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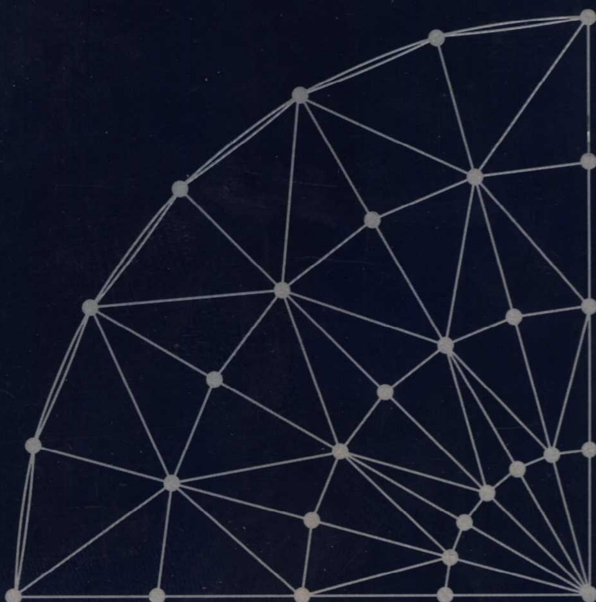
Fundamentals of Finite Element Analysis

有限元分析基础

[美] David V. Hutton 著

武建华　缩编



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缩 编 说 明

《Fundamentals of Finite Element Analysis》是美国华盛顿州立大学 David V. Hutton 教授为大学本科生编写的教材。在浩繁的有限元著作中，本书是为数不多得到了广泛推崇和认可的佳作。全书共 10 章，前 6 章阐述了有限元基本理论，后 4 章为有限元法在不同工程领域中的应用。本书的特点是：

1. 不需要高深的数学力学知识，所用到的预备知识都是大学本科生学过和熟习的，例如线性代数、微积分和常微分方程以及静力学、动力学和材料力学。

2. 从多种角度和层次推导了有限元法的基本理论：有浅显易懂的直接法，有基于卡氏第一定理和最小势能原理的能量法，也有更为普遍的基于加权余量法的伽辽金有限单元法，为学生提供了宽阔的基础，便于有限元法在不同条件、不同问题中的应用。

3. 在重视基本理论的同时，教材还包含了有限元法在热传递、流体力学、固体力学及结构动力学中的实际应用，充分显示了有限元法是一个功能强大的计算工具。

4. 本教材不仅语言流畅，简明易懂，便于中国学生阅读；同时，论述思路清晰，能把复杂难懂的问题用简单的方法解释清楚，还安排了较多的例题来帮助学生掌握其原理和应用。

5. 每章都有小结，概括了本章所解决的问题，并引出下一章的内容。

6. 附录中通过网络提供了有限元 PC 计算机程序(FEPC)及用计算机求解的练习题。

有限元法作为边值问题的近似计算方法，随着计算机的迅猛发展，已广泛地应用于各个领域。该方法作为一门课程也正逐渐从研究生层次向本科生层次过渡，我国许多高校也已在本科生阶段开设了《有限单元法》课程。无论作为双语教学的教材，还是教学参考书，本书都是一个上等的选择。

为了更适于中国学生学习，我们对本书进行了缩编，在保持原书风貌和整体性的原则下，根据中国教学内容结构和专业学习要求，删去了第 7 章在热传递中的应用及第 8 章在流体力学中的应用，同时在附录中删去中国学生在前期课程中学过的矩阵数学和线性代数方程组解法两部分，使教材更加精炼、紧凑，适合中国大学的实际情况。缩编后的内容包括：有限元法的基本概念；刚度矩阵，弹簧与杆单元；桁架结构：直接刚度法；弯曲单元；加权余量法；一般单元列式的插值函数；在固体力学中的应用；结构动力学。共 8 章及三个附录部分。同时，给出了目录的中文翻译和英汉对照词汇表。

