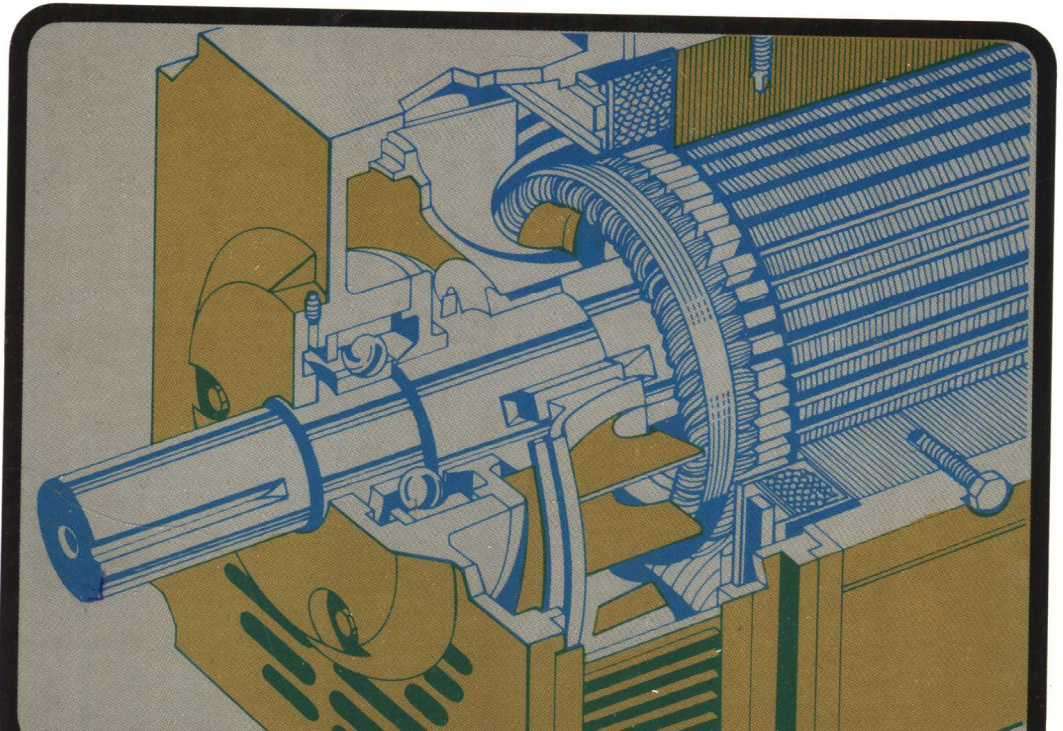


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Electromechanical Energy Conversion



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PREFACE

This book is intended primarily for use with a first course in electromechanical energy conversion. Sufficient material is presented for a two-semester sequence. Traditional subjects are covered in Chapters 1 through 5. In instances where programs allow for only a one-semester coverage, these first five chapters will satisfy most requirements. Additional supplementary material from the remaining chapters may then be appropriate depending on the interests of the instructor.

The emphasis of this book is on development of machine characteristics from as fundamental a framework as is practical. This emphasis is chosen in order to provide an appreciation of the subject beyond that which results from an approach based solely on equivalent circuits. The authors have been gratified to find that considerable student interest exists in development of an understanding of fundamental capabilities and limitations from a broader perspective than that presented when other approaches are used. In order to meet this need, this book starts with first principles but recognizes that many students will not have developed a strong feel for those aspects of field theory which are crucial to an understanding of electromechanical energy conversion. The authors feel that definitions and clear explanations in terms of fundamental relationships are critical. The first chapter reflects this viewpoint for what follows in the later chapters. This permits a development that does not later restrict treatment to special cases. Models are developed in such a way that steady-state operation is seen to be simply a special case of device behavior under certain assumed conditions.

It also permits presentation of design considerations where the intent is not to provide a detailed design manual but rather to develop an appreciation for fundamental limitations and capabilities.

Chapter 1 starts with a very deliberate treatment of a simple magnetic circuit. Definitions and fundamental principles are introduced *as required*. Maxwell's equations in integral form are stressed as fundamental to the concept. Consideration is given to field variation in all three dimensions and the role of highly permeable core material is emphasized fully. Examples are presented in a very deliberate step-by-step format. Phasor concepts are introduced where they are appropriate in order to examine steady-state ac behavior. Notation is chosen to emphasize the distinction between a phasor and its magnitude. The section on design considerations emphasizes practical aspects of transformer design without getting bogged down in detailed case-specific design procedures.

