PILE FOUNDATIONS SETTLEMENT ANALYSIS AND ITS APPLICATION FOR PRACTICE

Zhaoran XIAO

YELLOW RIVER WATER CONSERVANCY PRESS

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内 容 提 要

本书作者在多年对桩—土—台共同作用研究的基础上，提出了一种用于单桩、群桩非线性分析的半解析半数值的方法，该方法尤其适应于大型群桩的桩—土—台共同工作的非线性分析，是一种极具应用前景的新方法。针对桩—土界面处的非线性特性，建立了用于分析单桩、群桩荷载位移的传递函数，用于模拟单桩和群桩的荷载传递的非线性特性，并同时提出了有效可行的反分析方法，用于确定桩—土界面处的非线性特性土层参数。在试验研究的基础上，探讨了桩—土—台共同作用的内在本质和规律，研究桩周土、桩端土的力学参数和桩的布置方式对桩—土—台共同作用的影响。本书可供有关科研、设计和工程部门的科技工作者参考，亦可作为高等院校岩土工程专业研究生的教学参考书。

图书在版编目(CIP)数据

桩基础沉降分析及其工程应用/肖昭然编著．
—郑州：黄河水利出版社，2002.9
ISBN 7-80621-600-6

Ⅰ．桩…　Ⅱ．肖…　Ⅲ．桩基础-沉降分析法
Ⅳ．TU473.1

中国版本图书馆 CIP 数据核字(2002)第 067871 号

责任编辑　马广州　　　　封面设计　谢　萍
责任校对　张晓霞　　　　责任监制　常红昕

出 版 社：黄河水利出版社
地址：河南省郑州市金水路 11 号　　邮政编码：450003
发行单位：黄河水利出版社
发行部电话及传真：0371-6022620
E-mail：yrcp@public2.zz.ha.cn
承印单位：黄河水利委员会印刷厂
开本：787 mm×1 092 mm　1/16
印张：8.75
字数：200 千字　　印数：1—1 100
版次：2002 年 9 月第 1 版　　印次：2002 年 9 月第 1 次印刷

书号：ISBN 7-80621-600-6/TU·26　　定价：25.00 元

Contents

Chapter 1 Introduction....................1
1.1 Background and Motivation....................1
1.2 Scope and Objectives....................2
1.3 Organization of the Book....................2
Chapter 2 Literature Review....................4
2.1 Introduction....................4
2.2 Vertically Loaded Single Pile....................5
2.3 Vertically Loaded Pile-Group....................12
2.4 Summary....................23
Chapter3 A New Analytical Model for Settlement Analysis of Single Pile in Multi-layered Soil....................25
3.1 Introduction....................25
3.2 Mathematical Formulation and Solution Technique of the New Model....................26
3.3 Comparison with the Finite Element Method and Observed Results....................33
3.4 Summary....................39
Chapter 4 Simplified Nonlinear Modeling of Vertically Loaded Single Pile....................42
4.1 Introduction....................42
4.2 Nonlinear Transfer Function for Single Friction Piles....................43
4.3 Algorithm of Load-settlement Analysis of Single Piles by Using the Transfer Function and Verification....................54
4.4 The Basic Factors Affecting the Shaft Load Transfer Relationship....................62
4.5 Determination of Parameters for the Proposed Load-Transfer Function Back-analysis Method....................68
4.6 Determination of K_{si} Parameters and Correlationship with Soil Conditions an Example from Shanghai Test Results....................74
4.7 Load Transfer Behavior of Single Piles....................82
4.8 Summary....................88
Chapter5 A Simplified Nonlinear Approach for Pile-group Analysis and the Behaviours of Large Pile-groups under Vertical Loads....................89
5.1 Introduction....................89
5.2 Developing a Load-Transfer Function for Pile-groups....................90
5.3 Algorithm for Load-settlement Analysis of Pile-group by the Proposed Transfer Function....................96

5.4 Comparison with Boundary Integral Solutions and Elastic Approximate Approach and Field Measured Results......103
5.5 Results of Parametric Study......111
5.6 Summary......125
References......128