为使读者对原书有一个全面的了解，仍保留了英文版的序言。

本教材适用于土木工程、工程力学及机械工程等专业，也可作为工程技术人员的参考书。

限于水平，缩编取舍难免有不妥之处，恳请读者批评指正。

重庆大学　武建华

2006 年 7 月

PREFACE

Fundamentals of Finite Element Analysis is intended to be the text for a senior-level finite element course in engineering programs. The most appropriate major programs are civil engineering, engineering mechanics, and mechanical engineering. The finite element method is such a widely used analysis-and-design technique that it is essential that undergraduate engineering students have a basic knowledge of the theory and applications of the technique. Toward that objective, I developed and taught an undergraduate "special topics" course on the finite element method at Washington State University in the summer of 1992. The course was composed of approximately two-thirds theory and one-third use of commercial software in solving finite element problems. Since that time, the course has become a regularly offered technical elective in the mechanical engineering program and is generally in high demand. During the developmental process for the course, I was never satisfied with any text that was used, and we tried many. I found the available texts to be at one extreme or the other; namely, essentially no theory and all software application, or all theory and no software application. The former approach, in my opinion, represents training in using computer programs, while the latter represents graduate-level study. I have written this text to seek a middle ground.

Pedagogically, I believe that training undergraduate engineering students to use a particular software package without providing knowledge of the underlying theory is a disservice to the student and can be dangerous for their future employers. While I am acutely aware that most engineering programs have a specific finite element software package available for student use, I do not believe that the text the students use should be tied only to that software. Therefore, I have written this text to be software-independent. I emphasize the basic theory of the finite element method, in a context that can be understood by undergraduate engineering students, and leave the software-specific portions to the instructor.

As the text is intended for an undergraduate course, the prerequisites required are statics, dynamics, mechanics of materials, and calculus through ordinary differential equations. Of necessity, partial differential equations are introduced but in a manner that should be understood based on the stated prerequisites. Applications of the finite element method to heat transfer and fluid mechanics are included, but the necessary derivations are such that previous coursework in those topics is not required. Many students will have taken heat transfer and fluid mechanics courses, and the instructor can expand the topics based on the students' background.

Chapter 1 is a general introduction to the finite element method and includes a description of the basic concept of dividing a domain into finite-size subdomains. The finite difference method is introduced for comparison to the finite element method. A general procedure in the sequence of model definition, solution, and interpretation of results is discussed and related to the generally accepted terms of preprocessing, solution, and postprocessing. A brief history of the finite element method is included, as are a few examples illustrating application of the method.

Chapter 2 introduces the concept of a finite element stiffness matrix and associated displacement equation, in terms of interpolation functions, using the linear spring as a finite element. The linear spring is known to most undergraduate students so the mechanics should not be new. However, representation of the spring as a finite element *is* new but provides a simple, concise example of the finite element method. The premise of spring element formulation is extended to the bar element, and energy methods are introduced. The first theorem of Castigliano is applied, as is the principle of minimum potential energy. Castigliano's theorem is a simple method to introduce the undergraduate student to minimum principles without use of variational calculus.

Chapter 3 uses the bar element of Chapter 2 to illustrate assembly of global equilibrium equations for a structure composed of many finite elements. Transformation from element coordinates to global coordinates is developed and illustrated with both two- and three-dimensional examples. The direct stiffness method is utilized and two methods for global matrix assembly are presented. Application of boundary conditions and solution of the resultant constraint equations is discussed. Use of the basic displacement solution to obtain element strain and stress is shown as a postprocessing operation.

Chapter 4 introduces the beam/flexure element as a bridge to continuity requirements for higher-order elements. Slope continuity is introduced and this requires an adjustment to the assumed interpolation functions to insure continuity. Nodal load vectors are discussed in the context of discrete and distributed loads, using the method of work equivalence.

Chapters 2, 3 and 4 introduce the basic procedures of finite-element modeling in the context of simple structural elements that should be well-known to the student from the prerequisite mechanics of materials course. Thus the emphasis in the early part of the course in which the text is used can be on the finite element method without introduction of new physical concepts. The bar and beam elements can be used to give the student practical truss and frame problems for solution using available finite element software. If the instructor is so inclined, the bar and beam elements (in the two-dimensional context) also provide a relatively simple framework for student development of finite element software using basic programming languages.