Chapter 2 develops the fundamental force and torque relationships. The emphasis is on examples of a practical nature. State-variable formulation of the equations required to describe a system is stressed. The basic action of conventional induction machines, synchronous machines, and dc machines is introduced.

Chapters 3 and 4 cover induction and synchronous machines, respectively. The coupled-coil viewpoint is used to develop mathematical models in each case. Relevant assumptions are explicitly stated. Models are then developed in a rigorous but straightforward manner and are sufficiently general to permit the later analysis of machine transients (Chapter 6) and ac variable-speed drives (Chapter 8). The emphasis is on steady-state characteristics in Chapters 3 and 4, and appropriate models are obtained by assuming sinusoidal time variation in the more general relationships. Examples are presented in a deliberate step-by-step format and are in many cases based on the authors' personal experiences in industry.

Chapter 5 covers the dc machine. Motor and generator characteristics for the basic dc machine types are developed. Emphasis is on modern applications of dc machinery. The dynamics of dc machinery are treated with a state-variable formulation as well as with the more traditional block diagram approach.

Chapter 6 treats the analysis of ac machine transients. Models developed in Chapters 3 and 4 are examined for the general case, that is, without the assumption of steady-state sinusoidal time variation. The classical qdo transformation is introduced. It is presented as a mathematical tool that simplifies the necessary machine equations and provides additional insights on machine behavior. Numerous examples are presented early in the development to aid the student in making the transition from steady-state analysis to a more general analysis. Simplifying assumptions are made to permit analytical solutions (by hand), which are later compared to computer-assisted solutions (Chapter 8). Examples are based on practical applications and stress an intuitive understanding of the phenomena investigated.

Chapter 7 treats a variety of special-purpose electromechanical devices not generally regarded as conventional ac or dc rotating machines. The emphasis is on a general description of each device and its basic operating characteristics. The major thrust of the chapter is to develop an understanding that unusual and sometimes highly complex systems and devices can be developed based on the

fundamental framework learned earlier. Topics covered range from servo systems and computer control to magnetically levitated, "super-speed" trains.

Chapter 8 is devoted to advanced applications. The transformer is reexamined with a view toward model refinements required in order to examine various phenomena not generally covered in introductory courses. Examples are chosen to illustrate practical problems encountered. State-variable formulation of the necessary mathematical relationships is stressed throughout, and computer-assisted solutions are presented. The important area of ac machine variable-speed drives is introduced with emphasis on development of analytical methods suitable for computer implementation.

Finally, because of the shifting emphasis in many engineering and science programs, it is no longer possible to assume that all students will have sufficient background in the various analytical methods required. The material presented in the Appendixes provides a useful review of those techniques which are used throughout the text.

A complete solutions manual, with solutions to all of the text problems, is available from the publisher.

We wish to acknowledge the substantial contribution to this work made by our colleagues and students and Debra Hayden for her assistance in typing the manuscript through its many drafts. Additionally, we would like to thank the reviewers of the manuscript during its preparation, including John Grainger, A. G. Potter, Milton R. Johnson, and F. W. Schott. Finally, we would like to thank our families for their continued and patient support throughout this project.

D.R.B.
E.P.H.III

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Fundamental Concepts

1.1

INTRODUCTION

Electromechanical energy conversion involves the interaction of current conducting materials and magnetic fields. Maxwell's equations describe such phenomena and provide the starting point for analysis and design of energy conversion devices.

1.2

MAGNETIC CIRCUITS

The magnetic circuit concept is helpful in understanding the behavior of many practical electromagnetic devices. A simple magnetic circuit is illustrated in Figure 1.1. The magnetic circuit in Figure 1.1 consists of a core having a rectangular cross section wrapped with N turns of wire. We will use this relatively simple device to review some fundamental relationships and definitions.

We start by recalling Ampère's law:[†]

$$\oint \bar{H} \cdot d\bar{\ell} = \int_s \bar{J} \cdot d\bar{s} \quad (1.1)$$

[†]Displacement current terms are negligible in most electromechanical energy conversion devices and transformers because time variation is at relatively low frequencies.

