

2008

BARRON'S SAT II 数学

Subject Test Math Level 2 2008 8th Edition

Richard Ku, M.A. and Howard P. Dodge, M.A.

考点透析 全真试题
诊断测试 透视强弱项
备考策略 指点高分方案

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Introduction 绪论

The purpose of this book is to help you prepare for the SAT Level 2 Mathematics Subject Test. This book can be used as a self-study guide or as a textbook in a test preparation course. It is a self-contained resource for those who want to achieve their best possible score.

Because the SAT Subject Tests cover specific content, they should be taken as soon as possible after completing the necessary course(s). This means that you should register for the Level 2 Mathematics Subject Test in June after you complete a precalculus course.

You can register for SAT Subject Tests at the College Board's web site, www.collegeboard.com; by calling (866) 756-7346, if you previously registered for an SAT Reasoning Test or Subject Test; or by completing registration forms in the SAT Registration Booklet which can be obtained in your high school guidance office. You can also write to:

College Board SAT Program
P.O. Box 025505
Miami, FL 33102

You may register for up to three Subject Tests at each sitting.

Colleges use SAT Subject Tests to help them make both admission and placement decisions. Because the Subject Tests are not tied to specific curricula, grading procedures, or instructional methods, they provide uniform measures of achievement in various subject areas. This way, colleges can use Subject Test results to compare the achievement of students who come from varying backgrounds and schools.

You can consult college catalogs and web sites to determine which, if any, SAT Subject Tests are required as part of an admissions package. Many "competitive" colleges require the Level 1 Mathematics Test.

If you intend to apply for admission to a college program in mathematics, science, or engineering, you may be required to take the Level 2 Mathematics Subject Test. If you have been generally successful in high school mathematics courses and want to showcase your achievement, you may want to take the Level 2 Subject Test and send your scores to colleges you are interested in even if it isn't required.

NEW IN THE 8TH EDITION 第8版与旧版的不同

The 8th edition is the result of revisions that reflect both the organization and content of the current Level 2 Subject Test. The Review of Major Topics in Part 2 is now grouped into four chapters:

- Functions
- Geometry and Measurement
- Numbers and Operations
- Data Analysis, Statistics, and Probability

These correspond to the four content areas identified by the College Board for both the Level 1 and Level 2 Subject Tests.

Some content that was in earlier editions of this book has been deleted. These include:

- The relation between the zeros and coefficients of cubic and higher-degree polynomials
- Trigonometric formulas for the sum and difference of angles, and half-angle formulas
- Trigonometric form of complex numbers

- DeMoivre's theorem
- Three-dimensional coordinate geometry concepts of trace, direction numbers, and direction cosines

Other content has been added or enhanced:

- Transformations and symmetry
- Counting (e.g., permutations and combinations)
- Complex numbers
- Matrices
- Recursive sequences
- Statistical concepts of standard deviation and linear least squares regression

Also new to the 8th edition are two model tests on CD-ROM that appear in books with a CD-ROM.

STRUCTURE OF THIS BOOK 本书框架

The overall structure of the 8th edition remains essentially the same as its predecessor. A Diagnostic Test in Part 1 follows this introduction. This test will help you quickly identify your weaknesses and gaps in your knowledge of the topics. You should take it under test conditions (in 1 quiet hour), complete the self-evaluation chart at the end, and read the solution explanations for the problems that you got wrong. These explanations include a code for calculator use (which is explained later), the correct answer choice, and the location of the relevant topic in the Part 2 "Review of Major Topics." For your convenience, a self-evaluation chart is also keyed to these locations.

Instructional material on the use of graphing calculators can be found in Part 3. The majority of those taking the Level 2 Mathematics Subject Test are accustomed to using graphing calculators. Where appropriate, explanations of problem solutions are based on their use. (Secondary explanations rely on algebraic techniques, where possible.) The material in Part 3 is designed to fill gaps in the graphing calculator experience of Level 2 test takers.

Part 4 contains six model tests. The breakdown of test items by topic approximately reflects the nominal distribution established by the College Board. The percentage of questions for which calculators are required or useful on the model tests is also approximately the same as that specified by the College Board. The model tests are self-contained. Each has an answer sheet and a complete set of directions. Each test is followed by an answer key, explanations such as those found in the Diagnostic Test, and a self-evaluation chart.