Chapter 1 Introduction

1.1 Background and Motivation

Pile foundations are used extensively around the world to support both inland and offshore structures including important structures such as nuclear plants and oil-drilling platforms. They are used mainly at construction sites, where the presence of soft soil layers would cause excessive deformation or failure of more conventional types of foundations. Piles can be used as isolated units in supporting a single footing or, most commonly, in groups with variable configurations and pile-to-pile spacing, in order to increase the total load carrying capacity of the foundation. The piles in a group may behave as isolated ones (large spacing), or they may interact significantly with each other (small spacing) through the interconnecting soil, leading to a group behaviour that is substantially different from that of an aggregate of individual piles. For a pile-group, in practice, the pile cap usually is constructed as contacting with ground soil or free standing pile-group. For the former pile-group type, a significant proportion of the structural load may then be transmitted directly from the pile raft to the ground.

Many numerical approaches and various closed-form solutions have been proposed for analysis of single piles and, more particularly, for pile-groups, and for pile-soil-raft interaction. However, for nonlinear analysis of a larger pile-group, it is rarely practicable, and in many cases impossible. The use of rigorous numerical analysis, due to limitations in computing capacity, and time and cost constraints, which make the use of rigorous numerical methods for nonlinear analysis of large pile-group very difficult, if not impossible at the moment. Therefore a number of hybrid load-transfer numerical approaches have been proposed, which take advantage of the strength of numerical and analytical solutions to produce a complete numerical analysis. Such approaches are generally more efficient than other methods currently available. These approaches are based on the superposition assumption in that the displacement is due to the shaft shear stress developed in the soils which is exactly the same as the displacement in the pile. This assumption could lead to overestimation of the corner pile load and underestimation of the center pile load and makes the load distribution among the piles highly nonuniform. Furthermore, this assumption may result in the total stiffness of the pile-group system being significantly lower than the actual stiffness of the pile-group system. It is necessary to propose an efficient approach, which is based on a nonlinear relationship between the pile shaft shear stress and the local shear displacement, to analysis the pile-soil-raft system.

1.2 Scope and Objectives

Due to the large variations in geometries, the construction method, loading conditions and soil properties associated with pile foundations, it is demonstrated that the behaviour of a single pile or pile-group is affected by these factors. Existing methods of analysis for single pile and pile-groups are generally based on several simplifying assumptions, such as perfect bonding between pile and soil, linear elastic and homogeneous soil, etc. In this research work, it is intended to relax many of such simplifying assumptions in modeling of single pile and pile-groups with the focus on the following factors: the interaction between the pile and soil, the interaction between pile to pile in the pile-groups, the interaction between the ground reaction pressure generated under the raft with surrounding piles.

The aims of this research, therefore, are to achieve the following:

1. To develop an analytical solution for single piles constructed in the multilayered soil profile under vertical loading based on the variational model.
2. To develop a transfer function and corresponding algorithm for the analysis of nonlinear behaviour of single piles. At the same time, a back-analysis method is proposed for determination of the model parameters based on field test results.
3. To develop a transfer function for a pile-group, which can consider all the pile to pile interaction among the pile-group, and to develop the corresponding algorithm.
4. To develop a load-transfer function to model the pile-soil-raft interaction of piled-raft systems.

The nonlinear behaviours of single piles and pile-group, pile-groups, and rigidly capped piled-raft systems will be examined.

1.3 Organization of the Book

The present book consists of the following:

1. The current existing techniques used for analysing single piles, pile-groups, and pile-soil-raft interaction are reviewed in chapter 2, which covers the performance of a single pile subjected to vertical loading and a pile-group subjected to vertical loading, with particular attention being paid to pile-soil-raft interaction.
2. In chapter 3, an extension of the variational model proposed by Vallabhan and Mustafa (1996) for the settlement analysis of an axially loaded pile embedded in a multi-layered soil profile is presented. Comparisons were made with the results of finite element analysis and field pile-load tests.
3. In chapter 4, two theoretical models were adopted to form a nonlinear load-transfer function for vertical loaded single piles. Results of analysis are presented for the nonlinear load-displacement behavior of single piles embedded in multilayered soils. A back-analysis method is proposed to determine the required model parameters based on the field pile-load test results.