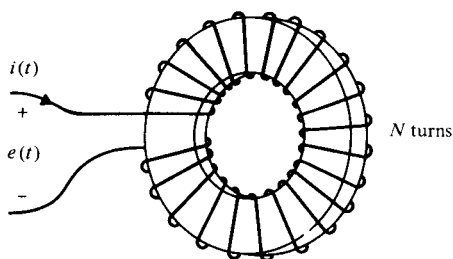


FIGURE 1.1 Simple Magnetic Circuit

In words, the line integral of magnetic field intensity \vec{H} around any closed path is equal to the surface integral of current density \vec{J} over the surface bounded by the closed path. The vector quantities involved are illustrated in Figure 1.2. In SI units, \vec{H} is in amperes per meter, \vec{J} is in amperes per square meter, $\vec{\ell}$ is path length in meters, and \vec{s} is surface area in square meters. All quantities are vector quantities having direction as well as magnitude. Note that the integral on the right-hand side of (1.1) gives the total current through the surface bounded by the path of integration for \vec{H} and it is a scalar quantity.

The magnetic circuit depicted in Figure 1.1 is a symmetrical configuration and permits a straightforward application of Ampère's law. Figure 1.3 is a cross-sectional view of the magnetic circuit showing appropriate dimensions.

A cylindrical coordinate system is chosen and unit vectors \vec{a}_r , \vec{a}_θ , and \vec{a}_z are shown in Figure 1.3 at radius r and at an arbitrary angle θ . A current direction is assumed in the winding such that for $r < r_i$ current is directed into the page and for $r > r_o$ current is directed out of the page. We use “ \times ” and “ \cdot ” symbols to indicate current direction into and out of the page, respectively. If the winding consists of many turns of relatively fine wire where the turns of the wire are spaced closely together, we can assume uniform current densities for those spaces actually occupied by winding material. This assumption results in a symmetrical configuration with respect to the angle θ . The magnetic field intensity in the azimuthal (θ) direction inside the winding (totally within the core) is obtained by direct application of Ampère's law. For a circular path of integration at radius r we have on the left-hand side of (1.1):

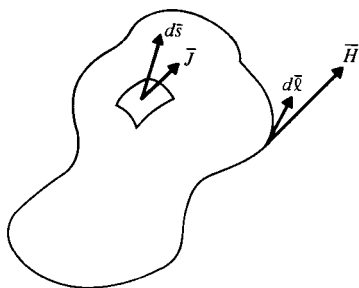


FIGURE 1.2 Vector Quantities Associated with Ampère's Law

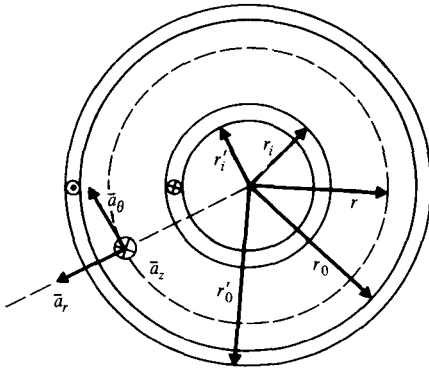


FIGURE 1.3 Cross-Sectional View of Core

$$\oint \bar{H} \cdot d\bar{\ell} = \oint (H_r \bar{a}_r + H_\theta \bar{a}_\theta + H_z \bar{a}_z) \cdot (r d\theta \bar{a}_\theta) \quad (1.2)$$

$$= \int_0^{2\pi} H_\theta r d\theta = 2\pi r H_\theta$$

where $\bar{a}_r \cdot \bar{a}_\theta = \bar{a}_z \cdot \bar{a}_\theta = 0$, $\bar{a}_\theta \cdot \bar{a}_\theta = 1$, and H is invariant with θ based on symmetry considerations. On the right-hand side of (1.1), we have for a path of integration within the core:

$$\int \bar{J} \cdot d\bar{s} = Ni \quad (1.3)$$

If we drop the θ subscript, the magnetic field intensity in the θ direction within the core must be

$$H = \frac{Ni}{2\pi r} \quad (1.4)$$

By choosing paths of integration at various radii, we can similarly determine H in the θ direction for the other regions illustrated in Figure 1.3. If we exclude the end-turn regions, the current density in the region $r'_i \leq r \leq r_i$ is $Ni/[\pi(r_i^2 - r'^2_i)]$ directed into the page. Similarly, the current density in the region $r_o \leq r \leq r'_o$ is $Ni/[\pi(r'^2_o - r_o^2)]$ directed out of the page. Thus, excluding the end-turn regions, we find

$$\begin{aligned} H &= 0 & r &\leq r'_i \\ H &= \frac{Ni}{2\pi r} \frac{r_i^2 - r'^2_i}{r_i^2 - r'^2_i} & r'_i &\leq r \leq r_i \\ H &= \frac{Ni}{2\pi r} & r_i &\leq r \leq r_o \\ H &= \frac{Ni}{2\pi r} \left(1 - \frac{r^2 - r_o^2}{r'^2_o - r_o^2} \right) & r_o &\leq r \leq r'_o \\ H &= 0 & r &\geq r'_o \end{aligned} \quad (1.5)$$