Chapter 5 is the springboard to more advanced concepts of finite element analysis. The method of weighted residuals is introduced as the fundamental technique used in the remainder of the text. The Galerkin method is utilized exclusively since I have found this method is both understandable for undergraduate students and is amenable to a wide range of engineering problems. The material in this chapter repeats the bar and beam developments and extends the finite element concept to one-dimensional heat transfer. Application to the bar and beam elements illustrates that the method is in agreement with the basic mechanics approach of Chapters 2-4. Introduction of heat transfer exposes the student to additional applications of the finite element method that are, most likely, new to the student.

Chapter 6 is a stand-alone description of the requirements of interpolation functions used in developing finite element models for *any* physical problem. Continuity and completeness requirements are delineated. Natural (serendipity) coordinates, triangular coordinates, and volume coordinates are defined and used to develop interpolation functions for several element types in two- and three-dimensions. The concept of isoparametric mapping is introduced in the context of the plane quadrilateral element. As a precursor to following chapters, numerical integration using Gaussian quadrature is covered and several examples included. The use of two-dimensional elements to model three-dimensional axisymmetric problems is included.

Chapter 7 uses Galerkin's finite element method to develop the finite element equations for several commonly encountered situations in heat transfer. One-, two- and three-dimensional formulations are discussed for conduction and convection. Radiation is not included, as that phenomenon introduces a nonlinearity that undergraduate students are not prepared to deal with at the intended level of the text. Heat transfer with mass transport is included. The finite difference method in conjunction with the finite element method is utilized to present methods of solving time-dependent heat transfer problems.

Chapter 8 introduces finite element applications to fluid mechanics. The general equations governing fluid flow are so complex and nonlinear that the topic is introduced via ideal flow. The stream function and velocity potential function are illustrated and the applicable restrictions noted. Example problems are included that note the analogy with heat transfer and use heat transfer finite element solutions to solve ideal flow problems. A brief discussion of viscous flow shows the nonlinearities that arise when nonideal flows are considered.

Chapter 9 applies the finite element method to problems in solid mechanics with the proviso that the material response is linearly elastic and small deflection. Both plane stress and plane strain are defined and the finite element formulations developed for each case. General three-dimensional states of stress and axisymmetric stress are included. A model for torsion of noncircular sections is developed using the Prandtl stress function. The purpose of the torsion section is to make the student aware that all torsionally loaded objects are not circular and the analysis methods must be adjusted to suit geometry.

Chapter 10 introduces the concept of dynamic motion of structures. It is not presumed that the student has taken a course in mechanical vibrations; as a result, this chapter includes a primer on basic vibration theory. Most of this material is drawn from my previously published text *Applied Mechanical Vibrations*. The concept of the mass or inertia matrix is developed by examples of simple spring-mass systems and then extended to continuous bodies. Both lumped and consistent mass matrices are defined and used in examples. Modal analysis is the basic method espoused for dynamic response; hence, a considerable amount of text material is devoted to determination of natural modes, orthogonality, and modal superposition. Combination of finite difference and finite element methods for solving transient dynamic structural problems is included.

The appendices are included in order to provide the student with material that might be new or may be "rusty" in the student's mind.

Appendix A is a review of matrix algebra and should be known to the student from a course in linear algebra.

Appendix B states the general three-dimensional constitutive relations for a homogeneous, isotropic, elastic material. I have found over the years that undergraduate engineering students do not have a firm grasp of these relations. In general, the student has been exposed to so many special cases that the three-dimensional equations are not truly understood.

Appendix C covers three methods for solving linear algebraic equations. Some students may use this material as an outline for programming solution methods. I include the appendix only so the reader is aware of the algorithms underlying the software he/she will use in solving finite element problems.

