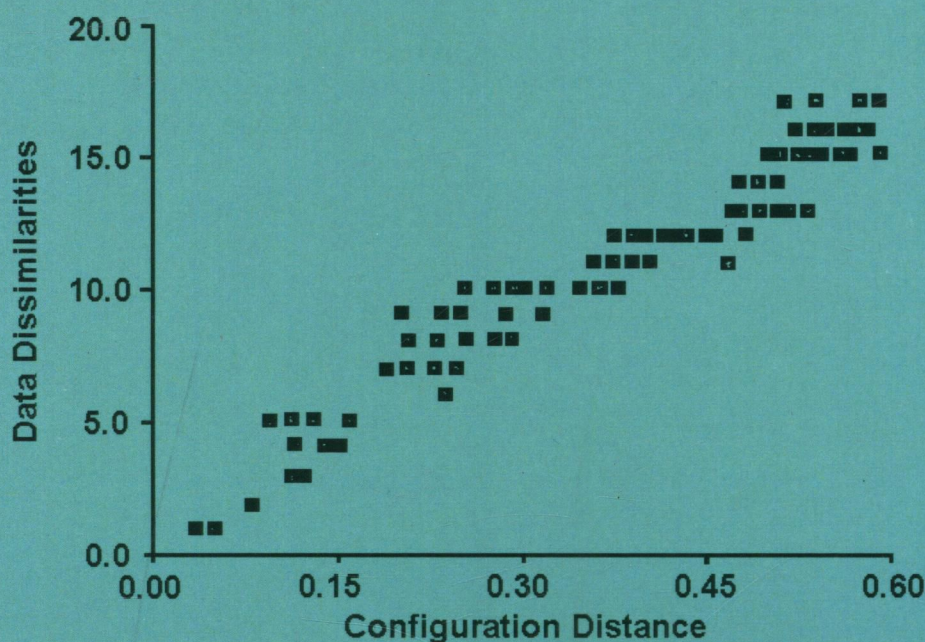


**Bryan F. J. Manly**  
**Jorge A. Navarro Alberto**

# **Multivariate Statistical Methods**

## **A Primer**

*Fourth Edition*



**CRC Press**  
Taylor & Francis Group

A CHAPMAN & HALL BOOK

## Multivariate Statistical Methods A Primer

Fourth Edition

Bryan F.J. Manly and Jorge A. Navarro Alberto

**Multivariate Statistical Methods: A Primer** provides an introductory overview of multivariate methods without delving too deeply into the mathematical details. This fourth edition is a revised and updated version of this bestselling introductory textbook. It retains the clear and concise style of the previous editions of the book and focuses on examples from biological and environmental sciences. The major update with this edition is that R code has been included for each of the analyses described, although in practice any standard statistical package can be used.

### Features

- Concise and accessible approach requiring minimal mathematical background
- Appeals to a broad range of quantitative scientists and applied statisticians
- Presents all the key topics for a course in multivariate statistics
- Updated with chapter appendices of R code for all analyses in the book
- Provides all the data used in the book on a companion website

The original idea of this book—to make it as short as possible and enable readers to begin using multivariate methods in an intelligent manner—still applies. With updated information on multivariate analyses, new references, and R code included, this book continues to provide a timely introduction to useful tools for multivariate statistical analysis.



