros, A. Nicolet
- Comparison of Two Different BEM Formulations for Nonlinear Eddy Currents 101
A. Kost

Procat System: Recent Advances and Future Developments in Numerical Simulation of Cathodic Protection Problems <i>W.J. Mansur, J.C.F. Telles, J.A.F. Santiago</i>	117
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SECTION 3: POTENTIAL TYPE PROBLEMS

Transient Saltwater Intrusion in Confined Aquifers using a B-Spline Boundary Element Formulation <i>J.J.S.P. Cabral, L.C. Wrobel</i>	135
Sound Radiation of a Cantilever Plate - Comparisons of BEM Results with Experimental Data <i>O. von Estorff, A. Homm, F. Bartels</i>	147

SECTION 4: STRESS ANALYSIS

Anisotropic Effective Stress Soil Behaviour by BEM <i>F. Lin, L.A. Wood</i>	161
The Use of a Foundation Element in a Three-Dimensional Structural Analysis Code <i>M.A. Ramalho, W.S. Venturini</i>	173
On the Use of Geometrical Symmetry in the Boundary Element Methods for 3D Elasticity <i>M. Bonnet</i>	185
The Multiple Reciprocity Method Applied to Thermoelastic Problems with Concentrated and Distributed Heat Sources <i>A.C. Neves, C.A. Brebbia</i>	201
Solving Plate Bending Problems using Boundary Elements Method <i>L.M. Castro, J. Pinho, P. Parreira</i>	215
Kernel Functions of Mindlin's Solution for Evaluation of Internal Stresses <i>Y.S. Jiang</i>	227
Boundary Element Formulation for Non-linear Problems using the Reissner's Theory <i>G.O. Ribeiro, W.S. Venturini</i>	239
Contact Analysis using BEASY: Theory and Application <i>S.M. Niku, R.A. Adey, J. Baynham</i>	253
Transient Dynamic Elastoplastic Analysis by the Boundary Element Method <i>J.A.M. Carrer, J.C.F. Telles</i>	265

A General Adjoint Approach to Shape Design Sensitivity Analysis	279
<i>J. Unzueta, E. Schaeidt, A. Longo, J.J. Anza</i>	

SECTION 5: NUMERICAL AND MATHEMATICAL TECHNIQUES

A new B.E. Formulation and its Applications in Engineering	295
<i>C.A. Brebbia, T. DeFigueiredo</i>	
Boundary Elements for Square-Root Singularities on Curves	309
<i>J.C. Mason</i>	
Improving the Accuracy of BEM in the use of Non-equidistant Elements	321
<i>M. Karamanoglu, G.E. Beswick</i>	
High Precision Numerical Integration Method for 3-D BEM and its Error Analysis using Complex Function Theory	335
<i>K. Hayami</i>	
Penalty-Combined Boundary Methods and Applications in Solving Elliptic Problems with Singularities	339
<i>Z.C. Li, T.D. Bui</i>	

SECTION 6: FRACTURE MECHANICS

Fracture Diagrams for Short Cracks at a Stress Concentration	357
<i>R.D. Stark, D.J. Cartwright</i>	
Cracks Grow in Cement Structures using Laplace Transform and Boundary Element Method	371
<i>M. Kassem</i>	
Dual Boundary Element Analysis of Pin-Loaded Lugs	381
<i>A. Portela, M.H. Aliabadi, D.P. Rooke</i>	
Three-Dimensional Weight Functions for Circular Cracks in Cylindrical Bars	393
<i>R. Bains, M.H. Aliabadi, D.P. Rooke</i>	
Stress Analysis of Cyclic Symmetric Structures by the New Type Boundary Integral Equation	407
<i>J.F. Xie, Y.L. Wu</i>	

SECTION 1: FLUID FLOW

Numerical Investigation of Fluid Flow with Natural Convection due to Non-linear Material Behavior and Double-Diffusion Effects

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ABSTRACT

The paper reports numerical simulation results of natural convection when the flow is driven by density variations due to combined heat and mass transfer. The Boussinesq approximation is used to take into account the temperature and concentration dependent density. Non-steady state solutions are performed for various combinations of boundary conditions and for different values of the thermal and mass transfer Grasshoff numbers. The influence of the fluid density inversion on the flow structure is also studied. Boundary-domain integral method has been applied.