4. In chapter 5, a simple analytical approach is proposed for the nonlinear analysis of load-transfer and load-displacement responses of a very large pile-group, whose raft is not contacted with ground soil. Based on the proposed load-transfer function for the pile-group, a numerical algorithm and a computer program for nonlinear analysis of pile-groups is developed and compared with well documented field pile-group test results. Factors affecting the pile-group behaviour will be explored.

Chapter 2 Literature Review

2.1 Introduction

Although pile foundations have been used extensively for many years to support a great variety of structures, their behaviour always cannot be well understood, and there is not a great deal in the way of guidance to the structural engineer on methods of analysis and design. This situation is not altogether surprising because piles and piled rafts are one of the most complex of all structural systems, and there are many different facets to the problem. Of particular interest are the load-settlement behaviour, the development of pile shaft and pile base capacity under the static load, and the distribution of pile load and soil load.

In their classic treatise on "Soil Mechanics in Engineering Practice: Terzaghi and Peck (1948) discuss the use of the pile load tests as a means to predict the pile settlement of large foundations. They conclude that such a prediction is unreliable, mainly because of the heterogeneities of the subsoil.

They also discuss the similar reasons why the settlement of a large piled foundation can differ significantly from that inferred from the settlement of a single pile, and present some examples to substantiate this settlement.

They finally state that: "any attempts to establish rules for the design of pile foundations necessarily involves radical simplifications, and the rules themselves are useful only as guides to judgments. For the same reasons, theoretical methods are useful only as guide to judgments. For the same reasons, theoretical refinements in dealing with pile problems, such as attempts to compute the distribution of load among the piles in a group by means of the theory of elasticity, are completely out of place and can be safely ignored. Even conclusions based on the results of small scale model tests may be far from reliable" .

Concerning the prediction of the settlement, the pessimistic opinion expressed by Terzaghi and Peck is still shared by many authoritative people, who gave substantial contributions to the understanding of the behaviour of piled foundations.

In his Rankine Lecture, Poulos (1989) states that: "the general tendency with all conventional methods of group analysis is to overestimate the interaction and to predict a settlement, which is too large".

The following literature review describes briefly the types of currently available methods for the analysis of pile foundations. It also points out areas where additional research is needed. Methods of analysis concerning the single piles, which subject to a vertical load, are reviewed first, and they are followed by a review of the available procedures for analyzing pile groups. Available reports on carefully conducted field tests on single piles and pile groups are listed separately, so that they can be used as a database for the evaluation of predictions by analytical models.

Analysis of piles can be broadly classified into (1) empirical methods, (2) numerical methods, (3) closed form solutions, and (4) a combination of these methods, (e.g. the hybrid method, which is a combination of (2) and (3)). Empirical methods and numerical approaches have been widely proposed, developed and refined.

2.2 Vertically Loaded Single Pile

The function of a pile foundation, as that of any other foundation, is to transmit the load of the superstructure to the foundation in a manner that will avoid objectionable settlement. While in practice piles are generally utilized in group-piles, it is essential first to understand the behavior of a single pile. The load transfer behavior for a single pile is of fundamental importance in understanding pile group behavior. Thus, the study of load transfer from an axially loaded pile to the surrounding soil is an important aspect of pile foundation engineering.

The problems of the load-settlement and load-transfer characteristics of friction piles and the computation of stresses in the surrounding soils have received considerable attention for almost half a century. The prediction of the response of single friction piles to axial loading involves an analysis of soil-structure interaction problems. Under ideal circumstances, a designer would be able to predict an entire load-settlement curve as well as load-transfer from pile to soil as a function of pile depth. Even now, several aspects of the behaviour of pile foundations remain poorly understood, for example, the non-linear behavior of pile load-transfer in multi-layered soils, how shaft friction is mobilized as the pile load increases, and the effects of pile geometry. Among the issues that need to be addressed are nonlinear behavior of the soils surrounding the piles, the interface between piles and soils, and even more importantly, the nature and magnitude of pile-soil-pile interaction under realistic conditions.

2.2.1 Empirical Correlations

Early research on single piles was directed mainly towards estimating their static load carrying capacity, implicitly assuming that the deformations will be acceptable if an appropriate safety factor is used in determining the allowable range of loads. The axial load carrying capacity of single piles is still determined in practice as the sum of the shaft resistance, due to the friction developed at the pile-soil interface, plus the tip resistance; both resistances are calculated by empirical functions of soil properties such as the undrained shear strength for cohesive soils and the friction angle for cohesionless soils, and the overburden pressure. The resultant resistances vary over a wide range.

