SECOND EDITION

COULSON & RICHARDSON'S

# CHEMICAL ENGINEERING

# 化学工程

第4卷 第2版

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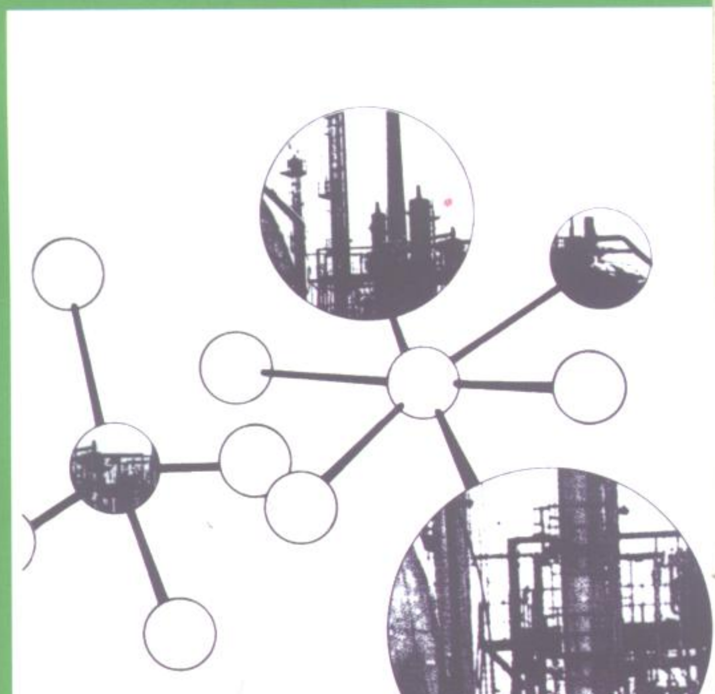
J H Harker



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VOLUME 4

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# CHEMICAL ENGINEERING

## VOLUME 4

**SECOND EDITION**

*Solutions to the Problems in Chemical Engineering Volume 1*

# Preface to Second Edition

With the publication of the Fourth Edition of *Chemical Engineering*, Volume 1, we felt it appropriate to prepare this Second Edition of Volume 4 and again, group the solutions to the problems in Volume 1 in sections corresponding to the chapters in that text and also update the reference to the equations and sources of data in that text. We have been encouraged in this by many readers who, in generating an amazing amount of correspondence and, praise be, in finding our previous volume of value, have pointed out a mercifully small number of errors and also made suggestions for improvement which have all been incorporated; particularly the suggestion that where two problems are very similar in the principles which they illustrate, then one or other might be omitted. The resulting, albeit modest, increase in space has enabled us to include additional problems relating to and illustrating the new material now incorporated in Volume 1 and here, we are more than grateful for the problems provided in this respect by Professor J. F. Richardson who has not only checked our efforts, but guided us, encouraged us and, at times, spurred us into action, by what seems to be his ever-increasing zest, understanding and enthusiasm for this series of texts. We owe him, our publishers and of course our colleagues at Newcastle a very great debt of gratitude indeed.

More than once, our readers have suggested that we might, where appropriate, offer solutions which make use of computer programs. Whilst we warm to this one, we feel that the prime aim of this volume, as indeed was the case with the First Edition, is to illustrate basic Chemical Engineering principles and, to this end, provide students with a series of worked examples using the various techniques involved. We firmly believe that, once a student has worked through these various problems 'by hand' as it were, then he or she will be uniquely equipped, with the computer by their side, to investigate further the vast range of solutions which are available from a variation of input parameters and iteration techniques and so be able to venture into all the complexities and joys of optimisation and both energy-efficient and cost-effective design procedures. In a real sense, our aim is to offer, not only a first-stage approach to problem solving, but also, dare we suggest, a way of thinking, firmly believing that little can be achieved until this has been grasped. Once it has been grasped however, then the possibilities are limitless.

Newcastle upon Tyne
1994

J. R. BACKHURST
J. H. HARKER

# Preface to First Edition

In the preface to the first edition of *Chemical Engineering*, Volume 1, Coulson and Richardson include the following paragraph:

"We have introduced into each chapter a number of worked examples which we believe are essential to a proper understanding of the methods of treatment given in the text. It is very desirable for a student to understand a worked example before tackling fresh practical problems himself. Chemical engineering problems require a numerical answer and it is essential to become familiar with the different techniques so that the answer is obtained by systematic methods rather than by intuition."

It is with these aims in mind that the present book, which in essence is a collection of solutions to the problems in the third edition of *Chemical Engineering*, Volume 1, has been prepared. The scope of the book is, of course, that of Volume 1, and the solutions are grouped in sections corresponding to the chapters in that text. The book has been written co-currently with the preparation of the new edition of Volume 1, and extensive reference has been made to the equations and sources of data in that volume at all stages. In this sense the present book is complementary to Volume 1. The working throughout is in SI units and the format is that of the third edition.

