

Electronic Transport in Mesoscopic Systems

介观系统中的电子输运

SUPRIYO DATTA

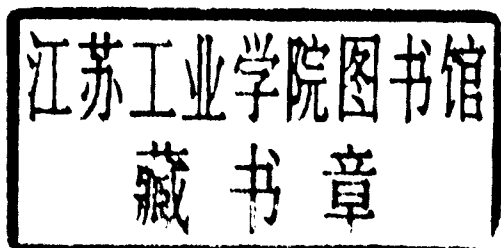
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Recent advances in technology have made it possible to fabricate structures whose dimensions are much smaller than the mean free path of an electron. This is the first text-book to give a thorough account of the theory of electronic transport in such mesoscopic systems. Important concepts are illustrated by reference to relevant experimental results.

The book begins with a chapter summarizing the necessary background material. The next chapter introduces the 'transmission formalism' which is widely used in describing mesoscopic transport. The applicability of this formalism to different transport regimes is examined and practical methods for evaluating the transmission function are discussed. This formalism is then used to describe three key topics in mesoscopic physics: quantum Hall effect, localization, and double-barrier tunneling. Optical analogies to mesoscopic phenomena are discussed briefly. The book closes with a simple intuitive description of the non-equilibrium Green's function formalism and its relation to the transmission formalism.

Emphasizing basic concepts and techniques throughout, and complete with problems and solutions, the book will be of great interest to graduate students as well as to established researchers interested in mesoscopic physics and nanoelectronics.

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ELECTRONIC TRANSPORT IN MESOSCOPIC SYSTEMS

TITLES IN THIS SERIES

- 1 Doping in III-V Semiconductors
E. F. Schubert
 - 2 The Physics and Applications of Resonant Tunnelling Diodes
H. Mizuta and T. Tanoue
 - 3 Electronic Transport in Mesoscopic Systems
S. Datta
-

To Anuradha, Manoshi and Malika

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A few common symbols

A	spectral function	$(\text{eV})^{-1}$
A	vector potential	V s/m
B	magnetic field	$10^4 \text{ G} = 1 \text{ T} = 1 \text{ V s/m}^2$
C	capacitance	F
D	diffusion coefficient	cm^2/s
e	electronic charge	$-1.6 \times 10^{-19} \text{ C}$
E	electric field	V/cm
E_f	equilibrium Fermi energy	eV
E_c	band-edge energy (bulk)	eV
E_s	band-edge energy (2-D)	eV
$f_0(E)$	equilibrium Fermi function	dimensionless
F_n	quasi-Fermi energy	eV
g	normalized conductance	dimensionless
G	conductance	Ω^{-1}
G^n	electron correlation function	
G^p	hole correlation function	$(\text{eV})^{-1}$ in a discrete representation
G^A	advanced Green's function	
G^R	retarded Green's function	
h	Planck's constant	$6.63 \times 10^{-34} \text{ J s}$
\hbar	$= h/2\pi$	
$i(E)$	current per unit energy	A/eV
I	current	A
J	current density	A/cm^2 (3-D) A/cm (2-D)
k_B	Boltzmann constant	0.087 meV/K
k_f	Fermi wavenumber	cm^{-1}

L	length	cm
L_m	mean free path	cm
L_φ	phase-relaxation length	cm
m	effective mass	for GaAs the standard value is $0.067 m_0$. We will generally use $0.07 m_0$ in our examples
m_0	free electron mass	9.1×10^{-31} kg
M	number of transverse modes	dimensionless
n_s	(areal) electron density	/cm ²
N_s	(2-D) density of states	$= m/\pi\hbar^2$ $\sim 2.9 \times 10^{10}/\text{cm}^2$ meV for GaAs
r	reflection amplitude	dimensionless
R	reflection probability	dimensionless
	resistance	Ω
S	area	cm ²
t	time	s
	transmission amplitude	dimensionless
T	temperature	K
	transmission probability	dimensionless
\bar{T}	transmission function	$= (\text{number of modes}) \times$ (transmission probability/mode)
U	potential energy	eV
v	velocity	cm/s
v_d	drift velocity	cm/s
v_f	Fermi velocity	cm/s
V	electrostatic potential	V
W	width	cm
Γ	energy broadening	eV
ϵ_n	cutoff energy	eV
ϑ	unit step function	$\vartheta(E) = 1$ if $E > 0$ $= 0$ if $E < 0$
λ_f	Fermi wavelength	cm
μ	mobility	cm ² /V s
	electrochemical potential	eV
ν	attempt frequency	s ⁻¹
	linear density of scatterers	cm ⁻¹
ρ	(2-D) resistivity	Ω

σ	(2-D) conductivity	Ω^{-1}
Σ^A	advanced self-energy	
Σ^R	retarded self-energy	(eV) in a discrete
Σ^{in}	inscattering function	representation
Σ^{out}	outscattering function	
τ_m	momentum-relaxation time	s
τ_φ	phase-relaxation time	s
$\omega_c/2\pi$	cyclotron frequency	s^{-1}

Please note that we have often used the terms ‘electrochemical potential’ (μ) and ‘quasi-Fermi energy’ (F_n) interchangeably.