Traditional methods of calculating the settlement of a pile rely on either an arbitrary assumption of the stress distribution along the pile and the use of conventional one-dimensional theory (Terzaghi, 1943), or on empirical correlations. Typical of these correlations are those proposed by Meyerhof (1959) for piles in sand and Focht (1967) for piles in clay. From an analysis of a number of load tests, Meyerhof (1959) has suggested that *at loads less than about* one third of the ultimate, the settlement of ρ of a pile could be estimated as follows (provided that no softer layers exist beneath the pile):

$$\rho = \frac{d_b}{30F} \tag{2.1}$$

where d_b is diameter of pile base and F is factor of safety (>3) on ultimate load;

Focht (1967) has examined data from a number of load tests and has related the observed settlement ρ, at the working load to the computed column deformation ρ_{col} at the working load. Focht (1967) has defined a "movement ratio" as ρ/ρ_{col}, and has found that for relatively long highly stressed piles having ρ_{col} >8mm, the movement ratio is on the order of 0.5, whereas for relatively rigid piles, having ρ_{col} <8mm, the movement ratio is larger, on the order of 1.0.

2.2.2 Load-Transfer Analysis

With the advent of computers, more-sophisticated methods of analysis have been developed to predict the settlement and load distribution in a single pile. The load-transfer analysis method is used to analyze the behavior of a single pile. This method is an uncoupled approach that treats the shaft and base as independent elastic springs. The behavior of the springs can be modeled as linear, nonlinear, or perfectly plastic spring and can be obtained by either empirical or theoretical relationships.

2.2.2.1 Experiment Load-transfer Curves

Seed and Reese (1957) proposed, what is now considered a classical work in the field, the load transfer mechanism for piles in soft clay by means of an empirical non-linear relation, called *t-z* curve, where *t* represents the shear stress transferred to the soil at the corresponding *z*-displacement of the pile. This method was suggested in practice by the American Petroleum institute for the design of offshore pile foundations (API 1986). In order to produce a complete load-displacement curve for a single pile, a finite difference approach was employed which involved dividing the pile into a number of cylindrical elements and applying displacement increments to the pile toe. Under each toe displacement increment, the displacements of the soil were iterated until they matched the elastic shortening of the pile over each element. Coyle & Reese (1966) used the results of Seed & Reese (1957) to compile a curve relating pile movement to the ratio of load transfer to soil shear strength (Fig.2.1), as determined from unconfined compression tests. This normalised curve was applicable at all points down the pile. However, they then noted that a single curve was not universally applicable and proposed a family of curves, which defined the load transfer behaviour of a clay soil depending on its stiffness. Coyle & Sulaiman (1967) correlated laboratory and field data to develop an analogous approach for steel piles in sand, using curves, which related skin friction to pile movement for different soil conditions. Various other researchers, such as Reese and O'Neill (1981), using data available from field pile-load tests attempted to modify these empirical *t-z* curves. They determined the *t-z* curve characteristics, which fit the measured data. These in turn are related to the undrained shear strengths of the soils.

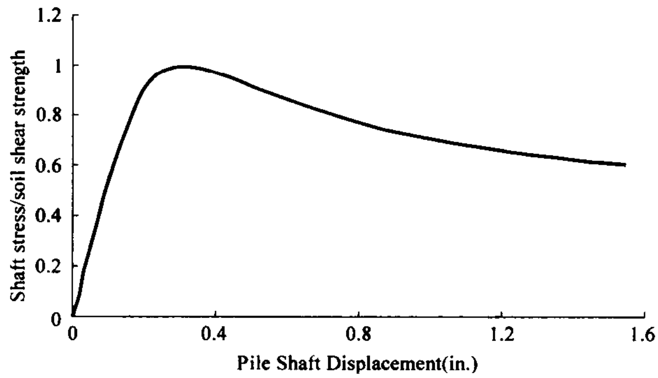

Figure 2.1 Typical shear stress vs. pile movement curve (after Coyle and Reese, 1966)