In common with countless students before us, we have battled with these problems for some two decades, and although our approach to the solutions has been refined over the years, we cannot claim to present the most elegant form of solution nor, indeed, always the most precise. Nevertheless, we hope that there is much to be learned from our efforts not only by the undergraduate but also by the professional engineer in industry.

This book could not have been written, of course, without the very real and longstanding contribution of Professors Coulson and Richardson to the profession. It is with considerable pleasure that we acknowledge our debt of gratitude, especially to Professor Coulson, who has guided our thoughts and encouraged our activities over so many years. We also acknowledge the help of our colleagues at the University of Newcastle upon Tyne and especially their forebearance during the preparation of this book.

Newcastle upon Tyne
1976

J. R. BACKHURST
J. H. HARKER

# Contents

SECTION 1

# Units and Dimensions

### Problem 1.1

98% sulphuric acid of viscosity 0.025 N s/m$^2$ and density 1840 kg/m$^3$ is pumped at 685 cm$^3$/s through a 25 mm line. Calculate the value of the Reynolds number.

### Solution

Cross-sectional area of line = $(\pi/4)0.025^2 = 0.00049\ \text{m}^2$.
Mean velocity of acid, $u = (685 \times 10^{-6})/0.00049 = 1.398\ \text{m/s}$.
∴ Reynolds number, $Re = du\rho/\mu = (0.025 \times 1.398 \times 1840)/0.025 = \underline{\underline{2572}}$

### Problem 1.2

Compare the costs of electricity at 1p per kWh and gas at 15 p per therm.

### Solution

Each cost is calculated in p/MJ.

$$1\ \text{kWh} = 1\ \text{kW} \times 1\ \text{h} = (1000\ \text{J/s})(3600\ \text{s}) = 3{,}600{,}000\ \text{J} \text{ or } 3.6\ \text{MJ}$$

$$1\ \text{therm} = 105.5\ \text{MJ}$$

∴
$$\text{cost of electricity} = 1\ \text{p}/3.6\ \text{MJ} \text{ or } (1/3.6) = \underline{\underline{0.28\ \text{p/MJ}}}$$

$$\text{cost of gas} = 15\text{p}/105.5\ \text{MJ} \text{ or } (15/105.5) = \underline{\underline{0.14\ \text{p/MJ}}}$$

### Problem 1.3

A boiler plant raises 5.2 kg/s of steam at 1825 kN/m$^2$ pressure using coal of calorific value 27.2 MJ/kg. If the boiler efficiency is 75%, how much coal is consumed per day? If the steam is used to generate electricity, what is the power generation in kilowatts assuming a 20% conversion efficiency of the turbines and generators?

### Solution

From steam tables, total enthalpy of steam at 1825 kN/m$^2$ = 2798 kJ/kg.

∴
$$\text{enthalpy of steam} = (5.2 \times 2798) = 14{,}550\ \text{kW}$$

Neglecting the enthalpy of the feed water, this must be derived from the coal. For an efficiency of 75%, heat provided by the coal = (14,550 × 100/75) = 19,400 kW.

For a calorific value of 27,200 kJ/kg, rate of coal consumption = (19,400/27,200) = 0.713 kg/s or: $(0.713 \times 3600 \times 24/1000) = \underline{\underline{61.6\ \text{Mg/day}\ (61.6\ \text{tonnes/day})}}$

20% of the enthalpy in the steam is converted to power or:

$$(14{,}550 \times 20/100) = 2910\ \text{kW} \ \text{ or } \ 2.91\ \text{MW} \ \text{ say } \ \underline{\underline{3\ \text{MW}}}$$

**Problem 1.4**

The power required by an agitator in a tank is a function of the variables:

(a) diameter of impeller,
(b) number of rotations of impeller per unit time,
(c) viscosity of liquid,
(d) density of liquid.

From a dimensional analysis, obtain a relation between the power and the four variables.

The power consumption is found experimentally to be proportional to the square of the speed of rotation. By what factor would the power be expected to increase if the impeller diameter were doubled?

