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Game Theory
Models, Numerical Methods and Applications

Dario Bauso

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Game Theory: Models, Numerical Methods and Applications

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Game Theory: Models, Numerical Methods and Applications

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Contents

Abstract

Game theory is the theory of "strategic thinking". Developed for military purposes and defense, in the past it has also been used as an alternative and complementary approach to deal with robustness in the presence of worst-case uncertainties or disturbances in many areas such as economics, engineering, computer science, just to name a few. However, game theory is recently gaining ground in systems and control engineering, mostly in engineered systems involving humans, where there is a trend to use game theoretic tools to design protocols that will provide incentives for people to cooperate. For instance, scientists tend to use game theoretic tools to design optimal traffic flows, or predicting or avoiding blackouts in power networks or congestion in cyber-physical networked controlled systems.

Incentives to cooperate are also crucial in dynamic resource allocation, multi-agent systems and social models (including social and economic networks). This paper assembles the material of two graduate courses given at the Department of Engineering Science of the University of Oxford in June-July 2013 and at the Department of Electrical and Electronic Engineering of Imperial College, in October-December 2013. The paper covers the foundations of the theory of noncooperative and cooperative games, both static and dynamic. It also highlights new trends in cooperative differential games, learning, approachability (games with vector payoffs) and mean–field games (large number of homogeneous players). The course emphasizes theoretical foundations, mathematical tools, modeling, and equilibrium notions in different environments.

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1

Introduction

This first chapter is introductory and streamlines the foundations of the theory together with seminal papers and applications. The chapter introduces different types of games, such as simultaneous and sequential games, and the corresponding representations. In addition, it makes a clear distinction between cooperative and noncooperative games. The introduction proceeds with the formalization of fundamental notions like pure and mixed strategy, Nash equilibrium and dominant strategy (strategic/normal representation and extensive/tree representation). In the second part of this chapter, we pinpoint seminal results on the existence of equilibria. The end of the chapter is devoted to the illustration of classical games such as the Cournot duopoly, as an example of infinite game, or other stylized games in strategic form known as the coordination game, the Hawk and Dove game or the Stag-Hunt game. We make use of the Cournot duopoly to briefly discuss the iterated dominance algorithm.

1.1 Historical note, definitions and applications

The foundations of game theory are in the book [von Neumann and Morgenstern, 1944] by the mathematician John Von Neumann and the economist Oskar Morgenstern,

> `Theory of games and economic behavior,`
> `Princeton University Press, 1944.`

The book builds on prior research by von Neumann published in German [von Neumann, 1928]: *Zur Theory der Gesellschaftsspiele, Mathematische Annalen, 1928.* Quoting from [Aumann, 1987], *Morgenstern was the first economist clearly and explicitly to recognize that economic agents must take the interactive nature of economics into account when making their decisions. He and von Neumann met at Princeton in the late Thirties, and started the collaboration that culminated in the Theory of Games.*

Forerunners of the theory are considered the french philosopher and mathematician Antoine Augustin Cournot, who first introduced the "duopoly model" in 1838, and the german economist Heinrich Freiherr von Stackelberg, who formulated the equilibrium concept named after him in 1934 [von Stackelberg, 1934].

Game theory intersects several disciplines, see e.g., Table 1.1, and conventionally involves multiple players each one endowed with its own payoff. Thus, game theory is different from optimization where one has one single player who optimizes its own payoff. Game theory also differs from multi-objective optimization, the latter characterized by one player and multiple payoffs. In the '60s another discipline was founded dealing with multiple decision makers with a common payoff, known as team theory [Ho, 1980, Marschak and Radner, 1972, Bauso and Pesenti, 2012].

The literature provides several formal definitions of game theory. For instance, Maschler, Solan, and Zamir say that game theory is a methodology using *mathematical tools to model and analyze situations involving several decision makers (DMs), called players* [Maschler et al., 2013]. According to Osborne and Rubinstein game theory is *a bag of analytical tools designed to help us understand the phenomena that we*