2.2.2.2 The Theoretical Load-transfer Curves

For the transfer method analysis, another way to determine the load transfer (*t*-*z*) curve was conducted by theoretical method. The first theoretical load transfer curve was developed by Reese *et al* (1969), which was based on the field testing of a bored pile. Computation of the load transfer at various depths was performed in a similar manner to that proposed by Coyle & Reese (1966), and an expression for the shape of the load transfer response was calculated by curve-fitting. The load transfer relationship depended on the pile geometry, and the results of unconfined compression tests and the standard penetration test. The following relationship between the load (adhesion) and pile movement was developed:

$$\tau_{az} = \tau_{a\max}.[2.0\sqrt{\frac{\rho}{s_o}} - (\frac{\rho}{s_o})] \qquad (2.2)$$

where τ_{az} is adhesion at depth z (tons/ft^2), τ_{amax} is maximum adhesion that can occur at any depth (tons/ft^2), ρ is downward movement of pile at depth z (in.), $s_o=2d\varepsilon$, d is the pile diameter, ε is average failure strain, in percent, obtained from stress-strain curves for unconfined compression tests run on soil samples near the pile tip.

Randolph & Wroth (1978) proposed an analytical approach to derive a closed form solution for the gradient of the load transfer curve in a linearly elastic soil (Eq.2.1). A major advantage of this relationship over the experimentally derived curve of Seed & Reese (1957) is that the pile head movement was defined in non-dimensional form. This greatly extended the applicability of the load-transfer method:

$$\frac{P_t}{G_L r_o w_t} = \frac{\left[\frac{4}{\eta(1-v_s)} + \frac{2\pi\rho_g L\tanh(\beta)}{\zeta r_o \beta}\right]}{\left[1 + \frac{4L\tanh(\beta)}{\pi\lambda(1-v_s)\pi\lambda r_o\beta}\right]} \qquad (2.3)$$

where, P_t is the applied load, w_t is the pile head displacement, L, r_o is pile length and pile radius, respectively, G_L is the shear modulus at depth z, v_s is the soil Poisson's ratio, respectively, η is coefficient to allow for embedment depth of pile base E_p/G_L, $\zeta = r_m/r_o$, r_m is the radius at which shear stress becomes negligible (r_m = 2.5l(1-ν)), $\beta = \sqrt{2/\zeta\lambda}L/r_o$, E_p is pile Young's modulus.

Other load-transfer functions proposed by other researcher are summarized in Table 2-1 and 2-2 for axial pile analysis. The coefficients governing these functions are adjusted to match the measured data.

Table 2.1 Shaft transfer function

Formulations	Explanation
Kezdi (1957) $\tau = \sigma \tan\varphi \left[1-\exp\left(-k\frac{w}{w_e - w}\right)\right]$ Non-linear elastic τ-w relation	τ is the shaft shear stress required for producing a displacement w at normal stress σ, φ is the angle of full shearing resistance; w_e is the shear displacement necessary for the development of full friction; k is the initial tangent of the τ/w versus w curve. The maximum ratio of τ/σ is equal to tanφ
Reese *et al.* (1969) $\tau = k\left[2\left(\sqrt{\frac{w}{w_e}} - \frac{w}{w_e}\right)\right]$ Non-linear elastic τ-w relation	τ is the shaft stress, k is the stress transfer factor, k=2.74N(kPa), N is the number of SPT value, $w_e = 2d\varepsilon_f$, ε_f is the average failure strain (%), obtained from unconfined compression tests on soil sample near the pile tip
Fujita (1976) $\frac{\tau}{w} = 4\bar{N}$ $(w \le w_e)$ $\tau = \tau_f$ $(w \le w_e)$ Ideal elastic-plastic τ-w curve	τ/w is the gradient of the load transfer curve, kPa/cm, $\bar{N}$ is the average N values, w_e is the shaft displacement at the transition depth from elastic to plastic stage, $\tau_f = 13\bar{N}$, maximum local stress, kPa.
Armaleh and Desai (1987) $\tau = \frac{(k_{os} - k_{fs})w}{M_s} + k_{fs}w$ where $M_s = \left(1 + \left\lvert\frac{(k_{os} - k_{fs})w}{p_{fs}}\right\rvert^{m_s}\right)^{1/m_s}$	K_{os}, k_{fs} are initial and final spring stiffness, respectively, $p_{fs} = K_h\sigma_v'\tan\varphi$, load at yield point, which equals to $\tau_{max}(z)$, K_h is the coefficient of earth pressure, σ_v' is effective normal stress and m_s is the order of the curve, taken as unity, $k_{fs}(z) = 0.005k_{os}(z)$, z is the depth below ground surface.
Hirayma (1990) $\frac{\tau}{w} = \frac{1}{(a_s + b_s w)}$ Hyperbolic τ-w curve	$a_s = 0.0025/\tau_f$, $b_s = 1/\tau_f$, where τ_f is correlation to SPT or CPT value for bored pile. For sand, τ_f = 10N or s_u ($\le$ 150)
Randolph and Worth (1978), Kraft *et al.* (1981)	Theoretical load transfer approach (Eq.(2.3))