A webpage has been set up at

<http://dynamo.ecn.purdue.edu/~datta/etms.html> or

<http://www.ece.purdue.edu/~datta/etms.html>

where additional information related to this book will be posted.

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Introductory remarks

It is well-known that the conductance (G) of a rectangular two-dimensional conductor is directly proportional to its width (W) and inversely proportional to its length (L); that is,

$$G = \sigma W/L$$

where the conductivity σ is a material property of the sample independent of its dimensions. How small can we make the dimensions (W and/or L) before this ohmic behavior breaks down? This question has intrigued scientists for a long time. During the 1980s it became possible to fabricate small conductors and explore this question experimentally, leading to significant progress in our understanding of the meaning of resistance at the microscopic level. What emerged in the process is a conceptual framework for describing current flow on length scales shorter than a mean free path. We believe that these concepts should be useful to a broad spectrum of scientists and engineers. This book represents an attempt to present these developments in a form accessible to graduate students and to non-specialists.

Small conductors whose dimensions are intermediate between the microscopic and the macroscopic are called 'mesoscopic'. They are much larger than microscopic objects like atoms, but not large enough to be 'ohmic'. A conductor usually shows ohmic behavior if its dimensions are much larger than each of three characteristic length scales: (1) the de Broglie wavelength, which is related to the kinetic energy of the electrons, (2) the mean free path, which is the distance that an electron travels before its initial momentum is destroyed and (3) the phase-relaxation length, which is the distance that an electron travels before its initial phase is destroyed. These length scales vary widely from one material to another and are also strongly affected by temperature,

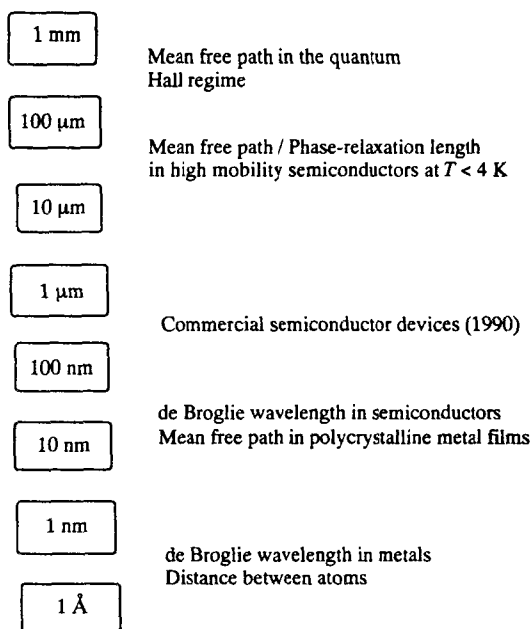


Fig. 0.1. A few relevant length scales. Note that
 $1\text{ }\mu\text{m} = 10^{-6}\text{ m} = 10^{-4}\text{ cm}$
 $1\text{ nm} = 10^{-9}\text{ m} = 10\text{ angstroms (}\text{\AA}\text{)}.$

magnetic field etc. (Fig. 0.1). For this reason, mesoscopic transport phenomena have been observed in conductors having a wide range of dimensions from a few nanometers to hundreds of microns (that is, micrometers).

Mesoscopic conductors are usually fabricated by patterning a planar conductor that has one very small dimension to start with. For example, Fig. 0.2 shows a ring-shaped conductor having dimensions $\sim 100\text{ nm}$, patterned out of a polycrystalline gold film $\sim 40\text{ nm}$ thick. This is the structure that was used for one of the landmark experiments in mesoscopic physics: the resistance of this ring was shown to oscillate as the magnetic field through it was changed because the magnetic field modified the interference between the electron waves traversing the two arms of the ring.

Although some of the pioneering experiments in this field were performed using metallic conductors, most of the recent work has been based on the gallium arsenide (GaAs)–aluminum gallium arsenide (AlGaAs) material system. Figure 0.3 shows a Hall bridge patterned out of a conducting layer $\sim 10\text{ nm}$ thick formed at a GaAs–AlGaAs inter-