INTRODUCTION

The transport phenomenon of natural convection in general fluid flow is included into governing equations through the Boussinesq approximation. In this paper two different problems of natural convection in fluids are studied.

In the first problem, natural convection motion due to simultaneous existence of temperature and concentration gradients in fluids is considered. A practical example of such complex phenomenon is a storage tank of liquified natural gas. Fluid properties of liquified natural gas ($Pr = 2.2$; $Sc = 130$; $Gr_t = 2500$; $Gr_m = -1000$) are taken into account in the numerical example of a closed cavity subject to different kinds of boundary condition.

In the second problem, the nonlinear Boussinesq approximation was considered. Neglecting the nonlinear term in the Boussinesq approximation for the same fluids may lead to a completely wrong picture of the real situation. Example of such fluid is water in the range of $0^{\circ}C$ to $8^{\circ}C$. Density of water is increasing to $4^{\circ}C$ and

4 Boundary Element Technology

then decreasing. Effect of such anomaly of water density is evident from the development of the temperature and velocity field.

The boundary-domain integral method is used for the numerical simulation of the time-dependent set of governing equations in velocity-vorticity formulations. The numerical results agree with those obtained from experiments [1] and calculations of other authors[2],[5].

GOVERNING EQUATIONS

The partial differential equations set, governing the motion of viscous incompressible fluid is known as nonlinear Navier-Stokes equations expressing the basic conservation balances of mass, momentum, energy and mass concentration. Introducing vorticity ω and stream function ψ of the solenoidal velocity field, the computation of the flow is divided into kinematics given by the Poisson's elliptic equation, written in plane $x-y$ by

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \omega = 0 \quad (1)$$

and into the kinetics described by the vorticity equation

$$\frac{\partial \omega}{\partial t} + v_x \frac{\partial \omega}{\partial x} + v_y \frac{\partial \omega}{\partial y} = \nu \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) + g_y \frac{\partial F}{\partial x} - g_x \frac{\partial F}{\partial y} \quad (2)$$

The buoyancy effect is included by energy and molar fraction equations

$$\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} = a \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \quad (3)$$

$$\frac{\partial C}{\partial t} + v_x \frac{\partial C}{\partial x} + v_y \frac{\partial C}{\partial y} = D \left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) \quad (4)$$

and Boussinesq approximation, given for linear normalised difference of density

$$\frac{\rho - \rho_o}{\rho_o} = F = -\beta_t (T - T_o) - \beta_m (C - C_o) \quad (5)$$

For water in the region of anomaly from $0^\circ C$ to $8^\circ C$ the normalized difference of density is given [5]

$$\frac{\rho - \rho_o}{\rho_o} = F = (0.066576 T - 0.008322 T^2) / \rho_o \quad (6)$$

BOUDARY-DOMAIN INTEGRAL EQUATIONS

The boundary domain integral statement for the flow kinematics can be derived from the vector elliptic equation for vector potential ψ_i [3] applying Green's theorem for the vector functions and the elliptic fundamental solution u^* , resulting in the following statement

$$c(\xi)v_x(\xi) + \int_{\Gamma} v_x \frac{\partial u^*}{\partial n} d\Gamma = \int_{\Gamma} v_y \frac{\partial u^*}{\partial t} d\Gamma - \int_{\Omega} \omega \frac{\partial u^*}{\partial y} d\Omega \quad (7)$$

$$c(\xi)v_y(\xi) + \int_{\Gamma} v_y \frac{\partial u^*}{\partial n} d\Gamma = - \int_{\Gamma} v_x \frac{\partial u^*}{\partial t} d\Gamma + \int_{\Omega} \omega \frac{\partial u^*}{\partial x} d\Omega \quad (8)$$

