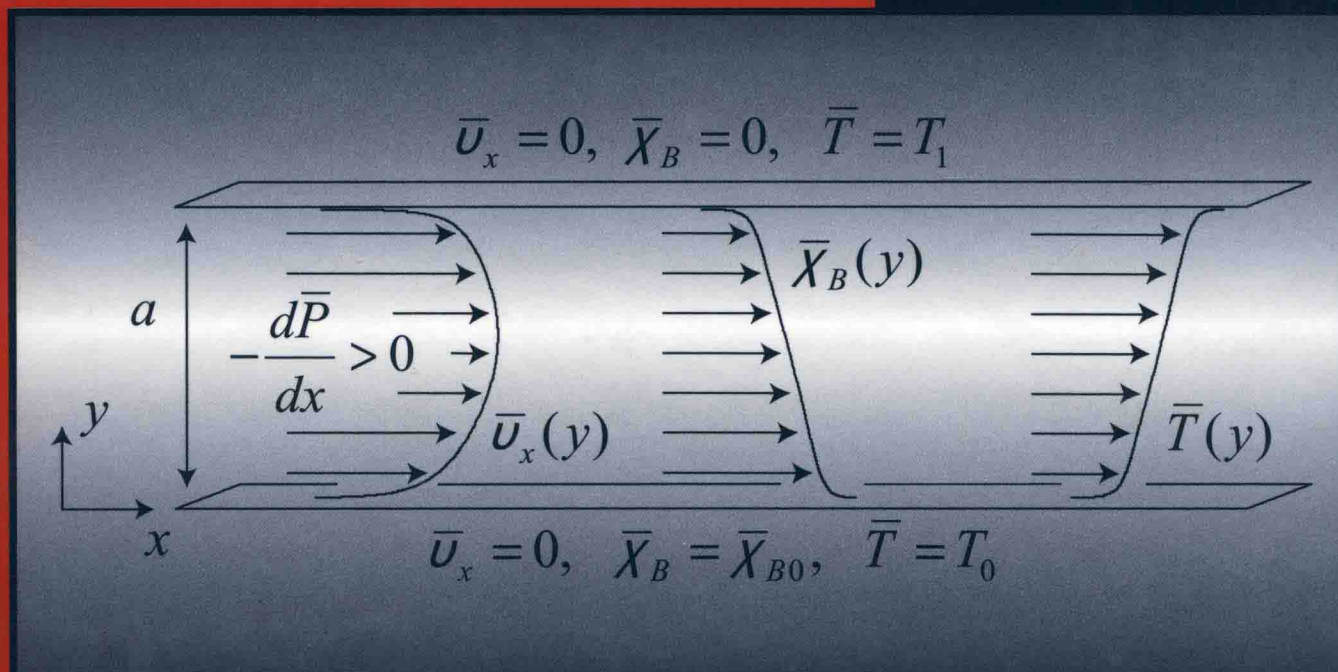


# Transport by Advection and Diffusion

Momentum, Heat, and Mass Transfer

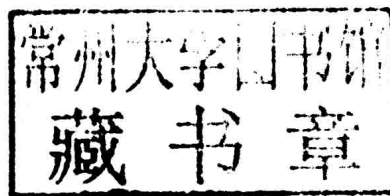


Ted D. Bennett

# Transport by Advection and Diffusion: Momentum, Heat, and Mass Transfer

**Ted D. Bennett**

*University of California, Santa Barbara*



**WILEY**

Vice President & Publisher *Don Fowley*  
Associate Publisher *Daniel Sayre*  
Assistant Editor *Alexandra Spicehandler*  
Marketing Manager *Christopher Ruel*  
Marketing Assistant *Ashley Tomeck*  
Senior Product Designer *Tom Kulesa*  
Media Specialist *Andre Legaspi*  
Senior Production Manager *Janis Soo*  
Associate Production Manager *Joel Balbin*  
Production Editor *Yee Lyn Song*  
Cover Designer *Seng Ping Ngieng*  
Cover Illustration *Ted D. Bennett*

This book was set in 9.5/11.5 Palatino by MPS Limited and printed and bound by Courier Kendallville. The cover was printed by Courier Kendallville.

