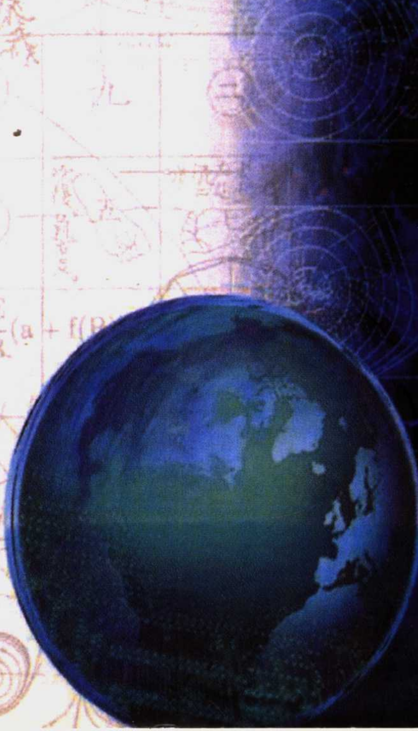
FIELD THEORY

A Path Integral Approach

场论的路径积分方法

Ashok Das



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FIELD THEORY: A PATH INTEGRAL APPROACH

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Introduction

Traditionally, field theory had its main thrust of development in high energy physics. Consequently, the conventional field theory courses are taught with a heavy emphasis on high energy physics. Over the years, however, it has become quite clear that the methods and techniques of field theory are widely applicable in many areas of physics. The canonical quantization methods, which is how conventional field theory courses are taught, do not bring out this feature of field theory. A path integral description of field theory is the appropriate setting for this. It is with this goal in mind, namely, to make graduate students aware of the applicability of the field theoretic methods to various areas, that the Department of Physics and Astronomy at the University of Rochester introduced a new one semester course on field theory in Fall 1991.

This course was aimed at second year graduate students who had already taken a one year course on nonrelativistic quantum mechanics but had not necessarily specialized into any area of physics and these lecture notes grew out of this course which I taught. In fact, the lecture notes are identical to what was covered in the class. Even in the published form, I have endeavored to keep as much of the detailed derivations of various results as I could - the idea being that a reader can then concentrate on the logical development of concepts without worrying about the technical details. Most of the concepts were developed within the context of quantum mechanics - which the students were expected to be familiar with - and subsequently these concepts were applied to various branches of physics. In writing these lecture notes, I have added some references at the end of every chapter. They are only intended to be suggestive. There is so much literature that is available in this subject that it would have been impossible to include all of them. The references are not meant to be complete and I apologize to many whose works I have not cited in the references. Since this was developed as a course for general students, the many interesting topics of gauge theories are also not covered in these lectures. It simply would have been impossible to do justice to these topics within a one semester course.

There are many who were responsible for these lecture notes. I would like to thank our chairman, Paul Slattery, for asking me to teach and design a syllabus for this course. The students deserve the most credit for keeping all the derivations complete and raising many issues which I, otherwise, would have taken for granted. I am grateful to my students Paulo Bedaque and Wen-Jui Huang as well as to Dr. Zhu Yang for straightening out many little details which were essential in presenting the material in a coherent and consistent way. I would also like to thank Michael Begel for helping out in numerous ways, in particular, in computer-generating all the figures in the book. The support of many colleagues was also vital for the completion of these lecture notes. Judy Mack, as always, has done a superb job as far as the appearance of the book is concerned and I sincerely thank her. Finally, I am grateful to Ammani for being there.

Ashok Das,
Rochester

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Chapter 1

Introduction

1.1 Particles and Fields

Classically, there are two kinds of dynamical systems that we encounter. First, there is the motion of a particle or a rigid body (with a finite number of degrees of freedom) which can be described by a finite number of coordinates. And then, there are physical systems where the number of degrees of freedom is nondenumerably (noncountably) infinite. Such systems are described by fields. Familiar examples of classical fields are the electromagnetic fields described by $\vec{E}(\vec{x},t)$ and $\vec{B}(\vec{x},t)$ or equivalently by the potentials $(\phi(\vec{x},t), \vec{A}(\vec{x},t))$. Similarly, the motion of a one-dimensional string is also described by a field $\phi(\vec{x},t)$, namely, the displacement field. Thus, while the coordinates of a particle depend only on time, fields depend continuously on some space variables as well. Therefore, a theory described by fields is usually known as a D+1 dimensional field theory where D represents the number of spatial dimensions on which the field variables depend. For example, a theory describing the displacements of the one-dimensional string would constitute a 1+1 dimensional field theory whereas the more familiar Maxwell's equations (in four dimensions) can be regarded as a 3+1 dimensional field theory. In this language, then, it is clear that a theory describing the motion of a particle can be regarded as a special case, namely, we can think of such a theory as a 0+1 dimensional field theory.