Describing the laminar transport of the vorticity, temperature and molar fraction in the integral statement, one has to take into account that the vorticity, temperature and mass concentration obey a nonhomogeneous parabolic equation[3],[6]. So the following boundary-domain integral formulations can be derived for the plane

$$\begin{aligned} c(\xi)\omega(\xi, t_F) + \nu \int_{\Gamma} \int_{t_{F-1}}^{t_F} \omega \frac{\partial u^*}{\partial n} dt d\Gamma &= \nu \int_{\Gamma} \int_{t_{F-1}}^{t_F} \frac{\partial \omega}{\partial n} u^* dt d\Gamma \\ &- \int_{\Gamma} \int_{t_{F-1}}^{t_F} (\omega v_n + g_t F) u^* dt d\Gamma \\ + \int_{\Omega} \int_{t_{F-1}}^{t_F} (\omega v_x + g_y F, \omega v_y - g_x F) &\left(\frac{\partial u^*}{\partial x}, \frac{\partial u^*}{\partial y} \right) dt d\Omega + \int_{\Omega} \omega_{F-1} u^*_{F-1} d\Omega \quad (9) \end{aligned}$$

$$\begin{aligned} c(\xi)T(\xi, t_F) + a \int_{\Gamma} \int_{t_{F-1}}^{t_F} T \frac{\partial u^*}{\partial n} dt d\Gamma &= a \int_{\Gamma} \int_{t_{F-1}}^{t_F} \frac{\partial T}{\partial n} u^* dt d\Gamma \\ &- \int_{\Gamma} \int_{t_{F-1}}^{t_F} T v_n u^* dt d\Gamma \\ - \int_{\Omega} \int_{t_{F-1}}^{t_F} (T v_x, T v_y) &\left(\frac{\partial u^*}{\partial x}, \frac{\partial u^*}{\partial y} \right) dt d\Omega + \int_{\Omega} T_{F-1} u^*_{F-1} d\Omega \quad (10) \end{aligned}$$

$$\begin{aligned} c(\xi)C(\xi, t_F) + D \int_{\Gamma} \int_{t_{F-1}}^{t_F} C \frac{\partial u^*}{\partial n} dt d\Gamma &= D \int_{\Gamma} \int_{t_{F-1}}^{t_F} \frac{\partial C}{\partial n} u^* dt d\Gamma \\ &- \int_{\Gamma} \int_{t_{F-1}}^{t_F} C v_n u^* dt d\Gamma \\ - \int_{\Omega} \int_{t_{F-1}}^{t_F} (C v_x, C v_y) &\left(\frac{\partial u^*}{\partial x}, \frac{\partial u^*}{\partial y} \right) dt d\Omega + \int_{\Omega} C_{F-1} u^*_{F-1} d\Omega \quad (11) \end{aligned}$$

NUMERICAL RESULTS

Water anomaly in a closed cavity

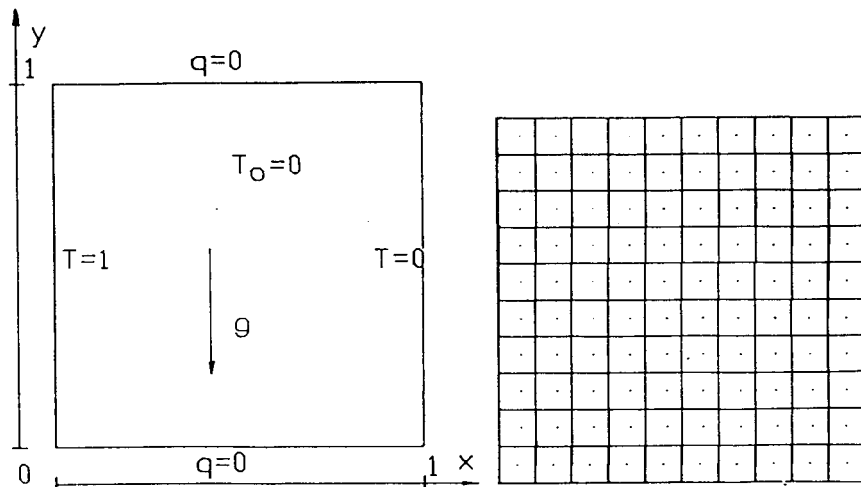


Fig. 1: Problem geometry and boundary conditions

Geometry and boundary condition for buoyancy driven laminar flow in a closed square cavity, filled by water exhibiting an anomaly of density is shown on Fig.1. A mesh of 40 elements (80 nodes) and 100 internal cells has been used. A model was taken as in [4] where linear Boussinesq approximation for air has been used. [1],[5].