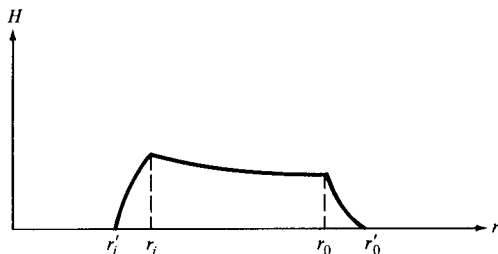


FIGURE 1.4 Variation of Magnetic Field Intensity with Radius

The H field in the θ direction excluding the end-turn regions might appear as shown in Figure 1.4.

The behavior internal to the end-turn regions is somewhat more complicated in that the H field in the θ direction is a function of both z and r . It does, however, reduce to zero outside the winding. This can be surmised by applying Ampère's law for appropriately chosen circular paths of integration outside the winding.

Magnetic flux density \bar{B} in webers per square meter or teslas (Wb/m^2 or T; $1.0 \text{ Wb}/\text{m}^2 = 1.0 \text{ T}$) is related to magnetic field intensity \bar{H} by the permeability of the material:

$$\bar{B} = \mu \bar{H} \quad (1.6)$$

where

$$\mu = \mu_r \mu_0 \quad (1.7)$$

In (1.7), μ_0 is the permeability of free space, $4\pi \times 10^{-7}$ henry per meter (H/m), and μ_r is the relative permeability. We are assuming here that the material is isotropic, which means that it has the same properties in all directions. The value of μ_r for ferromagnetic materials used in machines and transformers is high, say on the order of 10^3 to 10^5 . If the core material in our magnetic circuit is ferromagnetic, flux densities within the winding material (regions where $r'_i < r < r_i$ and $r_o < r < r'_o$ and internal to the end-turn regions) will be negligible compared to flux densities within the core. This is illustrated in Figure 1.5.

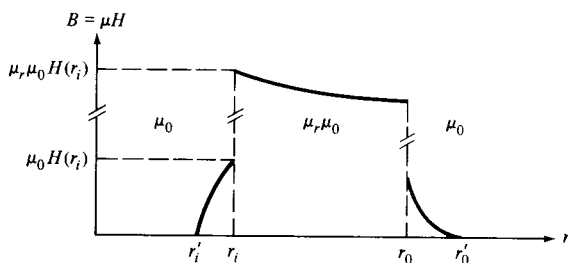


FIGURE 1.5 Variation of Magnetic Flux Density with Radius

Ferromagnetic materials in reality exhibit nonlinear characteristics, where the value of μ_r depends on the magnitude of H and its past behavior. We will examine some aspects of this behavior later, but as we will show, a relatively high value of μ_r permits us to assume linear behavior in many applications.

Magnetic flux ϕ in webers is obtained by integrating magnetic flux density \vec{B} over some particular surface area.

$$\phi = \int_s \vec{B} \cdot d\vec{s} \quad (1.8)$$

The vector quantities involved are illustrated in Figure 1.6. Note that magnetic flux is a scalar quantity.

To obtain flux in the core of our magnetic circuit example, we must integrate flux density in the core over the rectangular cross section. If we evaluate H at $r = (r_i + r_o)/2$ and assume this value over the entire cross section, we do not incur significant error (see Problem 1.2). Defining this value as H_c , (1.5) yields

$$H_c = \frac{Ni}{2\pi\left(\frac{r_i + r_o}{2}\right)} = \frac{Ni}{\ell_c} \quad (1.9)$$

where ℓ_c is the distance around the core at its midpoint. Then flux density is given by

$$B_c = \mu H_c = \frac{\mu Ni}{\ell_c} \quad (1.10)$$

and the flux is

$$\phi = \int_s \vec{B} \cdot d\vec{s} = B_c A_c = \frac{\mu Ni A_c}{\ell_c} = \frac{Ni}{\ell_c / \mu A_c} \quad (1.11)$$

where A_c is the cross-sectional area of the core. We can view (1.11) as a cause-and-effect relationship where an exciting current i causes a flux ϕ in the core.