Appendix D describes the basic computational capabilities of the FEPC software. The FEPC (FEPfinite element program for the PCpersonal computer) was developed by the late Dr. Charles Knight of Virginia Polytechnic Institute and State University and is used in conjunction with this text with permission of his estate. Dr. Knight's programs allow analysis of two-dimensional programs using bar, beam, and plane stress elements. The appendix describes in general terms the capabilities and limitations of the software. The FEPC program is available to the student at www. mhhe. com/hutton.

Appendix E includes problems for several chapters of the text that should be solved via commercial finite element software. Whether the instructor has available ANSYS, ALGOR, COSMOS, etc. , these problems are oriented to systems having many degrees of freedom and not amenable to hand calculation. Additional problems of this sort will be added to the website on a continuing basis.

The textbook features a Web site (www.mhhe.com/hutton) with finite element analysis links and the FEPC program. At this site, instructors will have access to PowerPoint images and an Instructors' Solutions Manual. Instructors can access these tools by contacting their local McGraw-Hill sales representative for password information.

I thank Raghu Agarwal, Rong Y. Chen, Nels Madsen, Robert L. Rankin, Joseph J. Rencis, Stephen R. Swanson, and Lonny L. Thompson, who reviewed some or all of the manuscript and provided constructive suggestions and criticisms that have helped improve the book.

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David V. Hutton
Pullman, WA

CONTENTS

CHAPTER 1

Basic Concepts of the Finite Element Method

1.1 INTRODUCTION

The finite element method (FEM), sometimes referred to as *finite element analysis* (FEA), is a computational technique used to obtain approximate solutions of boundary value problems in engineering. Simply stated, a boundary value problem is a mathematical problem in which one or more dependent variables must satisfy a differential equation everywhere within a known domain of independent variables and satisfy specific conditions on the boundary of the domain. Boundary value problems are also sometimes called *field* problems. The field is the domain of interest and most often represents a physical structure. The *field variables* are the dependent variables of interest governed by the differential equation. The *boundary conditions* are the specified values of the field variables (or related variables such as derivatives) on the boundaries of the field. Depending on the type of physical problem being analyzed, the field variables may include physical displacement, temperature, heat flux, and fluid velocity to name only a few.

1.2 HOW DOES THE FINITE ELEMENT METHOD WORK?

The general techniques and terminology of finite element analysis will be introduced with reference to Figure 1.1. The figure depicts a volume of some material or materials having known physical properties. The volume represents the domain of a boundary value problem to be solved. For simplicity, at this point, we assume a two-dimensional case with a single field variable $\phi(x, y)$ to be determined at every point $P(x, y)$ such that a known governing equation (or equations) is satisfied exactly at every such point. Note that this implies an exact mathematical solution is obtained; that is, the solution is a closed-form algebraic expression of the independent variables. In practical problems, the domain may be geometrically complex as is, often, the governing equation and the likelihood of obtaining an exact closed-form solution is very low. Therefore, approximate solutions based on numerical techniques and digital computation are most often obtained in engineering analyses of complex problems. Finite element analysis is a powerful technique for obtaining such approximate solutions with good accuracy.