OVERVIEW OF THE LEVEL 2 SUBJECT TEST

Level 2 考试概述

The SAT Mathematics Level 2 Subject Test is one hour in length and consists of 50 multiple-choice questions, each with five answer choices. The test is aimed at students who have had two years of algebra, one year of geometry, and one year of trigonometry and elementary functions. According to the College Board, test items are distributed over topics as follows:

- Numbers and Operation: 5–7 questions
Operations, ratio and proportion, complex numbers, counting, elementary number theory, matrices, sequences, series, and vectors
- Algebra and Functions: 24–26 questions
Work with equations, inequalities, and expressions; know properties of the following classes of functions: linear, polynomial, rational, exponential, logarithmic, trigonometric and inverse trigonometric, periodic, piecewise, recursive, and parametric
- Coordinate Geometry: 5–7 questions
Summarize transformations, coordinate geometry, and coordinate systems

- **Three-dimensional Geometry:** 2–3 questions
Volume and surface area of solids (prisms, cylinders, pyramids, cones, and spheres); coordinates in 3 dimensions
- **Trigonometry:** 6–8 questions
Radian measure; laws of sines and law of cosines; Pythagorean theorem, cofunction, and double-angle identities
- **Data Analysis, Statistics, and Probability:** 3–5 questions
Measures of central tendency and spread; graphs and plots; least squares regression (linear, quadratic, and exponential); probability

CALCULATOR USE 计算器使用须知

As noted earlier, most taking the Level 2 Mathematics Subject Test will use a graphing calculator. In addition to performing the calculations of a scientific calculator, graphing calculators can be used to analyze graphs and to find zeros, points of intersection of graphs, and maxima and minima of functions. Graphing calculators can also be used to find numerical solutions to equations, generate tables of function values, evaluate statistics, and find regression equations. Although a graphing calculator may provide an advantage on certain questions, you should use the calculator with which you are most comfortable.

You should always read a question carefully and decide on a strategy to answer it before deciding whether a calculator is necessary. You may find that you need a calculator only to evaluate some expression that must be determined based solely on your knowledge about how to solve the problem.

Graphing calculators and newer scientific calculators are very user friendly. Because they follow order of operations and expressions can be entered using several levels of parentheses, there is never a need to round and write down the result of an intermediate calculation and then rekey that value as part of another calculation. Premature rounding can result in choosing a wrong answer if the answer choices are close in value.

On the other hand, graphing calculators can be troublesome or even misleading. For example, if you have difficulty finding a window for a graph, perhaps there is a better way to solve a problem. Piecewise functions, functions with restricted domains, and functions having asymptotes provide other examples where the usefulness of a graphing calculator may be limited.

Calculators have popularized a multiple-choice problem-solving technique called backsolving, where answer choices are entered into the problem to see which works. In problems where decimal answer choices are rounded, none of the choices may work satisfactorily. Be careful not to overuse this technique.

Three calculator codes are used in the Diagnostic Test and sample tests in this book. Questions that are best approached without a calculator are labeled inactive (i). These are typically problems that contain only variables or where the computations are very simple. Questions labeled active (a) require the use of a calculator, for example to get decimal approximations for trigonometric ratios or radical expressions. Questions for which a graphing calculator is appropriate are labeled g. Many of these questions can be solved without a graphing calculator, and alternate solutions are given in these cases.

The College Board has established rules governing the use of calculators on the Mathematics Subject Tests:

- You may bring extra batteries or a backup calculator to the test. If you wish, you may bring both scientific and graphing calculators.
- Test centers are not expected to provide calculators, and test takers may not share calculators.
- Notify the test supervisor to have your score cancelled if your calculator malfunctions during the test and you do not have a backup.

TIP

Leave your cell phone at home in our locker, or in your car!

- Certain types of devices that have computational power are **not permitted**: cell phones, pocket organizers, powerbooks and portable handheld computers, and electronic writing pads. Calculators that require an electrical outlet, make noise or “talk,” or use paper tapes are also prohibited.
- You do not have to clear a graphing calculator memory before or after taking the test. However, any attempt to take notes in your calculator about a test and remove it from the room will be grounds for dismissal and cancellation of scores.