These methods listing in the Table 2-1 and 2-2 yield good prediction of pile behavior in cohesive soil. However, the following theoretical and practical limitations should be noted: (a) In using the load-transfer curves, it is inherently assumed that the movement of the pile at any point is related only to the shear stress at that point and is independent of the stresses elsewhere on the pile, i.e. this method essentially is based on the classical Winkler model. This inherent assumption is equivalent to that made when the theory of subgrade reaction is used to analyze laterally loaded piles. Thus, no proper account is taken of the continuity of the soil mass. (b) The load-transfer method, because of its inherent disregard for continuity of the soil, is not suitable for analyzing load-settlement characteristics of pile groups. (c) To obtain the load transfer curves, extensive instrumentation of a pile subjected to a load test is required. Furthermore, extrapolation of test data from one site to another is not always successful.

Table 2.2 Base Transfer Function

Formulations	Explanation
Fujita (1976) $P_b = A_b k_s W_b^n$, For linear case, n=1 , for nonlinear case, n=0.5 (1) Linear case k_s=80 $\bar{N}_b^{1.5}$, for nonlinear case k_s = 80 $\bar{N}_b$, (2) Linear case, k_s =100 $\bar{N}_b$, for nonlinear case, k_s = $\bar{N}_b^{1.5}$	A_b = base area, k_s is pile tip resistant factor, kPa/cm, for nonlinear case, kPa/cm$^{0.5}$, N_b is average N values for 3 meter above the tip. When shaft stiffness factor is correlated with the average SPT value over the pile, formulas shown in (1) factor is relate to pile length, formulas I (2) should be used.
Armaleh and Desai (1987) $P_b = \frac{(k_{ob}-k_{fb})w_b}{\left(1+\left\|\frac{(k_{ob}-k_{fb})w_b}{P_{fb}}\right\|^{m_b}\right)^{1/m_b}}$ where $k_{ob} = \lambda_b \frac{4r_o G_i}{(1-v_s)\omega}$	K_{ob}, k_{fb} represent the initial and final spring stiffness respectively, k_{fb} = 0.005k_{ob}, P_b is pile tip resistance and m_b =1, the order of the curve, λ_b=2.6, for very dense sand when L/D≥20 or for very loose sand when L/D≥10, the yield point P_{fb} is estimate by $P_{fb} = q_f A_p$, otherwise, $P_{fb}=N_q^* \sigma_v' A_p$, $N_q^*$is bearing capacity for deep circular or square foundation.
Wang (1987) $k_b = 00.267\bar{g}_c\sqrt{\frac{0.3}{d}}$	$\bar{g}_c$ average tip friction from CPT between the depth 4D above pile tip level and one d below the level. D is pile diameter, m
Hirayma (1990) $\frac{P_b}{w_b} = \frac{1}{(a_b + b_b w_b)}$ Hyperbolic P_b-w_b curve	For sandy layer, a_b = 0.25D_b/P_{bult}, P_{bult} has been relate to SPT and / or CPT value
Randolph and Wroth (1978), Chow (1986) $k_b = \frac{4G_b r_0 (1-P_b R_{fb}/P_{fb})^2}{1-v_s}$	The case for R_{fb}=0 was proposed by Randolph and Wroth (1978).