**Solution**

If the power $P = \phi(DN\rho\mu)$, then a typical form of the function is $P = kD^a N^b \rho^c \mu^d$, where $k$ is a constant. The dimensions of each parameter in terms of **M**, **L**, and **T** are: power, $P = \mathbf{ML^2/T^3}$, density, $\rho = \mathbf{M/L^3}$, diameter, $D = \mathbf{L}$, viscosity, $\mu = \mathbf{M/LT}$, and speed of rotation, $N = \mathbf{T}^{-1}$

Equating dimensions:

**M**: $1 = c + d$
L: $2 = a - 3c - d$
T: $-3 = -b - d$

Solving in terms of $d$: $a = (5 - 2d)$, $b = (3 - d)$, $c = (1 - d)$

$$\therefore \qquad P = k\left(\frac{D^5}{D^{2d}}\ \frac{N^3}{N^d}\ \frac{\rho}{\rho^d}\ \mu^d\right)$$

or: $P/D^5N^3\rho = k(D^2N\rho/\mu)^{-d}$ that is: $N_P = k\,Re^m$

Thus the power number is a function of the Reynolds number to the power $m$. In fact $N_P$ is also a function of the Froude number, $DN^2/g$. The above equation may be written as:

$$P/D^5N^3\rho = k(D^2N\rho/\mu)^m$$

Experimentally: $P \propto N^2$

From the equation, $P \propto N^m N^3$, that is $m + 3 = 2$ and $m = -1$

Thus for the same fluid, that is the same viscosity and density,

$$(P_2/P_1)(D_1^5N_1^3/D_2^5N_2^3) = (D_1^2N_1/D_2^2N_2)^{-1} \text{ or } (P_2/P_1) = (N_2^2D_2^3)/(N_1^2D_1^3)$$

In this case, $N_1 = N_2$ and $D_2 = 2D_1$.

$$\therefore \qquad (P_2/P_1) = 8D_1^3/D_1^3 = \underline{\underline{8}}$$

A similar solution may be obtained using the Recurring Set method as follows:

$$P = \phi(D, N, \rho, \mu), \ f(P, D, N, \rho, \mu) = 0$$

Using **M**, **L** and **T** as fundamentals, there are five variables and three fundamentals and therefore by Buckingham's $\pi$ theorem, there will be two dimensionless groups.

Choosing $D$, $N$ and $\rho$ as the recurring set, dimensionally:

$$\left.\begin{array}{l} D \equiv \mathbf{L} \\ N \equiv \mathbf{T}^{-1} \\ \rho \equiv \mathbf{ML}^{-3} \end{array}\right] \quad \text{Thus:} \quad \left[\begin{array}{l} \mathbf{L} \equiv D \\ \mathbf{T} \equiv N^{-1} \\ \mathbf{M} \equiv \rho\mathbf{L}^3 = \rho D^3 \end{array}\right.$$

First group, $\pi_1$, is $P(\mathbf{ML}^2\mathbf{T}^{-3})^{-1} \equiv P(\rho D^3 D^2 N^3)^{-1} \equiv \dfrac{P}{\rho D^5 N^3}$

Second group, $\pi_2$, is $\mu(\mathbf{ML}^{-1}\mathbf{T}^{-1})^{-1} \equiv \mu(\rho D^3 D^{-1} N)^{-1} \equiv \dfrac{\mu}{\rho D^2 N}$

Thus:
$$f\left(\frac{P}{\rho D^5 N^3}, \frac{\mu}{\rho D^2 N}\right) = 0$$

Although there is little to be gained by using this method for simple problems, there is considerable advantage when a large number of groups are involved.

### Problem 1.5

It is found experimentally that the terminal settling velocity $u_0$ of a spherical particle in a fluid is a function of:

particle diameter, $d$; buoyant weight of particle (weight of particle – weight of displaced fluid), $W$; fluid density, $\rho$, and fluid viscosity, $\mu$.

Obtain a relationship for $u_0$ using dimensional analysis.

Stokes established from theoretical considerations that for small particles which settle at very low velocities, the settling velocity is independent of the density of the fluid except in so far as this affects the buoyancy. Show that the settling velocity *must* then be inversely proportional to the viscosity of the fluid.

### Solution

If: $u_0 = k d^a W^b \rho^c \mu^d$, then working in dimensions of **M**, **L** and T:

$$(\mathbf{L}/\mathbf{T}) = k(\mathbf{L}^a(\mathbf{ML}/\mathbf{T}^2)^b(\mathbf{M}/\mathbf{L}^3)^c(\mathbf{M}/\mathbf{LT})^d)$$

Equating dimensions:

$$\begin{array}{ll} \mathbf{M}: & 0 = b + c + d \\ \mathbf{L}: & 1 = a + b - 3c - d \\ \mathbf{T}: & -1 = -2b - d \end{array}$$

Solving in terms of $b$:

$$a = -1, \; c = (b-1), \text{ and } d = (1-2b)$$

$\therefore$ $u_0 = k(1/d)(W^b)(\rho^b/\rho)(\mu/\mu^{2b})$ where $k$ is a constant,

or: $\underline{\underline{u_0 = k(\mu/d\rho)(W\rho/\mu^2)^b}}$

Rearranging:

$$(du_0\rho/\mu) = k(W\rho/\mu^2)^b$$

where $(W\rho/\mu^2)$ is a function of a form of the Reynolds number.

For $u_0$ to be independent of $\rho$, $b$ must equal unity and $u_0 = kW/d\mu$

Thus, for constant diameter and hence buoyant weight, the settling velocity is inversely proportional to the fluid viscosity.