1.2 Metric and Other Notations

In these lectures, we will discuss both nonrelativistic as well as relativistic theories. For the relativistic case, we will use the Bjorken-Drell convention. Namely, the contravariant coordinates are assumed to be

$$x^\mu = (t, \vec{x}) \qquad \mu = 0, 1, 2, 3 \tag{1.1}$$

while the covariant coordinates have the form

$$x_\mu = \eta_{\mu\nu} x^\nu = (t, -\vec{x}) \tag{1.2}$$

Here we have assumed the speed of light to be unity (c=1). The covariant metric can, therefore, be obtained to be diagonal with the signatures

$$\eta_{\mu\nu} = (+, -, -, -) \tag{1.3}$$

The inverse or the contravariant metric clearly also has the same form, namely,

$$\eta^{\mu\nu} = (+, -, -, -) \tag{1.4}$$

The invariant length is given by

$$x^2 = x^\mu x_\mu = \eta^{\mu\nu} x_\mu x_\nu = \eta_{\mu\nu} x^\mu x^\nu = t^2 - \vec{x}^2 \tag{1.5}$$

The gradients are similarly obtained from Eqs. (1.1) and (1.2) to be

$$\partial_\mu = \frac{\partial}{\partial x^\mu} = (\frac{\partial}{\partial t}, \vec{\nabla}) \tag{1.6}$$

$$\partial^\mu = \frac{\partial}{\partial x_\mu} = (\frac{\partial}{\partial t}, -\vec{\nabla}) \tag{1.7}$$

so that the D'Alembertian takes the form

$$\Box = \partial^\mu \partial_\mu = \eta^{\mu\nu} \partial_\mu \partial_\nu = \frac{\partial^2}{\partial t^2} - \vec{\nabla}^2 \tag{1.8}$$

1.3 Functionals

In any case, it is evident that in dealing with dynamical systems, we are dealing with functions of continuous variables. In fact, most of the times, we are really dealing with functions of functions which are otherwise known as functionals. If we are considering the motion of a particle in one dimension in a potential, then the Lagrangian is given by

$$L(x, \dot{x}) = \frac{1}{2}m\dot{x}^2 - V(x) \tag{1.9}$$

where $x(t)$ and $\dot{x}(t)$ denote the coordinate and the velocity of the particle and the simplest functional we can think of is the action functional defined as

$$S[x] = \int_{t_i}^{t_f} dt\; L(x, \dot{x}) \tag{1.10}$$

Note that unlike a function whose value depends on a particular point in the coordinate space, the value of the action depends on the entire trajectory along which the integration is carried out.

Thus, a functional has the generic form

$$F[f] = \int dx\; F(f(x)) \tag{1.11}$$

where, for example, we may have

$$F(f(x)) = (f(x))^n \tag{1.12}$$

Sometimes, one loosely also says that $F(f(x))$ is a functional. The notion of a derivative can be extended to the case of functionals in a natural way through the notion of generalized functions. Thus, one defines the functional derivative or the Gateaux derivative from the linear functional

$$F'[v] = \frac{d}{d\epsilon}F[f+\epsilon v]\bigg|_{\epsilon=0} = \int dx\,\frac{\delta F[f]}{\delta f(x)}\,v(x) \tag{1.13}$$

Equivalently, from the working point of view, this simply corresponds to defining

$$\frac{\delta F(f(x))}{\delta f(y)} = \lim_{\epsilon\to 0}\frac{F(f(x)+\epsilon\delta(x-y)) - F(f(x))}{\epsilon} \tag{1.14}$$

It now follows from Eq. (1.14) that

$$\frac{\delta f(x)}{\delta f(y)} = \delta(x-y) \tag{1.15}$$

The functional derivative satisfies all the properties of a derivative, namely, it is linear and associative,

$$\begin{aligned}\frac{\delta}{\delta f(x)}(F_1[f]+F_2[f]) &= \frac{\delta F_1[f]}{\delta f(x)} + \frac{\delta F_2[f]}{\delta f(x)}\\ \frac{\delta}{\delta f(x)}(F_1[f]F_2[f]) &= \frac{\delta F_1[f]}{\delta f(x)}\,F_2[f] + F_1[f]\,\frac{\delta F_2[f]}{\delta f(x)}\end{aligned} \tag{1.16}$$

It also satisfies the chain rule of differentiation. Furthermore, we now see that given a functional $F[f]$, we can Taylor expand it in the form

$$\begin{aligned}F[f] &= \int dx\; P_0(x) + \int dx_1 dx_2\; P_1(x_1,x_2)\; f(x_2)\\ &\quad + \int dx_1 dx_2 dx_3\; P_2(x_1,x_2,x_3)\; f(x_2)f(x_3) + \cdots\end{aligned} \tag{1.17}$$