The quantity Ni in (1.11) is sometimes referred to as *magnetomotive force* (mmf). The units of mmf are ampere-turns ($A \cdot t$) or, alternatively, amperes if N is considered dimensionless. The ratio of mmf to flux is defined as *reluctance*:

$$\mathcal{R} = \frac{Ni}{\phi} \quad A \cdot t / \text{Wb} \quad (1.12)$$

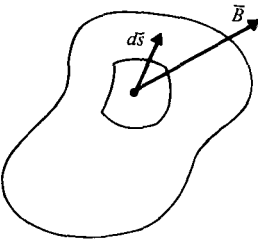


FIGURE 1.6 Vector Quantities Associated with Equation (1.8)

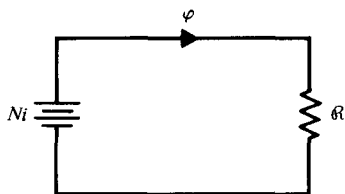


FIGURE 1.7 An Analogous Electrical Circuit to the Magnetic Circuit of Figure 1.1

The reciprocal of reluctance is defined as *permeance*:

$$\mathcal{P} = \frac{1}{\mathcal{R}} = \frac{\Phi}{Ni} \quad \text{Wb/A} \cdot \text{t} \quad (1.13)$$

For the magnetic circuit we are considering we see from (1.11) that

$$\mathcal{R} = \frac{\ell_c}{\mu A_c} \quad (1.14)$$

This expression is similar to the expression for the resistance of a length of wire in terms of its resistivity ρ , its length ℓ , and its cross-sectional area A :

$$R = \frac{\rho \ell}{A} \quad (1.15)$$

An analogy is suggested between our magnetic circuit and a simple direct-current (dc) circuit as shown in Figure 1.7.

The analogy is sometimes useful in conceptualizing a magnetic circuit. To see this, we will consider an example.

EXAMPLE 1.1 Find the mmf required to produce a flux density of 0.5 Wb/m^2 in the air gap of the magnetic circuit depicted in Figure 1.8. Dimensions and relative permeability are

ℓ_g	gap length is 0.1 cm
A_c	core cross-sectional area is 1 cm^2
ℓ_c	mean length of core is 6 cm
μ_r	relative permeability of core is 10^4

The example differs considerably from the symmetrical situation we encountered with the magnetic circuit configuration of Figure 1.1. The high permeability of the core results in negligible flux external to the core when compared to flux within the core. (There will

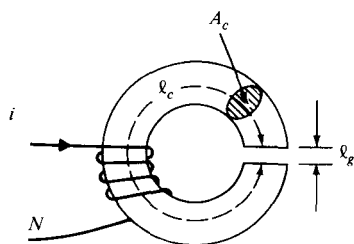


FIGURE 1.8 Magnetic Circuit For Example 1.1

be flux in the air gap.) If we neglect all flux except core flux and air-gap flux, the continuity law for magnetic flux;

$$\oint \vec{B} \cdot d\vec{s} = 0 \quad (1.16)$$

requires that our core flux density B_c , assumed constant over any cross section, be the same at any point along the flux path within the core, since the core cross-sectional area is the same at any point along the path. That is, $\oint \vec{B} \cdot d\vec{s} = B_{c1}A_c - B_{c2}A_c = 0$, which requires that $B_{c1} = B_{c2}$ for any two points along the flux path ℓ_c within the core. This in turn requires that $H_c = B_c/\mu$ be constant along the length of the core. In the vicinity of the gap we will have some fringing, as illustrated in Figure 1.9. This can be accounted for (approximately) by increasing the effective cross-sectional area of the air gap over that of the core. We will assume that

$$A_g = 1.10A_c$$

If the effective cross-sectional area in the air gap A_g is assumed to apply for the entire length of the air gap ℓ_g , the continuity law for flux would require that flux density in the gap B_g be constant for the entire length of the air gap. Thus $H_g = B_g/\mu_0$ would be constant for the entire length of the air gap. Moreover, continuity of flux would require that $B_gA_g = B_cA_c = \Phi$. This implies a discontinuity in B at the transition from core to air gap since $A_g \neq A_c$. In reality, the normal component of B at a boundary must be continuous, and it will be in our magnetic circuit (see Figure 1.9). The use of an effective air-gap cross-sectional area A_g is obviously a simplifying assumption.

