Tsinghua University Academic Treatise

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清华大学学术专著

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混凝土坝-地基-库水系统的数值模拟

ZHANG CHUHAN et al.

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内 容 简 介

本书内容包括：1. 无限地基的数值模拟；2. 结构-地基动力相互作用与拱坝地震反应；3. 混凝土坝-库水动力相互作用；4. 混凝土坝静动力断裂分析；5. 高坝边坡与地下结构的动力反应五部分，共收入论文 31 篇，是从作者及其合作者在国际上发表的论著中选择具有代表性的论文编辑而成的，它代表了作者及其合作者 20 年来研究成果的结晶，其中在无限边界单元模式的提出，无限地基的模拟，拱坝-地基动力相互作用的时域模型，库水与坝体的动力相互作用模型以及混凝土坝静动力断裂分析方法等方面均有所创新和发展。本书可供水工结构与岩土地基工程师与研究工作者在从事高坝与地基结构静、动力分析与抗震设计研究中作参考并可作为从事有限元、边界元、离散元结构数值分析的科技人员与高等院校有关研究生的参考用书。

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FOREWORD

for 《Numerical Modeling of Concrete Dam—Foundation—Reservoir Systems》

I am pleased to have this opportunity to write the "Foreword" for this volume edited by my friend Zhang Chuhan in celebration of 35 years of interaction in the field of Hydraulic Structures between the Tsinghua University of Beijing and the University of California at Berkeley. This number of years represents the time from when I first met Professor K. T. Zhang at the 1964 Arch Dams Conference in Southampton, England during which he invited me to come to Tsinghua University to give a series of lectures on (a) Earthquake Engineering and (b) Finite Element Analysis of Elastic Solids. However, because my typical projects at Berkeley did not provide sufficient funds for me to travel to Beijing, I was not able to present the lectures until 1980 at which time the "U. S. - China Protocol for Cooperation in Earthquake Studies" was initiated.

Although my presentation of those lectures provided my first opportunity for extended contact with Tsinghua University, it might be said that the interaction between U. C. Berkeley and Tsinghua University began much earlier - when K. T. Zhang arrived in Berkeley in the early 1930's to undertake Graduate Studies on Concrete Dams working under Professor Raymond E. Davis. In any case, a close relationship has been maintained since I met Professor K. T. Zhang for the second time in 1978 when I visited Tsinghua University as a member of the U. S. Earthquake Engineering Delegation sent to China by the U. S. National Science Foundation.

During the subsequent close collaboration of U. C. Berkeley with Tsinghua University and IWHR, we have collaborated on the field test study of the vibration properties and the predicted earthquake performance of four major arch dams in China - Xiang Hong Dian (1984), Quanshui (1984). Dong Jiang (1993), and Longyangxia (1999); in addition the Monticello Arch Dam in California was studied by the two groups in a similar way. Another collaboration of these two groups was the International Arch Dams Conference that they organized at Tsinghua University in 1987. The most recent interaction between these groups was the Symposium held in Beijing in November 2000 to celebrate "Twenty Years of U. S. - China Cooperation in Earthquake Studies". Although this Symposium took account of the major part of Tsinghua - Berkeley interaction, it is evident that our relationship really is much older than twenty years.

Ray W. Clough
Byron and Elvira Nishkian Professor of
Structural Engineering Emeritus
University of California at Berkeley

PREFACE

Numerical modeling and earthquake behavior of the concrete dam-foundation-reservoir system is an important and challenging topic in earthquake engineering and structural dynamics. Today, it becomes even more crucial due to the rapid development of hydropower and dam construction in China. A number of high dams, up to 250 ~ 300m in height, located in hazardously seismic-active regions under design or in pre-construction stages are still in need of much support research. Because of the complexity of multi-coupling features of the problem the topic has received much attention resulting in significant progress in research during the past several decades. Yet many problems remain unsolved.

The Tsinghua Group of Earthquake Resistance on Dams started research on earthquake behavior of concrete dams in the mid-1960s when Xingfengjiang dam experienced severe damage due to a reservoir impounding induced earthquake. A large-scale model test for strengthening purpose was then conducted at the site under the supervision of Professor Zhang Guangdou. In the second stage, from 1978—1983, two research personnel, first Zhang Chuhan and then Wang Guanglun were sent to the University of California at Berkeley as visiting scholars under the supervision of Professor R. W. Clough. Research on seismic cavitation of gravity dams and earthquake response of Xianghong Dian dam were conducted during this period of study. Since mid-1980s this group has carried out a continuous research on arch dam-foundation-reservoir interaction, incorporating the design and feasibility studies of several arch dams in China. Most research was accomplished during the past fifteen years. During this period, the cooperation with Professor O. A. Pekau of Concordia University, Canada starting in 1988 has also resulted in major achievements in the field. Now the most representative works accomplished have been selected and edited into this book. It is hoped that it will serve as a reference for further research in developing numerical modeling methods for dams in earthquake engineering.

