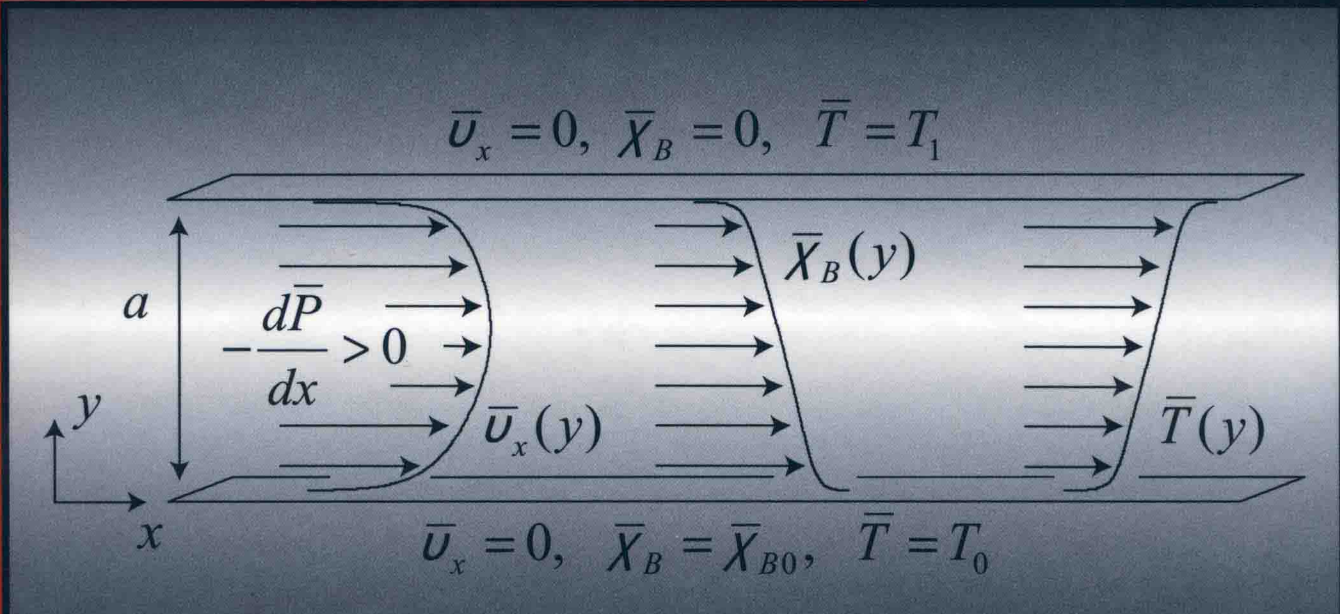
Transport by Advection and Diffusion

Momentum, Heat, and Mass Transfer



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Transport by Advection and Diffusion: Momentum, Heat, and Mass Transfer

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Transport by Advection and Diffusion: Momentum, Heat, and Mass Transfer

To Lei, Annette, and Ava

Preface

This text covers material for an introductory-level graduate course or advanced undergraduate course that draws attention to the intellectual coherence of transport. While not intended to replace specialized treatises that introduce terminology and organization of subject matter for a narrow benefit, this text provides a broad treatment of transport phenomena in the coverage of a wide array of topics. A general framework for transport phenomena is revealed through the development of differential equations that employ transport principles and conservation laws. In application, these governing equations must be solved. Therefore, significant attention is given to the mathematical treatment of these equations, which is a powerful, if not essential, way to build understanding of the associated physics.

The common features of transport phenomena provide the basis for simultaneous development of momentum, heat, and mass transport. This commonality is emphasized throughout the text for maximum pedagogical benefit. For example, the momentum equation is derived from the basic elements of diffusion and advection transport, rather than using the traditional approach that follows from Newton's second law. The essential difference is whether viscous effects in the flow are treated as diffusion of momentum or as stresses imparting momentum on the fluid. Both descriptions are equivalent, but in this text, the transport perspective is advanced first to emphasize an essentially equivalent treatment given to all the transport equations.

This text is organized into relatively short chapters that address concise topics. The first three chapters are devoted to some preliminary subjects: Chapter 1 reviews some fundamental thermodynamics; Chapter 2 introduces basic transport principles; and a cursory overview of index notation is given in Chapter 3. The next two chapters are devoted to developing transport equations from the principles of conservation laws and transport phenomena. In Chapter 4, transport equations are developed to reveal the common advection and diffusion transport terms. However, significant differences between various transport equations are exposed by the addition of source terms, which are considered in Chapter 5. Chapter 6 reviews some elementary aspects of problem formulation and solution requirements associated with differential equations.