This book is printed on acid free paper ∞.

Founded in 1807, John Wiley & Sons, Inc. has been a valued source of knowledge and understanding for more than 200 years, helping people around the world meet their needs and fulfill their aspirations. Our company is built on a foundation of principles that include responsibility to the communities we serve and where we live and work. In 2008, we launched a Corporate Citizenship Initiative, a global effort to address the environmental, social, economic, and ethical challenges we face in our business. Among the issues we are addressing are carbon impact, paper specifications and procurement, ethical conduct within our business and among our vendors, and community and charitable support. For more information, please visit our website: [www.wiley.com/go/citizenship](http://www.wiley.com/go/citizenship).

Copyright © 2013 John Wiley & Sons, Inc. All rights reserved. No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning or otherwise, except as permitted under Sections 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc. 222 Rosewood Drive, Danvers, MA 01923, website [www.copyright.com](http://www.copyright.com). Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030-5774, (201)748-6011, fax (201)748-6008, website <http://www.wiley.com/go/permissions>.

Evaluation copies are provided to qualified academics and professionals for review purposes only, for use in their courses during the next academic year. These copies are licensed and may not be sold or transferred to a third party. Upon completion of the review period, please return the evaluation copy to Wiley. Return instructions and a free of charge return mailing label are available at [www.wiley.com/go/returnlabel](http://www.wiley.com/go/returnlabel). If you have chosen to adopt this textbook for use in your course, please accept this book as your complimentary desk copy. Outside of the United States, please contact your local sales representative.

***Library of Congress Cataloging-in-Publication Data:***

Bennett, Ted D., 1965-

Transport by advection and diffusion : momentum, heat, and mass transfer / Ted D. Bennett.

pages cm

Includes bibliographical references and index.

ISBN 978-0-470-63148-5

1. Transport theory. 2. Diffusion processes. I. Title.

TP156.T7B465 2013

530.13'8—dc23

2012016769

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

# **Transport by Advection and Diffusion: Momentum, Heat, and Mass Transfer**



*To Lei, Annette, and Ava*



# Preface

---

This text covers material for an introductory-level graduate course or advanced undergraduate course that draws attention to the intellectual coherence of transport. While not intended to replace specialized treatises that introduce terminology and organization of subject matter for a narrow benefit, this text provides a broad treatment of transport phenomena in the coverage of a wide array of topics. A general framework for transport phenomena is revealed through the development of differential equations that employ transport principles and conservation laws. In application, these governing equations must be solved. Therefore, significant attention is given to the mathematical treatment of these equations, which is a powerful, if not essential, way to build understanding of the associated physics.

The common features of transport phenomena provide the basis for simultaneous development of momentum, heat, and mass transport. This commonality is emphasized throughout the text for maximum pedagogical benefit. For example, the momentum equation is derived from the basic elements of diffusion and advection transport, rather than using the traditional approach that follows from Newton's second law. The essential difference is whether viscous effects in the flow are treated as diffusion of momentum or as stresses imparting momentum on the fluid. Both descriptions are equivalent, but in this text, the transport perspective is advanced first to emphasize an essentially equivalent treatment given to all the transport equations.

This text is organized into relatively short chapters that address concise topics. The first three chapters are devoted to some preliminary subjects: Chapter 1 reviews some fundamental thermodynamics; Chapter 2 introduces basic transport principles; and a cursory overview of index notation is given in Chapter 3. The next two chapters are devoted to developing transport equations from the principles of conservation laws and transport phenomena. In Chapter 4, transport equations are developed to reveal the common advection and diffusion transport terms. However, significant differences between various transport equations are exposed by the addition of source terms, which are considered in Chapter 5. Chapter 6 reviews some elementary aspects of problem formulation and solution requirements associated with differential equations.