2.2.3 Numerical Analysis based on Mindlin's solution

2.2.3.1 Boundary Element Method (Poulos,1979)

The first systematic analytical study of load-displacement behavior of piles under static axial or lateral loading is due to Poulos and his co workers (Poulos and Davis 1968, Mattes and Poulos 1969, Poulos 1972, Poulos 1979) and is summarized in a his book (Poulos and Davis 1980). This work is based on linear elastic theory for both the pile and the surrounding soil and Mindlin's solutions for the soil displacements due to an embedded vertical point load within a half space, compatibility of pile and soil displacements is enforced at several discrete points along the line representing the pile. The vertical displacements of the pile at each location are expressed in terms of the unknown interaction stresses and the pile properties while the soil displacements are expressed in terms of the interaction stresses and the soil properties. If no slip occurs at the pile-soil interface, the expressions for pile-soil displacement can be equated and the resulting equations solved for the interaction stresses, the displacement along the pile can then be evaluated. The displacement influence factor may be evaluated by integration of the Mindlin's equation for vertical displacement due to a vertical subsurface point load acting within a half space. Results from Poulos's work have been cast in the form of design charts, and have been used widely by researchers and practicing engineers, forming the core of a whole class of elastic methods.

The earlier applications of this method have since been shown to produce unreliable results for very short piles, where a significant proportion of the load is transferred through the toe of the pile. This led Poulos (1972) to modify the analysis by changing the way in which the pressure distribution on the surface of the toe was modelled: previously it had been assumed to be uniform across the entire surface; the modified approach subdivided the toe into a series of circle of uniform contact pressure. A more rigorous boundary element analysis was developed by Butterfield and Banerjee (1971a), which allowed a general stress distribution over the surface of the pile toe, and was solved for the case of uniform displacement of the toe. The only significant difference this made was to piles of slenderness (length/diameter) ratio less than 10, where the refined analysis predicted a larger proportion of the load being carried by the base.

For the non-homogeneous condition, an equivalent value of shear modulus has been adopted, which is an average of the soil modulus at elements i and j. This analysis is generally consistent with that by boundary integral analysis (Banerjee and Davies, 1977), except for short piles. In fact, for short piles, the analysis method proposed by Banerjee and Davies (1977) is reported to overestimate the pile-head stiffness by 20% (Rajapakse, 1990). Shear stress distribution along a pile is considerably affected by the soil profile. However, for a stiff pile, the distribution of the shear stress down the pile is similar to the shear modulus profile, implying uniform shear strain with the depth. The settlement influence factor is generated for given pile-soil relative stiffness of various slenderness ratio in a soil layer of $\nu_s = 0.3$, $H/L = 2$ (H is depth of rigid layer).

The main difference between the various methods lies in the assumptions made regarding the distribution of shear stress along the pile. D'Applonia and Romualdi (1963), Thurman and D'Appolonia (1965) assume the shear stress at each element to be represented by a single point load acting on the axis at the center of each element. Poulos and Mattes (1974) consider a shear stress distributed uniformly around the circumference of the pile. The latter appears to be the most satisfactory of those mentioned, especially for shorter piles. However, for relatively slender piles, there is very little difference between solution based on the three above representations of shear stress.

2.2.3.2 Butterfield and Banerjee (1971)

The essence of the boundary element approach is to find a fictitious stress system which, when applied to the boundaries of the figure inscribed in the half space, will produce displacements of the boundaries which are identical to the specified boundary conditions of a real pile system of the same geometry and also satisfy identically the stress boundary conditions on the free surface of the half space. The stress ϕ are fictitious in that they are to be applied to the boundaries of the fictitious half space figure and are therefore not necessarily the actual stresses acting on the real pile surfaces. However, once the ϕ values have been determined it is a simple matter to calculate the actual stresses and displacements they produce anywhere in the half space, including those on the real pile boundaries. The total vertical and radial displacements at a point due to a pile loaded vertically are expressed through integral equations as functions of ϕ and coefficients derived from Mindlin's solution (Butterfield and Banerjee, 1971). Radial displacement compatibility is ignored, since it generally produces negligible effects on the total load required for a given settlement. The integral equations are then estimated numerically, in a way that the pile shaft is divided into n equal segments and the base into m rings. With this approach, Butterfield and Banerjee (1971) provided the relationships between pile-head stiffness and the pile slenderness ratios for single piles and different pile groups.