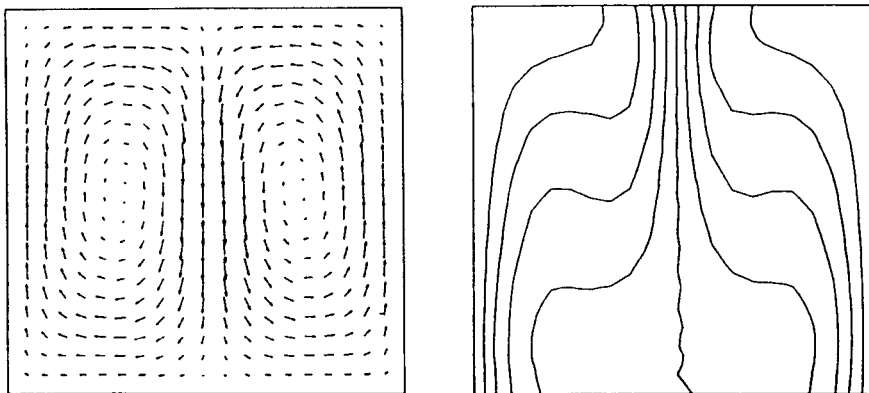


Fig. 2: Steady state results $Ra = 10^5$

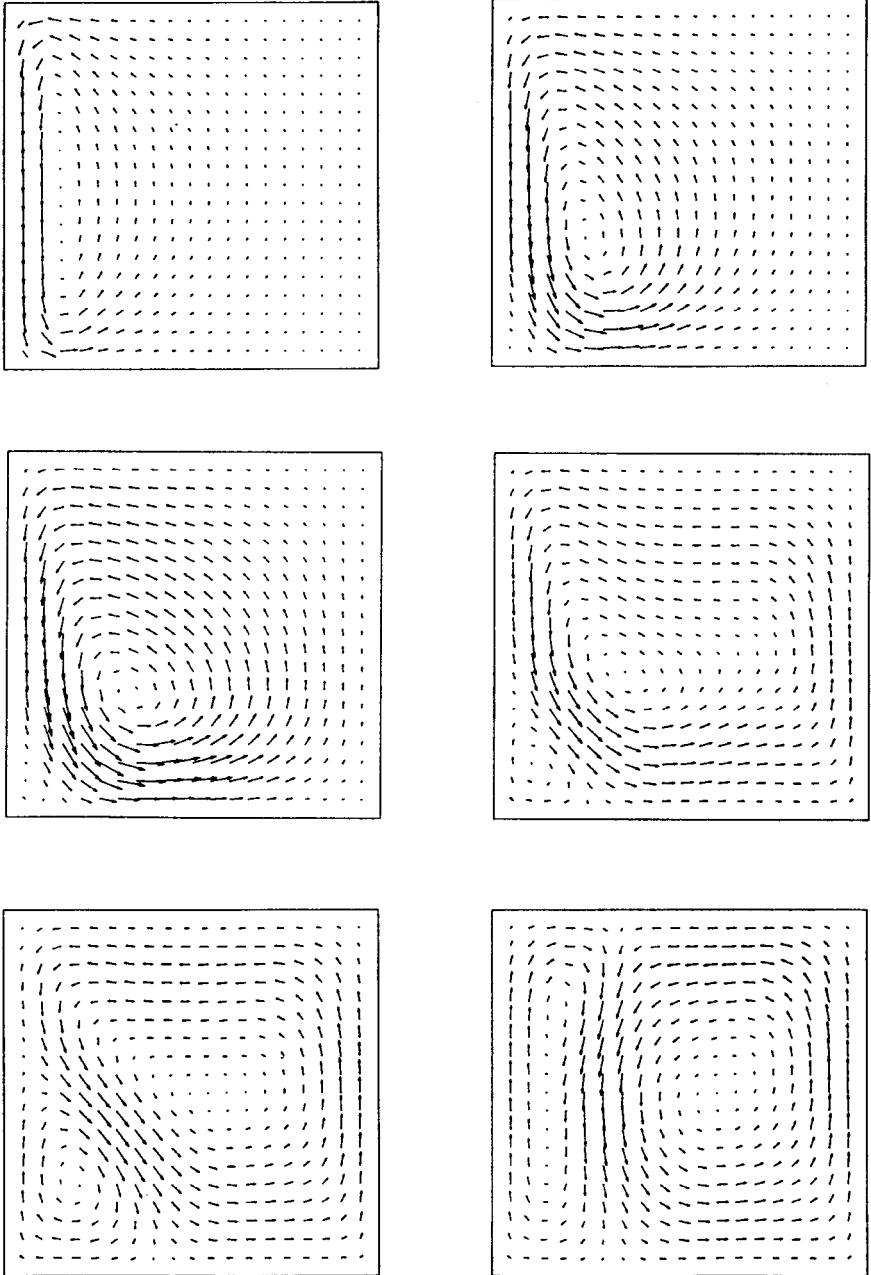


Fig. 3: Velocity field development, $Ra = 10^5$, $t = 0.2, 0.8, 1.2, 3.2, 6.4$ and $40s$