As will be seen, in many problems transport is dominated by either diffusion or advection, encouraging the insignificant process to be dropped from the description. In Chapters 7 through 13 of this text, problems in which diffusion describes the main features of transport are considered. Chapters 7 through 11 treat diffusion transport in transient one-dimensional and steady two-dimensional problems. The scope of diffusion transport is extended to moving boundary problems in Chapter 12 and lubrication theory in Chapter 13.

Chapters 14 through 22 of this text look at problems in which advection describes the main features of transport. Chapter 14 and 15 discuss ideal plane flows, which is applied to airfoil problems in Chapter 16. Two other important classes of advection problems are discussed: open-channel flows in Chapters 18 and 19, and high-speed gas dynamics in Chapters 20, 21, and 22.

Chapter 24 is the first chapter devoted to convection transport, in which both diffusion and advection play a comparable role. The topic of convection is carried throughout the remainder of the text. Through Chapter 29 transport is assumed to occur in laminar flows. Chapters 24, 25, and 26 are devoted to boundary layer problems, and



Chapters 27 and 28 are concerned with internal flows. Chapter 29 looks at the significance of nonconstant fluid properties on the solution to transport equations.

Some elementary concepts of turbulence are introduced in Chapters 30 and 32, in the context of the mixing length model. The mixing length model is used to solve the time-averaged transport equations for fully developed internal flows bounded by smooth surfaces in Chapters 31 and 33, and bounded by rough surfaces in Chapter 34. The mixing length model is also used to solve the turbulent boundary layer problem in Chapter 35. Finally, the k-epsilon model of turbulence is discussed in Chapter 36, and applied to fully developed transport in Chapter 37.

Interspersed among the main topics of this text are sections devoted to building the mathematical tools required to solve equations that govern problems of interest. For example, the method of separation of variables provides a systematic approach to solving linear partial differential equations. This topic is developed in Chapters 7 through 9, first in the context of transient diffusion and then steady-state diffusion in multiple spatial directions. In Chapter 14 it is demonstrated that some steady-state irrotational incompressible flows governed by advection also lead to a linear equation that can be solved by the method of separation of variables.

Flows governed by nonlinear advection prove to be among the most difficult transport equations to solve, and MacCormack integration is introduced in Chapter 17 as a numerical recipe to address some of these problems. MacCormack integration is used to solve open channel flow problems in Chapter 18 and 19, and to solve problems in gas dynamics in Chapters 20, 21, and 22.

Some problems can be solved using a similarity variable to transform linear and nonlinear governing equations into more easily solved ordinary differential equations. The similarity solution is introduced in Chapter 10, where it is applied to transient diffusion problems, and is applied to moving boundary problems in Chapter 12. This technique also finds great utility in solving laminar boundary layer convection problems treated in Chapters 24, 25, and 26. Similarity solutions of linear governing equations will give rise to linear ordinary differential equations with nonconstant coefficients that may be solved by the method of power series solutions, which is folded into Chapter 10. However, similarity solutions of nonlinear governing equations will give rise to nonlinear ordinary differential equations for which numerical solutions are required. Chapter 23 discusses fourth-order Runge-Kutta integration of ordinary differential equations that arise in convection transport treated in Chapter 25 and subsequent chapters.

A few numerical tasks in this text will require the use of finite differencing methods. For example, MacCormack integration is developed in Chapter 17 for application to equations describing advection transport. MacCormack integration is used to solve open-channel flows in Chapters 18 and 19, and high-speed gas flows in Chapters 20, 21, and 22. The finite differencing method is also applied to convection equations describing turbulent transport; the boundary layer equations are solved using the mixing length model in Chapter 35, and the equations for fully developed transport are solved using the k-epsilon model in Chapter 37.