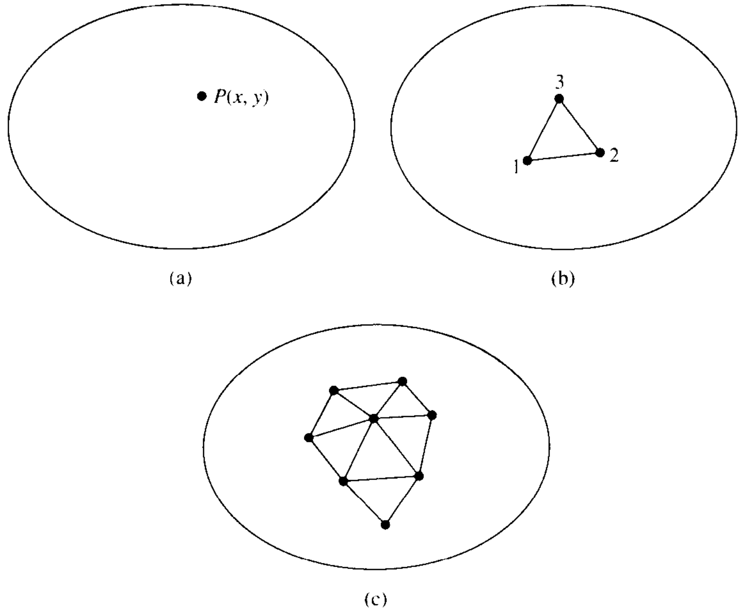


Figure 1.1
(a) A general two-dimensional domain of field variable $\phi(x, y)$.
(b) A three-node finite element defined in the domain.
(c) Additional elements showing a partial finite element mesh of the domain.

A small triangular element that encloses a finite-sized subdomain of the area of interest is shown in Figure 1.1b. That this element is *not* a differential element of size $dx \times dy$ makes this a *finite element.* As we treat this example as a two-dimensional problem, it is assumed that the thickness in the z direction is constant and z dependency is not indicated in the differential equation. The vertices of the triangular element are numbered to indicate that these points are nodes. A *node* is a specific point in the finite element at which the value of the field variable is to be explicitly calculated. *Exterior* nodes are located on the boundaries of the finite element and may be used to connect an element to adjacent finite elements. Nodes that do not lie on element boundaries are *interior* nodes and cannot be connected to any other element. The triangular element of Figure 1.1b has only exterior nodes.

If the values of the field variable are computed only at nodes, how are values obtained at other points within a finite element? The answer contains the crux of the finite element method: The values of the field variable computed at the nodes are used to approximate the values at nonnodal points (that is, in the element interior) by *interpolation* of the nodal values. For the three-node triangle example, the nodes are all exterior and, at any other point within the element, the field variable is described by the approximate relation

$$\phi(x,y) = N_1(x,y)\phi_1 + N_2(x,y)\phi_2 + N_3(x,y)\phi_3 \tag{1.1}$$

where ϕ_1, ϕ_2, and ϕ_3 are the values of the field variable at the nodes, and N_1, N_2, and N_3 are the *interpolation functions*, also known as *shape functions* or *blending functions*. In the finite element approach, the nodal values of the field variable are treated as unknown *constants* that are to be determined. The interpolation functions are most often polynomial forms of the independent variables, derived to satisfy certain required conditions at the nodes. These conditions are discussed in detail in subsequent chapters. The major point to be made here is that the interpolation functions are predetermined, *known* functions of the independent variables; and these functions describe the variation of the field variable within the finite element.

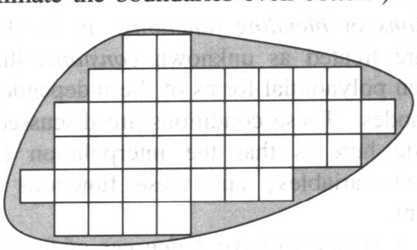
The triangular element described by Equation 1.1 is said to have 3 *degrees of freedom*, as three nodal values of the field variable are required to describe the field variable everywhere in the element. This would be the case if the field variable represents a scalar field, such as temperature in a heat transfer problem. If the domain of Figure 1.1 represents a thin, solid body subjected to plane stress (Chapter 7), the field variable becomes the displacement vector and the values of two components must be computed at each node. In the latter case, the three-node triangular element has 6 degrees of freedom. In general, the number of degrees of freedom associated with a finite element is equal to the product of the number of nodes and the number of values of the field variable (and possibly its derivatives) that must be computed at each node.

How does this element-based approach work over the entire domain of interest? As depicted in Figure 1.1c, every element is connected *at its exterior nodes* to other elements. The finite element equations are formulated such that, at the nodal connections, the value of the field variable at any connection is the same for each element connected to the node. Thus, continuity of the field variable at the nodes is ensured. In fact, finite element formulations are such that continuity of the field variable across interelement boundaries is also ensured. This feature avoids the physically unacceptable possibility of gaps or voids occurring in the domain. In structural problems, such gaps would represent physical separation of the material. In heat transfer, a "gap" would manifest itself in the form of different temperatures at the same physical point.