Now apply Ampère's law:

$$\begin{aligned} Ni &= \oint \vec{H} \cdot d\vec{\ell} \\ &= H_c\ell_c + H_g\ell_g \end{aligned}$$

where

$$\begin{aligned} H_g &= \frac{B_g}{\mu_0} = \frac{0.5}{4\pi \times 10^{-7}} = 3.98(10^5) \text{ A/m} \\ H_c &= \frac{B_c}{\mu_r\mu_0} = \frac{\Phi}{A_c\mu_r\mu_0} = \frac{B_gA_g}{A_c\mu_r\mu_0} \\ &= \frac{(0.5)(1.1)}{(10^4)(4\pi \times 10^{-7})} = 4.38(10^1) \text{ A/m} \end{aligned}$$

Thus

$$\begin{aligned} Ni &= 4.38(10)(0.06) + 3.98(10^5)(0.001) \\ &= 2.63 + 3.98(10^2) \\ &= 401 \text{ A or } A \cdot t \end{aligned}$$

Thus a 100-turn coil carrying approximately 4 A could be used.

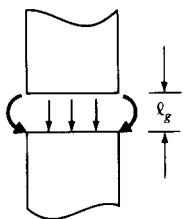


FIGURE 1.9 Typical Air-Gap Fringing

Note the relative magnitudes of H_g , the air-gap magnetic field intensity, and H_c , the core magnetic field intensity. Note also the relative magnitudes of $H_g \ell_g$ and $H_c \ell_c$. Ampère's law requires that these terms have the same units as the mmf Ni . $H_c \ell_c$ is sometimes referred to as the *mmf drop* in the core and $H_g \ell_g$ is referred to as the mmf drop in the air gap. The relative magnitudes of the quantities above suggest some approximations that will prove extremely useful in the future. To gain additional insights, we will solve the problem in a different way. Again starting with Ampère's law:

$$\begin{aligned} Ni &= \oint \vec{H} \cdot d\vec{\ell} \\ &= H_c \ell_c + H_g \ell_g \\ &= \frac{B_c}{\mu_c} \ell_c + \frac{B_g}{\mu_0} \ell_g \\ &= \phi \frac{\ell_c}{\mu_c A_c} + \phi \frac{\ell_g}{\mu_0 A_g} \\ &= \phi [\mathcal{R}_c + \mathcal{R}_g] \end{aligned}$$

Solving yields

$$\begin{aligned} \mathcal{R}_c &= \frac{0.06}{(10^4)(4\pi \times 10^{-7})(10^{-4})} = 4.77(10^4) \text{ A} \cdot \text{t/Wb} \\ \mathcal{R}_g &= \frac{0.001}{(4\pi \times 10^{-7})(1.1 \times 10^{-4})} = 7.23(10^6) \\ \phi &= B_g A_g = (0.5)(1.1 \times 10^{-4}) = 5.5(10^{-5}) \text{ Wb} \end{aligned}$$

Thus

$$\begin{aligned} Ni &= 5.5(10^{-5})[4.77(10^4) + 7.23(10^6)] \\ &= 400 \text{ A} \cdot \text{t} \end{aligned}$$

which is the same as our previous result except for round-off differences. Note the relative magnitudes of air-gap reluctance \mathcal{R}_g and core reluctance \mathcal{R}_c . The air-gap reluctance dominates. This is often the case. Errors we may have introduced by neglecting nonlinear effects of the core will be minimal if air-gap reluctance is significantly larger than core reluctance. Our use of the term “mmf drop” takes on added significance. It can easily be seen that

$$\begin{aligned} H_c \ell_c &= \phi \mathcal{R}_c \\ H_g \ell_g &= \phi \mathcal{R}_g \end{aligned}$$

Other concepts suggested by the electrical circuit analog follow. The electrical circuit analog for our example is shown in Figure 1.10. A “voltage division” effect is suggested, with the major portion of the total mmf drop Ni appearing across the air-gap reluctance \mathcal{R}_g .

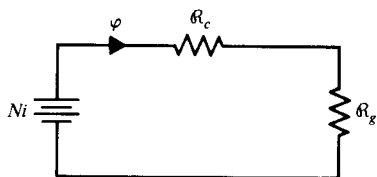


FIGURE 1.10 Electrical Circuit Analog