HOW THE TEST IS SCORED 如何评分

There are 50 questions on the Math Level 2 Subject Test. Your raw score is the number of correct answers minus one-fourth of the number of incorrect answers, rounded to the nearest whole number. For example, if you get 30 correct answers, 15 incorrect answers, and leave 5 blank, your raw score would be $30 - \frac{1}{4}(15) \approx 26$, rounded to the nearest whole number.

Raw scores are transformed into scaled scores between 200 and 800. The formula for this transformation changes slightly from year to year to reflect varying test difficulty. In recent years, a raw score of 44 was high enough to transform to a scaled score of 800. Each point less in the raw score resulted in approximately 10 points less in the scaled score. For a raw score of 44 or more, the approximate scaled score is 800. For raw scores less than 44, the following formula can be used to get an approximate scaled score on the Diagnostic Test and each model test:

$S = 800 - 10(44 - R)$, where S is the approximate scaled score and R is the rounded raw score less than 44.

The self-evaluation page for the Diagnostic Test and each model test includes spaces for you to calculate your raw score and scaled score.

STRATEGIES TO MAXIMIZE YOUR SCORE 如何取得高分

- **Budget your time.** Although most testing centers have wall clocks, you would be wise to have a watch on your desk. Since there are 50 items on a one-hour test, you have a little over a minute per item. Typically, test items are easier near the beginning of a test, and they get progressively more difficult. Don't linger over difficult questions. Work the problems you are confident of first, and then return later to the ones that are difficult for you.
- **Guess intelligently.** As noted above, you are likely to get a higher score if you can confidently eliminate two or more answer choices, and a lower score if you can't eliminate any.
- **Read the questions carefully.** Answer the question asked, not the one you may have expected. For example, you may have to solve an equation to answer the question, but the solution itself may not be the answer.
- **Mark answers clearly and accurately.** Since you may skip questions that are difficult, be sure to mark the correct number on your answer sheet. If you change an answer, erase cleanly and leave no stray marks. Mark only one answer; an item will be graded as incorrect if more than one answer choice is marked.
- **Change an answer only if you have a good reason for doing so.** It is usually not a good idea to change an answer on the basis of a hunch or whim.
- **As you read a problem, think about possible computational shortcuts to obtain the correct answer choice.** Even though calculators simplify the computational process, you may save time by identifying a pattern that leads to a shortcut.
- **Substitute numbers to determine the nature of a relationship.** If a problem contains only variable quantities, it is sometimes helpful to substitute numbers to understand the relationships implied in the problem.

- **Think carefully about whether to use a calculator.** The College Board's guideline is that a calculator is useful or necessary in about 60% of the problems on the Level 2 Test. An appropriate percentage for you may differ significantly from this, depending on your experience with calculators, especially graphing calculators. Even if you learned the material in a highly calculator-active environment, you may discover that a problem can be done more efficiently without a calculator than with one.
- **Check the answer choices.** If the answer choices are in decimal form, the problem is likely to require the use of a calculator.

STUDY PLANS 学习计划

Your first step is to take the Diagnostic Test. This should be taken under test conditions: timed, quiet, without interruption. Correct the test and identify areas of weakness using the cross-references to the Part 2 review. Use the review to strengthen your understanding of the concepts involved. Check the Part 3 graphing calculator applications to see when this approach might be optimal for you.

Ideally, you would start preparing for the test two to three months in advance. Each week, you would be able to take one sample test, following the same procedure as for the Diagnostic Test. Depending on how well you do, it might take you anywhere between 15 minutes and an hour to complete the work after you take the test. Obviously, if you have less time to prepare, you would have to intensify your efforts to complete the six sample tests, or do fewer of them.

The best way to use Parts 2 and 3 of this book is as reference material. You should look through this material quickly before you take the sample tests, just to get an idea of the range of topics covered and the level of detail. However, these parts of the book are more effectively used after you've taken and corrected a sample test.

PART 1
第一部分

DIAGNOSTIC TEST
诊断测试