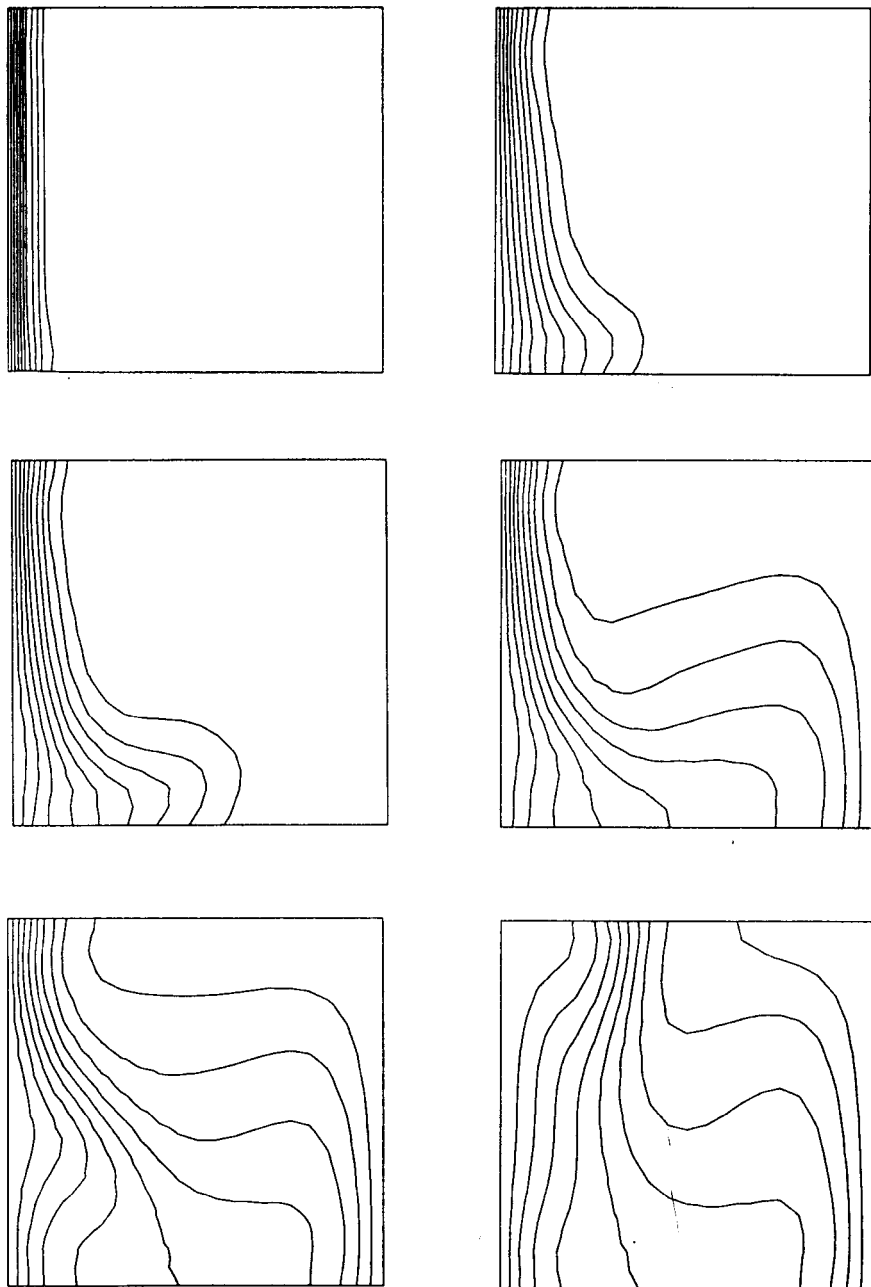


Fig. 4: Temperature field development $Ra = 10^5$, $t = 0.2, 0.8, 1.2, 3.2, 6.4$ and $40s$

Comparison of velocity (Fig. 3) and temperature field (Fig. 4) shows evident difference in formation and development of water transients with those obtained for air [4] in same range. Figure 4 gives the final steady state of both velocity and temperature distributions. There are two separated circulation zones observed, while in the middle a symmetry separation line developed.