**Problem 1.6**

A drop of liquid spreads over a horizontal surface. What are the factors which will influence:

(a) the rate at which the liquid spreads, and
(b) the final shape of the drop?

Obtain dimensionless groups involving the physical variables in the two cases.

**Solution**

(a) The rate at which a drop spreads, say $R$ m/s, will be influenced by: viscosity of the liquid, $\mu$; volume of the drop, $V$ expressed in terms of $d$, the drop diameter; density of the liquid, $\rho$; acceleration due to gravity, $g$ and possibly, surface tension of the liquid, $\sigma$. In this event: $R = f(\mu, d, \rho, g, \sigma)$. The dimensions of each variable are: $R = \mathbf{L/T}$, $\mu = \mathbf{M/LT}$, $d = \mathbf{L}$, $\rho = \mathbf{M/L^3}$, $g = \mathbf{L/T^2}$, and $\sigma = \mathbf{M/T^2}$. There are 6 variables and 3 fundamentals and hence $(6 - 3) = 3$ dimensionless groups. Taking as the recurring set, $d$, $\rho$ and $g$:

$$d \equiv \mathbf{L}, \qquad \mathbf{L} = d$$
$$\rho \equiv \mathbf{M/L^3} \qquad \therefore\ \mathbf{M} = \rho\mathbf{L}^3 = \rho d^3$$
$$g \equiv \mathbf{L/T^2} \qquad \therefore\ \mathbf{T}^2 = \mathbf{L}/g = d/g \text{ and } \mathbf{T} = d^{0.5}/g^{0.5}$$

Thus, dimensionless group 1: $\mathbf{RT/L} = Rd^{0.5}/dg^{0.5} = R/(dg)^{0.5}$
dimensionless group 2: $\mu\mathbf{LT/M} = \mu d(d^{0.5})/(g^{0.5}\rho d^3) = \mu/(g^{0.5}\rho d^{1.5})$
dimensionless group 3: $\sigma\mathbf{T}^2/\mathbf{M} = \sigma d/(g\rho d^3) = \sigma/(g\rho d^2)$

$$\therefore \qquad R/(dg)^{0.5} = f\left(\frac{\mu}{g^{0.5}\rho d^{1.5}}, \frac{\sigma}{g\rho d^2}\right)$$

or:

$$\frac{R^2}{dg} = f\left(\frac{\mu^2}{g\rho^2 d^3}, \frac{\sigma}{g\rho d^2}\right)$$

(b) The final shape of the drop as indicated by its diameter, $d$, may be obtained by using the argument in (a) and putting $R = 0$. An alternative approach is to assume the final shape of the drop, that is the final diameter attained when the force due to surface tension is equal to that attributable to gravitational force. The variables involved here will be: volume of the drop, $V$; density of the liquid, $\rho$; acceleration due to gravity, $g$, and the surface tension of the liquid, $\sigma$. In this case: $d = f(V, \rho, g, \sigma)$. The dimensions of each variable are: $d = \mathbf{L}$, $V = \mathbf{L}^3$, $\rho = \mathbf{M/L^3}$, $g = \mathbf{L/T^2}$, $\sigma = \mathbf{M/T^2}$. There are 5 variables and 3 fundamentals and hence $(5 - 3) = 2$ dimensionless groups. Taking, as before, $d$, $\rho$ and $g$ as the recurring set, then:

$$d \equiv \mathbf{L}, \qquad \mathbf{L} = d$$
$$\rho \equiv \mathbf{M/L^3} \qquad \therefore\ \mathbf{M} = \rho\mathbf{L}^3 = \rho d^3$$
$$g \equiv \mathbf{L/T^2} \qquad \therefore\ \mathbf{T}^2 = \mathbf{L}/g = d/g \text{ and } \mathbf{T} = d^{0.5}/g^{0.5}$$

Dimensionless group 1: $V/\mathbf{L}^3 = V/d^3$

Dimensionless group 2: $\sigma\mathbf{T}^2/\mathbf{M} = \sigma d/(g\rho d^3) = \sigma/(g\rho d^2)$

$$\therefore \qquad (d^3/V) = f\left(\frac{\sigma}{g\rho d^2}\right)$$

**Problem 1.7**

Liquid is flowing at a volumetric flowrate $Q$ per unit width down a vertical surface. Obtain from dimensional analysis the form of the relationship between flowrate and film thickness. If the flow is streamline, show that the volumetric flowrate is directly proportional to the density of the liquid.

**Solution**

The flowrate, $Q$, will be a function of the fluid density, $\rho$, and viscosity, $\mu$, the film thickness, $d$, and the acceleration due to gravity, $g$,

or: $Q = f(\rho, g, \mu, d)$, or: $Q = K\rho^a g^b \mu^c d^d$ where $K$ is a constant.