Although the text is not developed with the use of commercial computational software in mind, the mathematical attention given to solving transport equations could easily be coupled to such an activity. The material in this text has been developed with the idea that programming languages, using freely available compilers, can be employed for advanced problems in transport where analytical techniques are not feasible. Engaging the mathematical problems fully (whether the approach is analytical or numerical) can demystify the process of establishing solutions and provide an empowering experience.

Carrying out one's own solutions to problems encourages a healthy level of skepticism in the results, and the process of identifying wrong results will teach critical thinking skills. Not contemplating carefully the meaning of results that are accepted at face value is a tendency that the inexperienced can easily fall into with commercial software. Therefore, the pedagogical role of commercial software should be contemplated with the idea that the best method of solving a problem for the first time may be different from the tenth time.

Finally, it is hoped that the students who use this textbook to learn about transport phenomena will have the same experience of discovery as the author had in writing it.

TDB



# Organization of Text

## Transport fundamentals

Supporting	Core material	Supplementary
1: Thermodynamic preliminaries	2: Fundamentals of transport	
3: Index notation	4: Transport by advection and diffusion 5: Transport with source terms 6: Specification of transport problems	

## Diffusion transport

Supporting	Core material	Supplementary
7: Transient one-dimensional diffusion 8: Steady two-dimensional diffusion	9: Eigenfunction expansion	
10: Similarity solution	11: Superposition of solutions	
	12: Diffusion-driven boundaries 13: Lubrication theory	

## Advection transport

Supporting	Core material	Supplementary
Chapters 1–6, 8	14: Inviscid flow 15: Catalog of ideal plane flows	16: Complex variable methods
17: MacCormack integration	18: Open channel flow	19: Open channel flow with friction
	20: Compressible flow	21: Quasi-one-dimensional compressible flows 22: Two-dimensional compressible flows

## Convection transport

Supporting	Core material	Supplementary
Chapters 1–6, 8, 10 23: Runge-Kutta integration	24: Boundary layer convection 25: Convection into developing laminar flows	26: Natural convection
	27: Internal flow 28: Fully developed transport in internal flows	29: Influence of temperature-dependent properties
	30: Turbulence 31: Fully developed turbulent flow 32: Turbulent heat and species transfer 33: Fully developed transport in turbulent flows	34: Turbulence over rough surfaces 35: Turbulent boundary layer
	36: The k-epsilon model of turbulence	37: The k-epsilon model applied to fully developed flows



# Brief Contents

---

Chapter 1 Thermodynamic Preliminaries	1	Chapter 22 Two-Dimensional Compressible Flows	333
Chapter 2 Fundamentals of Transport	12	Chapter 23 Runge-Kutta Integration	344
Chapter 3 Index Notation	25	Chapter 24 Boundary Layer Convection	359
Chapter 4 Transport by Advection and Diffusion	36	Chapter 25 Convection into Developing Laminar Flows	376
Chapter 5 Transport with Source Terms	50	Chapter 26 Natural Convection	399
Chapter 6 Specification of Transport Problems	66	Chapter 27 Internal Flow	412
Chapter 7 Transient One-Dimensional Diffusion	82	Chapter 28 Fully Developed Transport in Internal Flows	429
Chapter 8 Steady Two-Dimensional Diffusion	103	Chapter 29 Influence of Temperature-Dependent Properties	447
Chapter 9 Eigenfunction Expansion	119	Chapter 30 Turbulence	465
Chapter 10 Similarity Solution	140	Chapter 31 Fully Developed Turbulent Flow	479
Chapter 11 Superposition of Solutions	159	Chapter 32 Turbulent Heat and Species Transfer	507
Chapter 12 Diffusion-Driven Boundaries	172	Chapter 33 Fully Developed Transport in Turbulent Flows	517
Chapter 13 Lubrication Theory	188	Chapter 34 Turbulence over Rough Surfaces	545
Chapter 14 Inviscid Flow	206	Chapter 35 Turbulent Boundary Layer	565
Chapter 15 Catalog of Ideal Plane Flows	224	Chapter 36 The K-Epsilon Model of Turbulence	581
Chapter 16 Complex Variable Methods	234	Chapter 37 The K-Epsilon Model Applied to Fully Developed Flows	589
Chapter 17 MacCormack Integration	249	Appendix A	606
Chapter 18 Open Channel Flow	265	Index	611
Chapter 19 Open Channel Flow with Friction	284		
Chapter 20 Compressible Flow	296		
Chapter 21 Quasi-One-Dimensional Compressible Flows	315		