Double-diffusion in closed cavity

On the same mesh as in previously case and similar boundary conditions except for vertical walls, where linear variation of temperature and molar fraction is imposed

$$X = 0, 1 \quad T = 1 - Y \quad C = 1 - Y$$

problem of double-diffusion was studied. The linear variation of molar fraction can be interpreted as diffusion of the component through the wall. Fluid was considered for which body forces due to heat and mass transfer are opposed, $Ra_t = 5.5 \cdot 10^3$ ($Gr_t = 2500$, $Pr = 2.2$) and $Ra_m = -1.3 \cdot 10^5$ ($Gr_m = -1000$, $Sc = 130$) [2]. Figure 5,6,7 show development of velocity, temperature and molar fraction.

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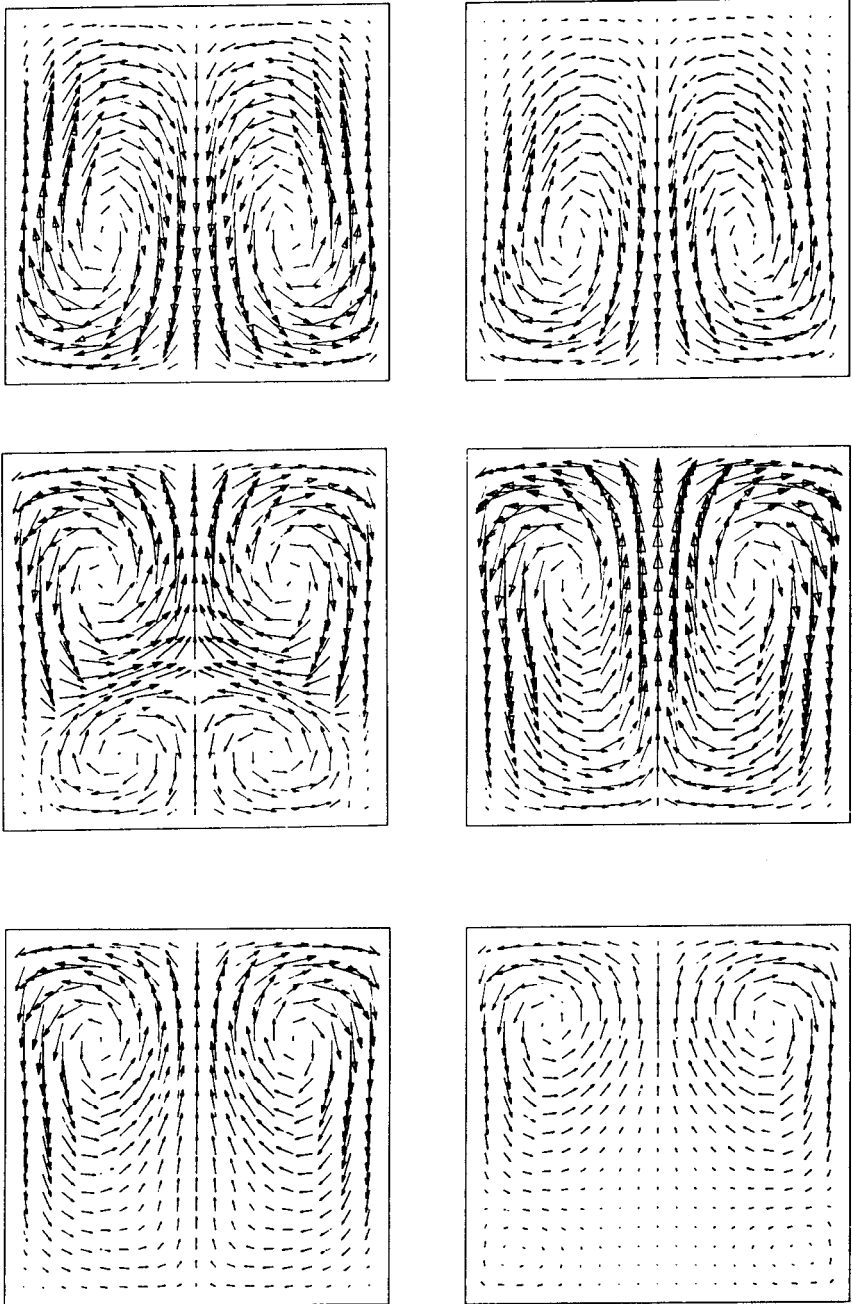