The dimensions of each variable are: $Q = \mathbf{L}^2/\mathbf{T}$, $\rho = \mathbf{M}/\mathbf{L}^3$, $g = \mathbf{L}/\mathbf{T}^2$, $\mu = \mathbf{M}/\mathbf{LT}$ and $d = \mathbf{L}$.
$\therefore$ Equating dimensions:

$$\begin{aligned} \mathbf{M}: &\quad 0 = a + c \\ \mathbf{L}: &\quad 2 = -3a + b - c + d \\ \mathbf{T}: &\quad -1 = -2b - c \end{aligned}$$

from which, $c = 1 - 2b$, $a = -c = 2b - 1$, and $d = 2 + 3a - b + c$
$= 2 + 6b - 3 - b + 1 - 2b = 3b$

$$\therefore \qquad Q = K(\rho^{2b-1} \,.\, g^b \,.\, \mu^{1-2b} \,.\, d^{3b})$$

$$\text{or} \qquad \frac{Q\rho}{\mu} = K(\rho^2 g d^3/\mu^2)^b \quad \text{and} \quad Q \propto \mu^{1-2b}.$$

For streamline flow, $Q \propto \mu^{-1}$

$$\therefore \qquad -1 = 1 - 2b \text{ and } b = 1$$

$$\therefore \qquad Q\rho/\mu = K(\rho^2 g d^3/\mu^2),\; Q = K(\rho g d^3/\mu)$$

and Q is directly proportional to the density, $\rho$

**Problem 1.8**

Obtain, by dimensional analysis, a functional relationship for the heat transfer coefficient for forced convection at the inner wall of an annulus through which a cooling liquid is flowing.

**Solution**

Taking the heat transfer coefficient, $h$, as a function of the fluid velocity, density, viscosity, specific heat and thermal conductivity, $u$, $\rho$, $\mu$, $C_p$ and $k$, respectively, and of the inside and outside diameters of the annulus, $d_i$ and $d_0$ respectively, then:

$$h = f(u, d_i, d_0, \rho, \mu, C_p, k)$$

The dimensions of each variable are: $h = \mathbf{H}/\mathbf{L}^2\mathbf{T}\boldsymbol{\theta}$, $u = \mathbf{L}/\mathbf{T}$, $d_i = \mathbf{L}$, $d_0 = \mathbf{L}$, $\rho = \mathbf{M}/\mathbf{L}^3$, $\mu = \mathbf{M}/\mathbf{LT}$, $C_p = \mathbf{H}/\mathbf{M}\boldsymbol{\theta}$, $k = \mathbf{H}/\mathbf{LT}\boldsymbol{\theta}$. There are 8 variables and 5 fundamental dimensions and hence there will be $(8 - 5) = 3$ groups. $\mathbf{H}$ and $\boldsymbol{\theta}$ always appear however as the grouping

$\mathbf{H}/\theta$ and in effect the fundamental dimensions are 4 (**M**, **L**, **T** and $\mathbf{H}/\theta$) and there will be $(8 - 4) = 4$ groups. For the recurring set, the variables $d_i$, $\mu$, $k$ and $\rho$ will be chosen:

$$d_i \equiv \mathbf{L}, \qquad \mathbf{L} = d_i$$
$$\rho \equiv \mathbf{M}/\mathbf{L}^3 \qquad \mathbf{M} = \rho\mathbf{L}^3 = \rho d_i^3$$
$$\mu \equiv \mathbf{M}/\mathbf{LT}, \qquad \mathbf{T} = \mathbf{M}/\mathbf{L}\mu = \rho d_i^3/d_i\mu = \rho d_i^2/\mu$$
$$k \equiv (\mathbf{H}/\boldsymbol{\theta})/\mathbf{LT}, \qquad (\mathbf{H}/\boldsymbol{\theta}) = k\mathbf{LT} = kd_i\rho d_i^2/\mu = k\rho d_i^3/\mu$$

Dimensionless group 1: $h\mathbf{L}^2\mathbf{T}/(\mathbf{H}/\boldsymbol{\theta}) = hd_i^2\rho d_i^2/\mu(k\rho d_i^3/\mu) = hd_i/k$

Dimensionless group 2: $uT/\mathbf{L} = u\rho d_i^2/\mu d_i = d_i u\rho/\mu$

Dimensionless group 3: $d_0/\mathbf{L} = d_0/d_i$

Dimensionless group 4: $C_p\mathbf{M}/(\mathbf{H}/\boldsymbol{\theta}) = C_p\rho d_i^3/k(\rho d_i^3/\mu) = C_p\mu/k$

$\therefore$ $\quad hd_i/k = f(d_i u\rho/\mu,\ C_p\mu/k,\ d_0/d_i)$ which is a form of equation 9.86.