Five topics on the subject are presented.

Part I, *Simulation of Infinite Foundations*, presents two numerical schemes for modeling infinite foundation. First, a novel element-infinite boundary element (IBE) is developed and coupled with the normal boundary element (BE) to deal with the half-plane and half-space problems, especially those having irregular boundaries. The essential of the IBE is to use analytical shape functions of exponential type in describing the amplitude attenuation and phase delay behaviors in the infinite directions of the domain. Another innovative scheme in modeling infinite foundation is the development and improvement of infinite element (IE), based on the pioneered work of P. Bettess. Effects of canyon topography and geological conditions on strong ground motion are also examined using FE-IE coupling.

In part Ⅱ, ten papers cover a wide range in *Soil-Structure Interaction and Earthquake Response of Arch Dams*. A time domain procedure of FE-BE-IBE coupling for seismic interaction for arch dams and canyons is first presented to study the effects of canyon radiation damping and nonuniform seismic free-field input on the response of arch dams. Nonlinear modeling of contraction joint opening of arch dams by G. Fenves can be incorporated with the modeling of dam-canyon interaction. There is also a model of joint reinforcement in arch dams for strengthening joint opening control. Detailed analysis of earthquake behaviors for the Ertan, Xiao-wan and Laxiwa arch dams using linear and nonlinear models have been done. In addition, a physical model test for verification of the linear dynamic behavior of the Jinshuitan arch dam is included.

In part Ⅲ, three papers describe some important aspects of *Dam-Reservoir Interaction*. The first paper is related to seismic cavitation effects on gravity dams. It was the first study of this nonlinear dam-reservoir interaction problem using a simplified and finite element procedure. Some interesting phenomena were found from the study. The second paper deals with simulation of reservoir sediments and effects on arch dam response. A visco-elastic model and a multiphase saturated porous model for simulation of wave reflection behaviors of sediments are presented and compared. In addition, analytical solutions for hydro-dynamic pressures on submerged rigid bodies are derived.

In part Ⅳ, seven papers cover the topic *Static and Seismic Fracture of Concrete Dams* by linear and nonlinear simulations. In linear modeling, boundary element technique, fracture mechanics and experimental verification are combined to study the fracture mechanism of 2-D gravity dams. A 3-D simplified through-crack model using boundary elements is also presented for reservoir and temperature loadings. The fracture damage of the Kolnbrein Dam in Austria is examined by the presented 3-D model. Also, some important aspects of linear fracture analysis for orthotropic media using time domain BEM are discussed. In the nonlinear modeling, based on the crack band theory by Bazant, the nonlinear fracture model considering the softening behavior of concrete is employed to analyze the seismic fracture process of Koyna dam during the 1967 Earthquake. An innovative aspect of the procedure is the presentation of a mesh realignment scheme of finite element to accommodate the crack profile development. Experimental tests for softening behavior of rolled compacted concrete (RCC) are conducted and used for prediction of seismic cracking of RCC dams.

In part Ⅴ, five papers deal with *Dynamic Response of Slopes and Underground Structures*, either by time domain BEM or by Discrete Element Method (DEM). By time domain BEM, wave propagation in isotropic and orthotropic medium in infinite rock with cavities are considered and effects of anisotropy on dynamic response of underground structures are examined. By DEM, dynamic stability analysis of high rock slopes in the Three Gorges ship locks are studied in detail. The unloading deformation of the slopes due to excavation and creep behavior is examined and compared with the field measurements.

The work reported herein has been adopted in aseismic design of a number of large dams in China, and thus was awarded a China State Natural Science Prize in 1999. The diversity and depth of the collection should appeal to researchers and engineers in this field for further development of numerical procedures of dams to resist earthquakes.