# Contents

---

## Chapter 1 Thermodynamic Preliminaries 1

---

- 1.1 The First and Second Laws of Thermodynamics 1
- 1.2 Fundamental Equations 2
  - 1.2.1 The Maxwell Relations 4
  - 1.2.2 Internal Energy Expressed in Measurable Variables 5
  - 1.2.3 Enthalpy Expressed in Measurable Variables 6
- 1.3 Ideal Gas 7
- 1.4 Constant Density Solid or Liquid 8
- 1.5 Properties of Mixtures 9
- 1.6 Summary of Thermodynamic Results 9
- 1.7 Problems 10

## Chapter 2 Fundamentals of Transport 12

---

- 2.1 Physics of Advection and Diffusion 12
- 2.2 Advection Fluxes 14
  - 2.2.1 Advection Transport in a Binary Mixture 15
  - 2.2.2 Summary of Advection Transport 16
- 2.3 Diffusion Fluxes 17
  - 2.3.1 Heat Diffusion 18
  - 2.3.2 Momentum Diffusion 18
  - 2.3.3 Species Diffusion 20
  - 2.3.4 Summary of Diffusion Laws 21
- 2.4 Reversible vs. Irreversible Transport 22
- 2.5 Looking Ahead 23
- 2.6 Problems 23

## Chapter 3 Index Notation 25

---

- 3.1 Indices 25
- 3.2 Representation of Cartesian Differential Equations 26
- 3.3 Special Operators 27
  - 3.3.1 Surface Normal Operator 27
  - 3.3.2 Kronecker Delta Operator 28
  - 3.3.3 Alternating Unit Tensor Operator 30
  - 3.3.4 Proof of a Vector Identity 30
- 3.4 Operators in Non-Cartesian Coordinates 31
- 3.5 Problems 34

## Chapter 4 Transport by Advection and Diffusion 36

---

- 4.1 Continuity Equation 37
- 4.2 Transport of Species 39
  - 4.2.1 Transport in a Binary Mixture 40
- 4.3 Transport of Heat 42
- 4.4 Transport of Momentum 43
- 4.5 Summary of Transport Equations without Sources 44
- 4.6 Conservation Statements from a Finite Volume 44
- 4.7 Eulerian and Lagrangian Coordinates and the Substantial Derivative 46
- 4.8 Problems 48

## Chapter 5 Transport with Source Terms 50

---

- 5.1 Continuity Equation 51
- 5.2 Species Equation 51
- 5.3 Heat Equation (without Viscous Heating) 52
- 5.4 Momentum Equation 54
- 5.5 Kinetic Energy Equation 55
- 5.6 Heat Equation (with Viscous Heating) 57
- 5.7 Entropy Generation in Irreversible Flows 58
- 5.8 Conservation Statements Derived from a Finite Volume 59
  - 5.8.1 Continuity 59
  - 5.8.2 Momentum 60
  - 5.8.3 Total Energy 61
- 5.9 Leibniz's Theorem 62
- 5.10 Looking Ahead 63
- 5.11 Problems 64

## Chapter 6 Specification of Transport Problems 66

---

- 6.1 Classification of Equations 66
- 6.2 Boundary Conditions 67
- 6.3 Elementary Linear Examples 69
  - 6.3.1 Gravity Driven Flow on an Inclined Surface 69
  - 6.3.2 Heat Transfer across a Liquid Film 70
  - 6.3.3 Groundwater Contamination 71