**Problem 1.9**

Obtain by dimensional analysis a functional relationship for the wall heat transfer coefficient for a fluid flowing through a straight pipe of circular cross-section. Assume that the effects of natural convection can be neglected in comparison with those of forced convection.

It is found by experiment that, when the flow is turbulent, increasing the flowrate by a factor of 2 always results in a 50% increase in the coefficient. How would you expect a 50% increase in density of the fluid to affect the coefficient, all other variables remaining constant?

**Solution**

For heat transfer for a fluid flowing through a circular pipe, the dimensional analysis is detailed in Section 9.4.2 and, for forced convection, the heat transfer coefficient at the wall is given by equations 9.57 and 9.53 which may be written as:

$$hd/k = f(du\rho/\mu,\ C_p\mu/k)$$

or

$$hd/k = K(du\rho/\mu)^n(C_p\mu/k)^m$$

$\therefore$

$$h_2/h_1 = (u_2/u_1)^n.$$

Increasing the flowrate by a factor of 2 results in a 50% increase in the coefficient, or:

$$1.5 = 2.0^n \text{ and } n = \ln 1.5/\ln 2.0 = 0.585.$$

Also:

$$h_2/h_1 = (\rho_2/\rho_1)^{0.585}$$

when $(\rho_2/\rho_1) = 1.50$, $h_2/h_1 = (1.50)^{0.585} = 1.27$ and the coefficient is increased by 27%

**Problem 1.10**

A stream of droplets of liquid is formed rapidly at an orifice submerged in a second, immiscible liquid. What physical properties would be expected to influence the mean size of droplet formed? Using dimensional analysis obtain a functional relation between the variables.

**Solution**

The mean droplet size, $d_p$, will be influenced by: diameter of the orifice, $d$; velocity of the liquid, $u$; interfacial tension, $\sigma$; viscosity of the dispersed phase, $\mu$; density of the dispersed phase, $\rho_d$; density of the continuous phase, $\rho_c$, and acceleration due to gravity, $g$. (It would also be acceptable to use the term $(\rho_d - \rho_c)g$ to take account of gravitational forces and there may be some justification in also taking into account the viscosity of the continuous phase.)

On this basis: $$d_p = f(d, u, \sigma, \mu, \rho_d, \rho_c, g)$$

The dimensions of each variable are: $d_p = \mathbf{L}$, $d = \mathbf{L}$, $u = \mathbf{L/T}$, $\sigma = \mathbf{M/T^2}$, $\mu = \mathbf{M/LT}$, $\rho_d = \mathbf{M/L^3}$, $\rho_c = \mathbf{M/L^3}$, and $g = \mathbf{L/T^2}$. There are 7 variables and hence with 3 fundamental dimensions, there will be (T − 3) = 4 dimensionless groups. The variables $d$, $u$ and $\sigma$ will be chosen as the recurring set and:

$$d \equiv \mathbf{L}, \qquad \mathbf{L} = d$$
$$u \equiv \mathbf{L/T}, \qquad \mathbf{T} = \mathbf{L}/u = d/u$$
$$\sigma \equiv \mathbf{M/T^2}, \qquad \mathbf{M} = \sigma\mathbf{T}^2 = \sigma d^2/u^2$$

Thus, dimensionless group 1: $\mu\mathbf{LT/M} = \mu d(d/u)/(\sigma d^2/u^2) = \mu u/\sigma$

dimensionless group 2: $\rho_d\mathbf{L^3/M} = \rho_d d^3/(\sigma d^2/u^2) = \rho_d du^2/\sigma$

dimensionless group 3: $\rho_c\mathbf{L^3/M} = \rho_c d^3/(\sigma d^2/u^2) = \rho_c du^2/\sigma$

dimensionless group 4: $g\mathbf{T^2/L} = g(d^2/u^2)/d = gd/u^2$

and the function becomes: $\underline{\underline{d_p = f(\mu u/\sigma, \rho_d du^2/\sigma, \rho_c du^2/\sigma, gd/u^2)}}$

**Problem 1.11**

Liquid flows under steady-state conditions along an open channel of fixed inclination to the horizontal. On what factors will the depth of liquid in the channel depend? Obtain a relationship between the variables using dimensional analysis.