Fig. 5: Velocity field development, $t = 4, 8, 19.2, 57.6, 124.8, 240$ and $480s$

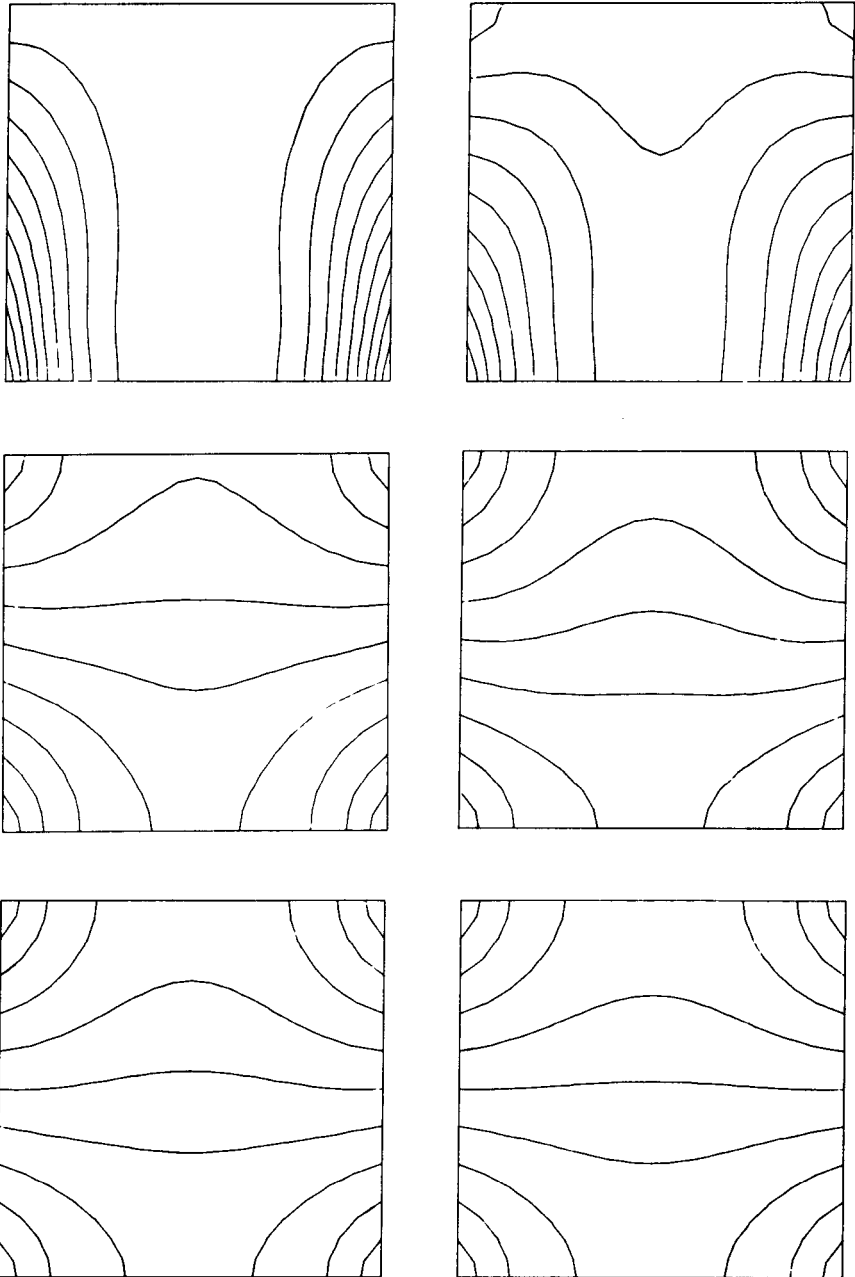


Fig. 6: Temperature field development, $t = 4, 8, 19.2, 57.6, 124.8, 240$ and $480s$