As a principal investigator, I would like to express our heartfelt gratitude to Professor Zhang Guangdou and Professor R. W. Clough for their guidance in the study. My thanks go to Professor Clough for his encourage ment of including our paper: "Seismic cavitation effects on gravity dams" in this volume. The encouragement and support by Professor Pan Jiazheng is gratefully appreciated. The fruitful cooperation given by Professor Wang Guanglun and Professor O. A. Pekau in the research is gratefully acknowledged. Our former students who made contributions to the research in various aspects are: Drs. Jin Feng, Song Chongmin, Zhao Chongbin, Feng Lingmin, Li Qingbin, Ren Yuntao, Yan Chengda, Dong Yuexing, Lu Jun, Chen Xinfeng, Liu Haixiao, Xu Yanjie and Messrs. Wang Shaomin, Sun Lixiang and Wang Gang. The financial support for the publication of this book provided by the Tsinghua Publishing Foundation is also acknowledged.

ZHANG Chuhan
Tsinghua University
June 2000

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PART Ⅰ

SIMULATION OF INFINITE FOUNDATIONS

BOUNDARY ELEMENT TECHNIQUE IN INFINITE AND SEMI-INFINITE PLANE DOMAIN

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SUMMARY
A modified Kelvin weighting function and infinite boundary element formulations are presented to solve semi-infinite plane domain problems having geometrically irregular boundaries. Results show that much fewer elements need to be prescribed and excellent accuracy can be obtained.

INTRODUCTION

In dealing with stress problems in practical engineering, very often encountered are infinite or semi-infinite plane problems in which unbounded regions or unlimited boundaries need to be considered. In solving problems of this sort, the boundary element method has more advantages when compared with the other numerical techniques. For the infinite plane domain, boundary integrations at infinity are always vanished when the Kelvin fundamental solutions are used. For the semi-infinite plane which has horizontally regular boundaries, Melan and Mindlin[1] solutions can be employed to satisfy the traction-free conditions over the surface of the semi-plane. However, problems in which boundaries are not geometrically regular from near to far field are sometimes encountered. Melan's method is no longer suitable for solving this kind of problem. In this case, either a large artificial boundary needs to be set up or the boundaries are simply truncated at a limited distance, neglecting the far field effects. Some authors presented alternative techniques[2] but the problems are not yet solved satisfactorily. Starting from Kelvin fundamental solutions, this paper discussed the convergent behaviors of the solution at infinity. A modified Kelvin solution was then presented to ensure the convergent conditions at infinity, and then an infinite boundary element formulation was obtained and coupled with the ordinary elements to solve the semi-infinite domain problems. This technique eliminates the integrations at infinity and takes the near to far field effects into account. Examples were given to demonstrate the accuracy of the technique and to show its engineering applications.

ANALYTICAL FORMULATIONS

1. Infinite plane domain

As shown in Figure 1, infinite plane domain is defined as the region enclosed by two

Boundary Elements, Proceedings of International Conference, Beijing, China, Du Qinghua edited,

boundaries Γ_1 and Γ_∞ when r_∞ tends to infinity. Thus, the integral equation has the form:

$$C_{kl}(S)V_l(S) + \left(\int_{\Gamma_1} + \int_{\Gamma_\infty}\right)V_l(Q)P_{kl}^*(Q,S)\mathrm{d}\Gamma = \left(\int_{\Gamma_1} + \int_{\Gamma_\infty}\right)P_l(Q)U_{kl}^*(Q,S)\mathrm{d}\Gamma \tag{1}$$

In which $V_l(Q)$ and $P_l(Q)$ are total displacements (with rigid body motions included) and tractions in l direction at field point Q, $U_{kl}^*(Q,S)$ and $P_{kl}^*(Q,S)$ represent the Kelvin solutions, which have the following characteristics:

$$U_{kl}^*(Q,S) \propto O[\ln r(Q,S)] \tag{2.1}$$

$$P_{kl}^*(Q,S) \propto O\left[\frac{1}{r(Q,S)}\right] \tag{2.2}$$

in which O[ ] represents the asymptotic behavior as $r \to \infty$.

The total displacements $V_l(Q)$ can be obtained by adding the relative displacements $U_l(Q)$ and rigid body displacements $d_l(Q)$ which are related to the selection of the reference point and have the form:

$$d_1(Q) = \omega X_2(Q) + U_1 \tag{3.1}$$

$$d_2(Q) = -\omega X_1(Q) + U_2 \tag{3.2}$$

Where U_1 and U_2 are the translations and ω is the rotation of rigid body motions.