Although continuity of the field variable from element to element is inherent to the finite element formulation, interelement continuity of gradients (i. e., derivatives) of the field variable does not generally exist. This is a critical observation. In most cases, such derivatives are of more interest than are field variable values. For example, in structural problems, the field variable is displacement but the true interest is more often in strain and stress. As *strain* is defined in terms of first derivatives of displacement components, strain is not continuous across element boundaries. However, the magnitudes of discontinuities of derivatives can be used to assess solution accuracy and convergence as the number of elements is increased, as is illustrated by the following example.

1.2.1 Comparison of Finite Element and Exact Solutions

The process of representing a physical domain with finite elements is referred to as *meshing*, and the resulting set of elements is known as the finite element *mesh*. As most of the commonly used element geometries have straight sides, it is generally impossible to include the entire physical domain in the element mesh if the domain includes curved boundaries. Such a situation is shown in Figure 1.2a, where a curved-boundary domain is meshed (quite coarsely) using square elements. A refined mesh for the same domain is shown in Figure 1.2b, using smaller, more numerous elements of the same type. Note that the refined mesh includes significantly more of the physical domain in the finite element representation and the curved boundaries are more closely approximated. (Triangular elements could approximate the boundaries even better.)


(a)

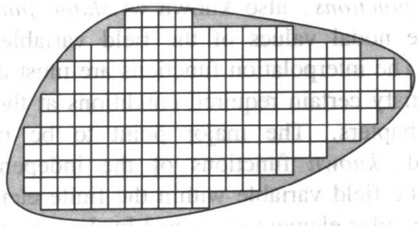

(b)

Figure 1.2
(a) Arbitrary curved-boundary domain modeled using square elements. Stippled areas are not included in the model. A total of 41 elements is shown.
(b) Refined finite element mesh showing reduction of the area not included in the model. A total of 192 elements is shown.

If the interpolation functions satisfy certain mathematical requirements (Chapter 6), a finite element solution for a particular problem converges to the exact solution of the problem. That is, as the number of elements is increased and the physical dimensions of the elements are decreased, the finite element solution changes incrementally. The incremental changes decrease with the mesh refinement process and approach the exact solution asymptotically. To illustrate convergence, we consider a relatively simple problem that has a known solution. Figure 1.3a depicts a tapered, solid cylinder fixed at one end and subjected to a tensile load at the other end. Assuming the displacement at the point of load application to be of interest, a first approximation is obtained by considering the cylinder to be uniform, having a cross-sectional area equal to the average area of the cylinder (Figure 1.3b). The uniform bar is a *link* or *bar* finite element (Chapter 2), so our first approximation is a one-element, finite element model. The solution is obtained using the strength of materials theory. Next, we model the tapered cylinder as two uniform bars in series, as in Figure 1.3c. In the two-element model, each element is of length equal to half the total length of the cylinder and has a cross-sectional area equal to the average area of the corresponding half-length of the cylinder. The mesh refinement is continued using a four-element model, as in Figure 1.3d, and so on. For this simple problem, the displacement of the end of the cylinder for each of the finite element models is as shown in Figure 1.4a, where the dashed line represents the known solution. Convergence of the finite element solutions to the exact solution is clearly indicated. On the other hand, if we plot displacement as a function of position along the length of the cylinder, we can observe convergence as well as the approximate nature of the finite element solutions. Figure 1.4b depicts the exact strength of materials solution and the displacement solution for the four-element models. We note that the displacement variation in each element is a linear approximation to the true nonlinear solution. The linear variation is directly attributable to the fact that the interpolation functions for a bar element are linear. Second, we note that, as the mesh is refined, the displacement solution converges to the nonlinear solution at *every point* in the solution domain.

The previous paragraph discussed convergence of the displacement of the tapered cylinder. As will be seen in Chapter 2, displacement is the primary field variable in structural problems. In most structural problems, however, we are interested primarily in stresses induced by specified loadings. The stresses must be computed via the appropriate stress-strain relations, and the strain components are derived from the displacement field solution. Hence, strains and stresses are referred to as *derived*