**Solution**

It seems likely that the depth of liquid, $d$, will depend on: density and viscosity of the liquid, $\rho$ and $\mu$; acceleration due to gravity, $g$; volumetric flowrate per unit width of channel, $Q$, and the angle of inclination, $\theta$,

or: $$d = f(\rho, \mu, g, Q, \theta).$$

Excluding $\theta$ at this stage, there are 5 variables and with 3 fundamental dimensions there will be (5 − 3) = 2 dimensionless groups. The dimensions of each variable are: $d = \mathbf{L}$, $\rho = \mathbf{M/L^3}$, $\mu = \mathbf{M/LT}$, $g = \mathbf{L/T^2}$, $Q = \mathbf{L^2/T}$, and, choosing $Q$, $\rho$ and $g$ as the recurring set:

$$Q = \mathbf{L^2/T} \qquad \mathbf{T} = \mathbf{L}^2/Q$$
$$g = \mathbf{L/T^2} \qquad \mathbf{L} = g\mathbf{T}^2 = g\mathbf{L}^4/Q^2,\ \mathbf{L}^3 = Q^2/g,\ \mathbf{L} = Q^{2/3}/g^{1/3} \text{ and } \mathbf{T} = Q^{4/3}/Qg^{2/3} = Q^{1/3}/g^{2/3}$$
$$\rho = \mathbf{M/L^3} \qquad \mathbf{M} = \mathbf{L}^3\rho = (Q^2/g)\rho = Q^2\rho/g$$

Thus, dimensionless group 1: $d/\mathbf{L} = dg^{1/3}/Q^{2/3}$ or $d^3g/Q^2$

dimensionless group 2: $\mu\mathbf{LT/M} = \mu(Q^{2/3}/g^{1/3})(Q^{1/3}/g^{2/3})/Q^2\rho g = \mu/Q\rho$

and the function becomes: $\underline{\underline{d^3g/Q^2 = f(\mu/Q\rho, \theta)}}$

**Problem 1.12**

Liquid flows down an inclined surface as a film. On what variables will the thickness of the liquid film depend? Obtain the relevant dimensionless groups. It may be assumed that the surface is sufficiently wide for edge effects to be negligible.

**Solution**

This is essentially the same as Problem 1.11, though here a solution will be attempted by equating indices.

If as before: $$d = K(\rho^a, \mu^b, g^c, Q^d, \theta^e)$$

then, excluding $\theta$ at this stage, the dimensions of each variable are: $d = \mathbf{L}$, $\rho = \mathbf{M/L^3}$, $\mu = \mathbf{M/LT}$, $g = \mathbf{L/T^2}$, $Q = \mathbf{L^2/T}$.
Equating dimensions:

$$\mathbf{M}: \quad 0 = a + b \quad \text{(i)}$$
$$\mathbf{L}: \quad 1 = -3a - b + c + 2d \quad \text{(ii)}$$
$$\mathbf{T}: \quad 0 = -b - 2c - d \quad \text{(iii)}$$

Solving in terms of $b$ and $c$,

from (i) $\quad a = -b$

from (iii) $\quad d = -b - 2c$

and in (ii) $\quad 1 = 3b - b + c - 2b - 4c$ or: $c = -1/3 \;\therefore\; d = 2/3 - b$

Thus:
$$d = K(\rho^{-b} \, . \, \mu^b \, . \, g^{-1/3} \, . \, Q^{2/3-b})$$
$$dg^{1/3}/Q^{2/3} = K(\mu/\rho Q)^b$$

$\therefore$
$$\underline{\underline{d^3 g/Q^2 = K(\mu/\rho Q)^b(\theta)^e}} \text{ as before.}$$

**Problem 1.13**

A glass particle settles under the action of gravity in a liquid. Upon which variables would you expect the terminal velocity of the particle to depend? Obtain a relevant dimensionless grouping of the variables. The falling velocity is found to be proportional to the square of the particle diameter when other variables are kept constant. What will be the effect of doubling the viscosity of the liquid? What does this suggest regarding the nature of the flow?

**Solution**

The variables expected to influence the terminal velocity, $u_0$, of a glass particle settling in a liquid are: particle diameter, $d$; density of the particle, $\rho_s$; density of the liquid, $\rho$; viscosity of the liquid, $\mu$ and the acceleration due to gravity, $g$. In this case, $u_0 = f(d, \rho_s, \rho, \mu, g)$. The dimensions of each variable are: $u_0 = \mathbf{L}/"\} \mathbf{T}$, $d = \mathbf{L}$, $\rho_s = \mathbf{M/L^3}$, $\rho = \mathbf{M/L^3}$, $\mu = \mathbf{M/LT}$é and $g = \mathbf{L/T^2}$. With 6 variables and 3 fundamental dimensions, $(6 - 3) = 3$ dimensionless groups are expected. Choosing $d$, $\rho$ and $\mu$ as the recurring set:

$$d \equiv \mathbf{L}, \qquad \mathbf{L} = d$$
$$\rho \equiv \mathbf{M/L^3}, \qquad \mathbf{M} = \rho\mathbf{L}^3 = \rho d^3$$
$$\mu \equiv \mathbf{M/LT}, \qquad \mathbf{T} = \mathbf{M}/\mu\mathbf{L} = \rho d^3/\mu d = \rho d^2/\mu$$