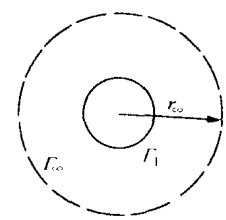

Figure 1. Infinite plane domain

In the following discussion, loading space Ω_p is assumed to be a limited region. Supposing the external resultant forces in X_1 and X_2 directions are equal to zero respectively, we have:

$$U_l(Q) \propto O\left[\frac{1}{r(Q,S_0)}\right] \tag{4.1}$$

$$P_l(Q) \propto O\left[\frac{1}{r^2(Q,S_0)}\right] \tag{4.2}$$

Where S_0 is an arbitrary point inside Ω_p.

If the external resultant forces in X_1 and X_2 directions are not equal to zero respectively, then:

$$U_l(Q) \propto O[\ln r(Q,S_0)] \tag{5.1}$$

$$P_l(Q) \propto O\left[\frac{1}{r(Q,S_0)}\right] \tag{5.2}$$

Based on above relations, we now analyze the integrations on boundary Γ_∞ in equation(1), i.e.

$$\int_{\Gamma_\infty} V_l(Q)P_{kl}^*(Q,S)\mathrm{d}\Gamma \text{ and } \int_{\Gamma_\infty} P_l(Q)U_{kl}^*(Q,S)\mathrm{d}\Gamma$$

As mentioned previously, $V_l(Q)=U_l(Q)+d_l(Q)$ we have:

$$\int_{\Gamma_\infty} V_l(Q)P_{kl}^*(Q,S)\mathrm{d}\Gamma = \int_{\Gamma_\infty} U_l(Q)P_{kl}^*(Q,S)\mathrm{d}\Gamma + \int_{\Gamma_\infty} d_l(Q)P_{kl}^*(Q,S)\mathrm{d}\Gamma \tag{6}$$

In equation (6), if the external resultant forces in X_1 and X_2 directions are equal to zero respectively, considering equations (2.1)(2.2) and (4.1) (4.2) leads to

$$\int_{\Gamma_\infty} U_l(Q)P_{kl}^*(Q,S)\mathrm{d}\Gamma = 0 \tag{7.1}$$

$$\int_{\Gamma_\infty} P_l(Q)U_{kl}^*(Q,S)\mathrm{d}\Gamma = 0 \tag{7.2}$$

If external resultants in X_1 and X_2 directions are not equal to zero respectively, then (7.1) and (7.2) are not satisfied respectively, but one can still prove:

$$\int_{\Gamma_\infty} (U_l(Q)P_{kl}^*(Q,S) - P_l(Q)U_{kl}^*(Q,S))\mathrm{d}\Gamma = 0 \tag{8}$$

Substituting equations (3. 1) (3. 2) into Kelvin solutions yields

$$\int_{\Gamma_\infty} d_l(Q)P_{kl}^*(Q,S)\mathrm{d}\Gamma = -U_k + C_k\omega \tag{9}$$

In which $U_k(k=1,2)$ represents translations of rigid body motion, C_k is a bounded value.

From the above expressions, equation(1) can be written as

$$C_{kl}(S)V_l(S) + \int_{\Gamma_1} V_l(Q)P_{kl}^*(Q,S)\mathrm{d}\Gamma - U_k + C_k\omega = \int_{\Gamma_1} P_l(Q)U_{kl}^*(Q,S)\mathrm{d}\Gamma \tag{10}$$

Equation (10) is the final formulation for infinite plane domain. It can be seen that boundary integrations on Γ_∞ are eliminated and only the boundary Γ_1 needs to be discretized. In calculation, if zero values for rigid body motions are assumed, i. e. $U_k=\omega=0$, then no restraint conditions at certain points need to be prescribed and the rigid body motions will be eliminated automatically.

2. Semi-infinite plane domain

For horizontally irregular boundaries as shown in Figure 2, a new technique is needed to treat this problem because Melan's solutions are no longer suitable. From Figure 2, semi-infinite plane domain may be viewed as the region enclosed by Γ_1, Γ_2, Γ_3 and Γ_∞. When r_∞ tends to infinity, Γ_2 and Γ_3 also extend to infinity. In this case, the integration equation has the form:

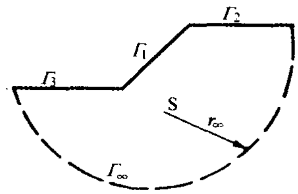

Figure 2. Irregular boundary

$$\begin{aligned} &C_{kl}(S)V_l(S) + \left(\int_{\Gamma_1} + \int_{\Gamma_2} + \int_{\Gamma_3} + \int_{\Gamma_\infty}\right)V_l(Q)P_{kl}^*(Q,S)\mathrm{d}\Gamma \\ &= \left(\int_{\Gamma_1} + \int_{\Gamma_2} + \int_{\Gamma_3} + \int_{\Gamma_\infty}\right)P_l(Q)U_{kl}^*(Q,S)\mathrm{d}\Gamma \end{aligned} \tag{11}$$