2.2.3.3 Boundary Integration Method (Banerjee and Davies ,1977)

Banerjee and Davies (1977) reported non-dimensional load displacement behaviour of axially loaded piles embedded in Gibson soil by utilising a boundary integral method (BI). They showed the substantial effect of soil profiles on pile-head stiffness, load distribution along the pile and pile-soil-pile interaction factors, and hence pile group behaviour. However, the approach, based on Mindlin's solutions (1936), is not strictly valid for a non-homogeneous, elastic half space.

2.2.3.4 Finite Element Analysis

The application finite-element analysis to pile foundations has been described by several investigators. This method is more likely to give a better insight into the pile foundation behavior, provided that adequate modeling of the soil and the pile-soil interface is taken. Ellison *et al.* (1971) have considered a multilinear soil stress-strain curve and have introduced special joint elements at the pile interface to allow for slip. A full three-dimensional model by Desai (1974) who used a hyperbolic constitutive law to model the soil as well as the pile-soil interface, The finite element method has also been used by Esu and Ottviani (1975) for analyzing a pile in clay. A very interesting result of their analysis is that the load-settlement behavior of a pile is substantially linear to a load well beyond half the failure load, despite the fact that the soil stress-strain response is nonlinear. This fact suggests that elastic theory, modified for slip as previously suggested, should provide an adequate basis for load-settlement prediction, provided appropriate values of soil modulus are used.

Valliappan *et al.* (1974) have done elastic parametric studies of the influence of soil layering on settlement behaviour. The superior accuracy of isoparametric elements over conventional elements is also demonstrated.

Balaam *et al.* (1975) have used a different type of analysis, in which the finite-element method is used to analyze the pile and soil mass separately and then compatibility conditions are imposed to determine the nodal forces and deflections. This approach is thus a generalization of the elastic approaches. The possibility of slip at the pile-interface is allowed for by specifying a limiting pile-soil shear strength, from which limiting values of nodal force can be calculated. Possible failure within the soil mass itself is allowed for by considering the soil as a bilinear elastic or elastic-plastic material. This type of approach appears to hold some advantage over the use of joint elements in that the rate of convergence of the solution is much more rapid when pile-soil slip or soil yield has occurred. It also overcomes problems that may arise when there are extreme differences between the modulus of the pile and the soil. Balaam *et al.* (1975) used this analysis to investigate the effects on settlement of non-homogeneity of the soil that might arise during installation of the pile.

2.3 Vertically Loaded Pile-Group

A large body of information is available for analyzing pile groups. Generally the performance of pile groups can be predicted by the following methods

1) Empirical methods;
2) Interaction factor approaches;
3) Elastic continuum based methods;
4) Hybrid load transfer approach;
5) Finite element method;
6) Block Method, Equivalent Pier or Raft Method;

2.3.1 Empirical Approaches

A number of non-dimensional parameters have been introduced to describe pile group behaviour. One of the parameters is the settlement ratio, R_s which was defined as the ratio of the average group settlement to the settlement of a single pile carrying the same average load (Poulos, 1968). For this particular parameter, empirical formulae were proposed by several researchers (e.g. Skempton, 1953; Meyerhof, 1959). The empirical formulae were generally established from the comparison of full-scale or model test results between the settlement of a single pile group and that of a single pile in sands (Skempton, 1953; Meyerhof, 1959), but only the group geometry was taken into account. In addition, test results (Kaniraj, 1993) show that the settlement ratios generally decrease as the pile spacing decreases, but the empirical formulae (Skempton, 1953; Meyerhof, 1959) indicated an opposite trend. Therefore, the empirical formulae may be used only for the cases where the overall conditions are similar to those on which these formulae are based.

Kaniraj (1993) modified the definition of settlement ratio by Poulos (1968), and defined a new settlement ratio as a ratio of the settlement of a pile group to that of a single pile when the average stress on their respective load transmitting area is identical. The load transmitting area is the area at the pile base level, estimated through the dispersion angle ($\approx 7°$ as reported by Berezantzev *et al.* 1961) as illustrated in Fig.2.2. This new settlement ratio was presented in the form of semi-empirical equations, and was compared with the measured values. The equations give better estimations of the settlement ratios than the previous empirical formulae (e.g. Skempton, 1953, Meyerhof, 1959), although generally the estimations are higher than the measured values.