Thus, dimensionless group 1: $u_0\mathbf{T/L} = u_0\rho d^2/(\mu d) = u_0\rho d/\mu$

dimensionless group 2: $\rho_s\mathbf{L^3/M} = \rho_s d^3/(\rho d^3) = \rho_s/\rho$

dimensionless group 3: $g\mathbf{T^2/L} = g\rho^2 d^4/(\mu^2 d) = g\rho^2 d^3/\mu^2$

and:
$$\underline{\underline{(u_0\rho d/\mu) = f(\rho_s/\rho, \; g\rho^2 d^3/\mu^2)}}$$

or:
$$(u_0\rho d/\mu) = K(\rho_s/\rho)^{n_1}(g\rho^2 d^3/\mu^2)^{n_2}$$

When $u_0 \propto d^2$, then $3n_2 - 1 = 2$ and $n_2 = 1$.

$\therefore$ $$(u_0\rho d/\mu) = K(\rho_s/\rho)^{n_1}(g\rho^2 d^3/\mu^2)$$

or: $$u_0 = K(\rho_s/\rho)^{n_1}(d^2\rho g/\mu)$$

and: $$u_0 \propto (1/\mu)$$

In this case, a doubling of the viscosity of the liquid will have the effect of halving the terminal velocity of the particle. This suggests in fact that the flow is described by Stokes' Law.

### Problem 1.14

Heat is transferred from condensing steam to a vertical surface and the resistance to heat transfer is attributable to the thermal resistance of the condensate layer on the surface.

What variables will be expected to affect the film thickness at a point?

Obtain the relevant dimensionless groups.

For streamline flow it is found that the film thickness is proportional to the one third power of the volumetric flowrate per unit width. Show that the heat transfer coefficient would be expected to be inversely proportional to the one third power of viscosity.

### Solution

For a film of liquid flowing down a vertical surface, the variables influencing the film thickness $\delta$, will include: viscosity of the liquid (water), $\mu$; density of the liquid, $\rho$; the flow per unit width of surface, $Q$, and the acceleration due to gravity, $g$. Thus: $\delta = f(\mu, \rho, Q, g)$. The dimensions of each variable are: $\delta = \mathbf{L}$, $\mu = \mathbf{M/LT}$, $\rho = \mathbf{M/L^3}$, $Q = \mathbf{L^2/T}$, and $g = \mathbf{L/T^2}$. Thus with 5 variables and 3 fundamental dimensions, $(5 - 3) = 2$ dimensionless groups are expected. Taking $\mu$, $\rho$ and $g$ as the recurring set, then:

$\mu \equiv \mathbf{M/LT}$, $\mathbf{M} = \mu\mathbf{LT}$

$\rho \equiv \mathbf{M/L^3}$, $\mathbf{M} = \rho\mathbf{L^3}$ $\therefore\ \rho\mathbf{L^3} = \mu\mathbf{LT}$, $\mathbf{T} = \rho\mathbf{L^2}/\mu$

$g \equiv \mathbf{L/T^2} = \mu^2\mathbf{L}/\rho^2\mathbf{L^4} = \mu^2/\rho^2\mathbf{L^3}$ $\therefore\ \mathbf{L^3} = \mu^2/\rho^2 g$ and $\mathbf{L} = \mu^{2/3}/(\rho^{2/3}g^{1/3})$

$\therefore$ $$\mathbf{T} = \rho(\mu^2/\rho^2 g)^{2/3}/\mu = \mu^{1/3}/(\rho^{1/3}g^{2/3})$$

and: $$\mathbf{M} = \mu(\mu^2/\rho^2 g)^{1/3}(\mu^{1/3}/(\rho^{1/3}g^{2/3})) = \mu^2/(\rho g)$$

Thus, dimensionless group 1: $Q\mathbf{T}/\mathbf{L}^2 = Q(\mu^{1/3}/(\rho^{1/3}g^{2/3}))/(\mu^{4/3}/(\rho^{4/3}g^{2/3})) = Q\rho/\mu$

dimensionless group 2: $\delta\mathbf{L} = \delta\mu^{2/3}/(\rho^{2/3}g^{1/3})$ or, cubing $= \delta^3\rho^2 g/\mu^2$

$\therefore$ $$\underline{\underline{(\delta^3\rho^2 g/\mu^2) = f(Q\rho/\mu)}}$$

This may be written: $$(\delta^3\rho^2 g/\mu^2) = K(Q\rho/\mu)^n$$

For streamline flow, $\delta \propto Q^{1/3}$ or $n = 1$

$\therefore$ $$(\delta^3\rho^2 g/\mu^2) = KQ\rho/\mu,\ \delta^3 = KQ\mu/(\rho g) \text{ and } \delta = (KQ\mu/\rho g)^{1/3}$$

As the resistance to heat transfer is attributable to the thermal resistance of the condensate layer which in turn is a function of the film thickness, then: $h \propto k/\delta$ where $k$ is the thermal