The behaviors of displacements and tractions at infinity under external loads described for infinite plane problems are also applicable to semi-infinite plane problems. Suppose the external loads acting on Γ_2 and Γ_3 satisfy:

$$P_l(Q) \propto O(1/r^n(Q,S_0)) \quad (n \geqslant 1) \tag{5. 3}$$

If external resultants in X_1 and X_2 directions are not equal to zero respectively, from (2. 1) (2. 2) (5. 1) (5. 2) integrations:

$$\int_{\Gamma_2} P_l(Q)U_{kl}^*(Q,S)\mathrm{d}\Gamma \text{ and } \int_{\Gamma_3} P_l(Q)U_{kl}^*(Q,S)\mathrm{d}\Gamma \quad (\text{when } n=1);$$

$$\int_{\Gamma_2} V_l(Q)P_{kl}^*(Q,S)\mathrm{d}\Gamma \text{ and } \int_{\Gamma_3} V_l(Q)P_{kl}^*(Q,S)\mathrm{d}\Gamma$$

are divergent at infinity. In this case, difficulties emerge if Kelvin fundamental solutions are still used as the weighting functions. The crux of the problem is that the $\ln r$ term makes the integration not convergent on infinite boundaries. The problem becomes solvable if one can find a modified weighting function which can make the integrations on Γ_∞ vanished and integrations on Γ_2 and Γ_3 convergent.

Based on the idea described above, let us consider an arbitrary point S_0 within the finite region as the reference point. When a unit force is acting at the load point S, another unit force acting in the opposite direction at point S_0 is prescribed to construct a self-equilibrated loading system. Using the displacements and tractions W_{kl}^* and T_{kl}^* caused by this loading system as the modified weighting functions, we have:

$$W^*_{kl} = U^*_{kl}(Q,S) - U^*_{kl}(Q,S_0) \quad (12.1)$$

$$T^*_{kl} = P^*_{kl}(Q,S) - P^*_{kl}(Q,S_0) \quad (12.2)$$

Where W^*_{kl} and T^*_{kl} may be called the modified Kelvin solutions. For this unit force system we have the following relations when the field point Q approaches infinity.

$$W^*_{kl} \propto O\left[\frac{1}{r(Q,S_0)}\right] \quad (13.1)$$

$$T^*_{kl} \propto O\left[\frac{1}{r^2(Q,S_0)}\right] \quad (13.2)$$

From (13.1) (13.2) (5.1) (5.2)

$$\int_{\Gamma_\infty} P_l(Q)W^*_{kl}\,d\Gamma = 0;$$

$$\int_{\Gamma_\infty} V_l(Q)T^*_{kl}\,d\Gamma = C_k\omega$$

$\int_{\Gamma_2,\Gamma_3} P_l(Q)W^*_{kl}\,d\Gamma$ and $\int_{\Gamma_2,\Gamma_3} V_l(Q)T^*_{kl}\,d\Gamma$ are convergent.

Equation (11) can be simplified as:

$$C_{kl}(S)V_l(S) - C_{kl}(S_0)V_l(S_0) + \left(\int_{\Gamma_1} + \int_{\Gamma_2} + \int_{\Gamma_3}\right)V_l(Q)T^*_{kl}\,d\Gamma + C_k\omega$$
$$= \left(\int_{\Gamma_1} + \int_{\Gamma_2} + \int_{\Gamma_3}\right)P_l(Q)W^*_{kl}\,d\Gamma \quad (14)$$

In equation (14), integrations on Γ_∞ are vanished and integrations on Γ_2 and Γ_3 are convergent. Thus, (14) can be solved without difficulty.

For solving equation (14) and evaluating the integrations on Γ_2 and Γ_3 which extend from near field to infinity, a technique called the infinite boundary element was developed by the authors and is briefly described as following:

As shown in Figure 3, in consideration of the nature of $U_l(Q)$ and $P_l(Q)$ at infinity, interpolation functions for coordinates, displacements and tractions in local system are derived.