6.4	Nonlinear Example	73
6.4.1	Steady-State Evaporation	73
6.5	Scaling Estimates	75
6.5.1	Scaling of Gravity-Driven Flow on an Inclined Surface	75
6.5.2	Scaling of Groundwater Contamination	76
6.5.3	Scaling Simplification to a Governing Equation	76
6.6	Problems	78

## Chapter 7 Transient One-Dimensional Diffusion 82

7.1	Separation of Time and Space Variables	83
7.1.1	Problem with Homogeneous Equation and Boundary Conditions	83
7.1.2	Demonstration of Orthogonality	86
7.1.3	Problem with Nonhomogeneous Equation and Boundary Conditions	87
7.2	Silicon Doping	89
7.3	Plane Wall With Heat Generation	93
7.4	Transient Groundwater Contamination	97
7.5	Problems	101

## Chapter 8 Steady Two-Dimensional Diffusion 103

8.1	Separation of Two Spatial Variables	103
8.2	Nonhomogeneous Conditions on Nonadjoining Boundaries	105
8.3	Nonhomogeneous Conditions on Adjoining Boundaries	107
8.3.1	Bar Heat Treatment	108
8.4	Nonhomogeneous Condition in Governing Equation	111
8.4.1	Steady Rectangular Duct Flow	111
8.5	Looking Ahead	115
8.6	Problems	115

## Chapter 9 Eigenfunction Expansion 119

9.1	Method of Eigenfunction Expansion	119
9.1.1	Species Transport (Silicon Doping)	120
9.1.2	Heat Transfer (Bar Heat Treatment)	122
9.1.3	Momentum Transport (Duct Flow)	125
9.2	Non-Cartesian Coordinate Systems	127
9.2.1	Cartesian Coordinates	128
9.2.2	Cylindrical Coordinates	128
9.2.3	Spherical Coordinates	130

9.3	Transport in Non-Cartesian Coordinates	130
9.3.1	Pin Fin Cooling	131
9.3.2	Transient Heat Transfer in a Sphere	136
9.4	Problems	139

## Chapter 10 Similarity Solution 140

10.1	The Similarity Variable	140
10.2	Laser Heating of a Semi-Infinite Solid	142
10.3	Transient Evaporation	146
10.4	Power Series Solution	148
10.5	Mass Transfer with Time-Dependent Boundary Condition	152
10.6	Problems	157

## Chapter 11 Superposition of Solutions 159

11.1	Superposition in Time	159
11.1.1	Duhamel's Theorem	161
11.1.2	Semi-Infinite Fluid Bounded by a Plate Set in Motion	162
11.2	Superposition in Space	164
11.2.1	Product-Superposition of Solutions	165
11.2.2	Method of Images	167
11.3	Problems	169

## Chapter 12 Diffusion-Driven Boundaries 172

12.1	Thermal Oxidation	172
12.2	Solidification of an Undercooled Liquid	174
12.3	Solidification of a Binary Alloy from an Undercooled Liquid	178
12.3.1	Heat Transfer	178
12.3.2	Species Transfer	179
12.3.3	Coupling Heat and Species Transport	182
12.4	Melting of a Solid Initially at the Melting Point	183
12.5	Problems	186

## Chapter 13 Lubrication Theory 188

13.1	Lubrication Flows Governed by Diffusion	188
13.2	Scaling Arguments for Squeeze Flow	189
13.2.1	Scaling Continuity	190
13.2.2	Scaling Momentum	190
13.3	Squeeze Flow Damping in an Accelerometer Design	191
13.3.1	Scaling Analysis	192
13.3.2	Flow Damping Coefficient	193
13.4	Coating Extrusion	194
13.4.1	Scaling Arguments	195
13.4.2	Final Coating Thickness	195