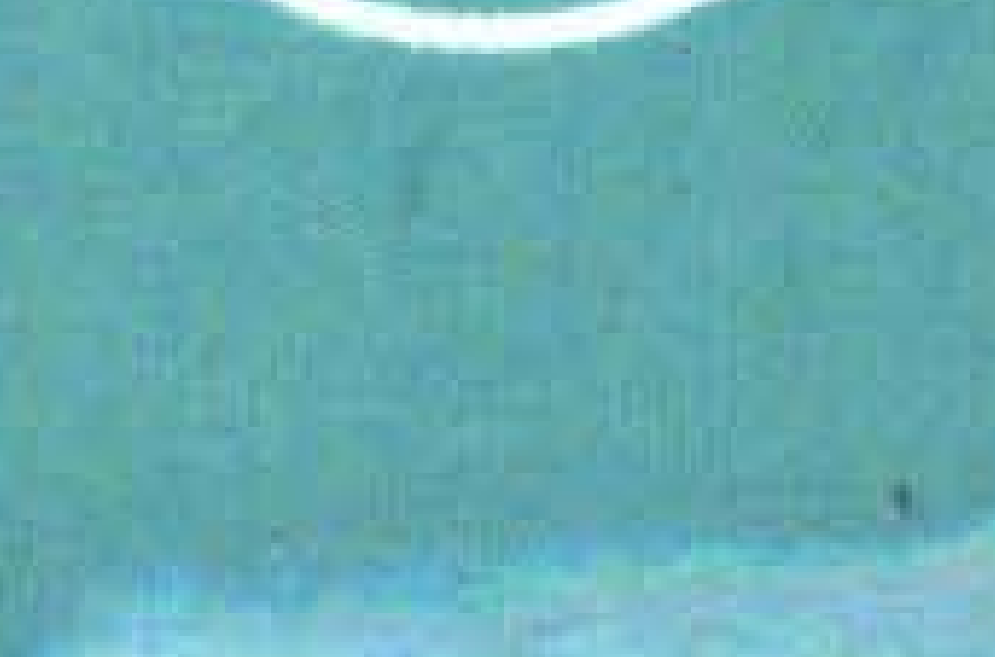
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**Navarro Albeerto**



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**Bryan F. J. Manly**

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Dunedin, New Zealand

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# **Multivariate Statistical Methods**

■ **A Primer** ■

*Fourth Edition*



# *Dedication*

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*To my multivariate (8-dimensional) space of caring  
and loving support: the Navarro-Contreras*

*J.N.A.*



A journey of a thousand miles begins with a single step

**Lao Tsu**



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# Preface

This is the fourth edition of the book *Multivariate Statistical Methods: a Primer*. The contents are similar to what was in the third edition of the book, with the main difference being the introduction of R code to do all of the analyses in the fourth edition. The version of R used for running the R-scripts (and the corresponding packages) is R 3.3.1. Also, the results obtained with the R code have been checked to ensure that they are the same as the results obtained from various other statistical packages.

The purpose of the book is to introduce multivariate statistical methods to nonmathematicians. It is not intended to be comprehensive. Rather, the intention is to keep the details to a minimum while still conveying a good idea about what can be done. In other words, it is a book to “get you going” in a particular area of statistical methods.

It is assumed that readers have a working knowledge of elementary statistics, including tests of significance using the normal,  $t$ , chi-squared, and  $F$  distributions, analysis of variance, and linear regression. The material covered in a typical first-year university course in statistics should be quite adequate in this respect. Some facility with algebra is also required to follow the equations in certain parts of the text, and understanding the theory of multivariate methods to some extent does require the use of matrix algebra. However, the amount needed is not great if some details are accepted on faith. Matrix algebra is summarized in Chapter 2, and anyone who masters this chapter will have a reasonable competency in this area.

To some extent, the chapters can be read independently of each other. The first five are preliminary reading, because they are mainly concerned with general aspects of multivariate data rather than with specific techniques. Chapter 1 introduces some examples with the aim of motivating the analyses covered in the book. Chapter 2 covers matrix algebra, Chapter 3 is about graphical methods of various types, Chapter 4 is about tests of significance, and Chapter 5 is about the measurement of distances between objects based on variables measured on those objects. It is recommended that these chapters should be reviewed before Chapters 6 through 12, which cover the most important multivariate procedures in

current use. The final Chapter 13 contains some general comments about the analysis of multivariate data.

The chapters in this fourth edition of the book are the same as those in the second and third editions, and in making changes we have continually kept in mind the original intention of the book, which was that it should be as short as possible and attempt to do no more than take readers to the stage where they can begin to use multivariate methods in an intelligent manner. An Appendix to Chapter 1 provides an introduction to the use of the R package, and the code for analyses is discussed in Appendices for Chapters 2 through 12.

We wish to thank the staff of Chapman and Hall/CRC Press for their work over the years in promoting this book and encouraging us to produce this fourth edition.

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