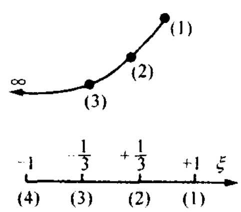

Figure 3. Mapping relationship

(1) Interpolation functions for coordinates:

$$X_l = a_1 + b_1/(1+\zeta) + c_1/(1+\zeta)^2 = \sum_{i=1}^{3}\varphi_i(\zeta)X_l(i) \quad (l = 1,2) \quad (15.1)$$

where

$$\varphi_1(\zeta) = \frac{1}{2}(9\zeta^2 - 1)/(1+\zeta)^2$$
$$\varphi_2(\zeta) = \frac{4}{3}(1-\zeta)(1+3\zeta)/(1+\zeta)^2 \quad (15.2)$$
$$\varphi_3(\zeta) = \frac{1}{6}(1-\zeta)(1-3\zeta)/(1+\zeta)^2$$

(2) Interpolation functions for boundary displacements:

$$U_l = a_2\ln(1+\zeta) + b_2 + c_2(1+\zeta) = \sum_{i=1}^{3}\varphi'_i(\zeta)U_l(i) \quad (l = 1,2) \quad (16.1)$$

where:

$$\varphi'_1(\zeta) = \frac{1}{\ln(4/3)}\left[-\ln 3 - \ln(1+\zeta) + \left(\frac{3}{2}\ln 2\right)(1+\zeta)\right]$$

$$\varphi'_2(\zeta) = \frac{1}{\ln(4/3)}\left[\ln\frac{27}{4} + 2\ln(1+\zeta) - \left(\frac{3}{2}\ln 3\right)(1+\zeta)\right] \quad (16.2)$$

$$\varphi'_3(\zeta) = \frac{1}{\ln(4/3)}\left[\ln\frac{26}{27} - \ln(1+\zeta) + \left(\frac{3}{2}\ln\frac{3}{2}\right)(1+\zeta)\right]$$

(3) Interpolation functions for boundary tractions

$$P_l = a_3(1+\zeta) + b_3(1+\zeta)^2 + c_3(1+\zeta)^3 = \sum_{i=1}^{3}\varphi''_i(\zeta)P_l(i) \quad (l=1,2) \quad (17.1)$$

where

$$\varphi''_1(\zeta) = \frac{9}{16}\left(\zeta-\frac{1}{3}\right)\left(\zeta+\frac{1}{3}\right)(1+\zeta)$$

$$\varphi''_2(\zeta) = \frac{27}{16}(1-\zeta)\left(\zeta+\frac{1}{3}\right)(1+\zeta) \quad (17.2)$$

$$\varphi''_3(\zeta) = \frac{-27}{16}(1-\zeta)\left(\zeta-\frac{1}{3}\right)(1+\zeta)$$

To eliminate the rigid body motions, translations and rotation at point S_0 may be prescribed to be zero. Using infinite boundary elements coupled with ordinary boundary elements, semi-infinite plane domain problems with irregular geometrical conditions can easily be solved.

APPLICATIONS AND EXAMPLES

By using the modified Kelvin solutions and infinite boundary elements(IBEM) developed in this paper, two examples are presented to compare the accuracy with analytical solutions or other techniques and the third one is given to demonstrate its application to real engineering problems.

1. Regular half-plane problem under uniform loading conditions

To compare the accuracy of the technique with exact solution and the ordinary boundary element method (BEM), problems with horizontally regular boundary conditions were chosen. The discretization of IBEM and BEM are shown in Figure 4(a),(b).

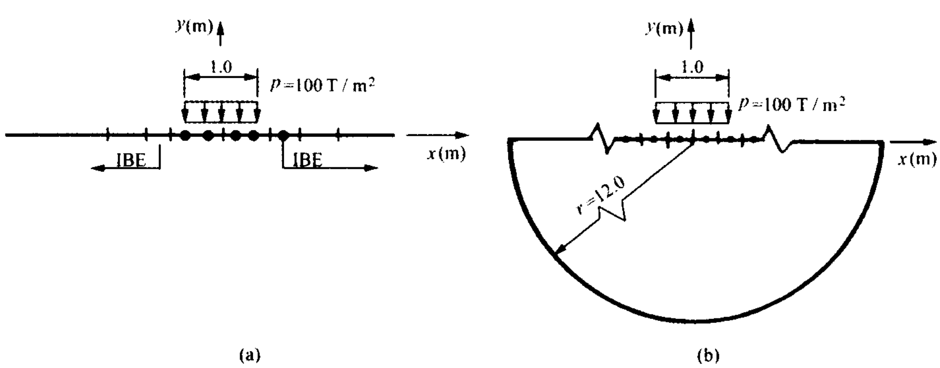

Figure 4. Discretization for half-plane