As will be seen, in many problems transport is dominated by either diffusion or advection, encouraging the insignificant process to be dropped from the description. In Chapters 7 through 13 of this text, problems in which diffusion describes the main features of transport are considered. Chapters 7 through 11 treat diffusion transport in transient one-dimensional and steady two-dimensional problems. The scope of diffusion transport is extended to moving boundary problems in Chapter 12 and lubrication theory in Chapter 13.

Chapters 14 through 22 of this text look at problems in which advection describes the main features of transport. Chapter 14 and 15 discuss ideal plane flows, which is applied to airfoil problems in Chapter 16. Two other important classes of advection problems are discussed: open-channel flows in Chapters 18 and 19, and high-speed gas dynamics in Chapters 20, 21, and 22.

Chapter 24 is the first chapter devoted to convection transport, in which both diffusion and advection play a comparable role. The topic of convection is carried throughout the remainder of the text. Through Chapter 29 transport is assumed to occur in laminar flows. Chapters 24, 25, and 26 are devoted to boundary layer problems, and

Chapters 27 and 28 are concerned with internal flows. Chapter 29 looks at the significance of nonconstant fluid properties on the solution to transport equations.

Some elementary concepts of turbulence are introduced in Chapters 30 and 32, in the context of the mixing length model. The mixing length model is used to solve the time-averaged transport equations for fully developed internal flows bounded by smooth surfaces in Chapters 31 and 33, and bounded by rough surfaces in Chapter 34. The mixing length model is also used to solve the turbulent boundary layer problem in Chapter 35. Finally, the k-epsilon model of turbulence is discussed in Chapter 36, and applied to fully developed transport in Chapter 37.

Interspersed among the main topics of this text are sections devoted to building the mathematical tools required to solve equations that govern problems of interest. For example, the method of separation of variables provides a systematic approach to solving linear partial differential equations. This topic is developed in Chapters 7 through 9, first in the context of transient diffusion and then steady-state diffusion in multiple spatial directions. In Chapter 14 it is demonstrated that some steady-state irrotational incompressible flows governed by advection also lead to a linear equation that can be solved by the method of separation of variables.

Flows governed by nonlinear advection prove to be among the most difficult transport equations to solve, and MacCormack integration is introduced in Chapter 17 as a numerical recipe to address some of these problems. MacCormack integration is used to solve open channel flow problems in Chapter 18 and 19, and to solve problems in gas dynamics in Chapters 20, 21, and 22.

Some problems can be solved using a similarity variable to transform linear and nonlinear governing equations into more easily solved ordinary differential equations. The similarity solution is introduced in Chapter 10, where it is applied to transient diffusion problems, and is applied to moving boundary problems in Chapter 12. This technique also finds great utility in solving laminar boundary layer convection problems treated in Chapters 24, 25, and 26. Similarity solutions of linear governing equations will give rise to linear ordinary differential equations with nonconstant coefficients that may be solved by the method of power series solutions, which is folded into Chapter 10. However, similarity solutions of nonlinear governing equations will give rise to nonlinear ordinary differential equations for which numerical solutions are required. Chapter 23 discusses fourth-order Runge-Kutta integration of ordinary differential equations that arise in convection transport treated in Chapter 25 and subsequent chapters.

A few numerical tasks in this text will require the use of finite differencing methods. For example, MacCormack integration is developed in Chapter 17 for application to equations describing advection transport. MacCormack integration is used to solve open-channel flows in Chapters 18 and 19, and high-speed gas flows in Chapters 20, 21, and 22. The finite differencing method is also applied to convection equations describing turbulent transport; the boundary layer equations are solved using the mixing length model in Chapter 35, and the equations for fully developed transport are solved using the k-epsilon model in Chapter 37.

Although the text is not developed with the use of commercial computational software in mind, the mathematical attention given to solving transport equations could easily be coupled to such an activity. The material in this text has been developed with the idea that programming languages, using freely available compilers, can be employed for advanced problems in transport where analytical techniques are not feasible. Engaging the mathematical problems fully (whether the approach is analytical or numerical) can demystify the process of establishing solutions and provide an empowering experience.

Carrying out one's own solutions to problems encourages a healthy level of skepticism in the results, and the process of identifying wrong results will teach critical thinking skills. Not contemplating carefully the meaning of results that are accepted at face value is a tendency that the inexperienced can easily fall into with commercial software. Therefore, the pedagogical role of commercial software should be contemplated with the idea that the best method of solving a problem for the first time may be different from the tenth time.

Finally, it is hoped that the students who use this textbook to learn about transport phenomena will have the same experience of discovery as the author had in writing it.

TDB

Organization of Text

Transport fundamentals

Supporting	Core material	Supplementary
1: Thermodynamic preliminaries		
	2: Fundamentals of transport	
3: Index notation		
	4: Transport by advection and diffusion	
	5: Transport with source terms	
	6: Specification of transport problems	

Diffusion transport

Supporting	Core material	Supplementary
	7: Transient one-dimensional diffusion	
	8: Steady two-dimensional diffusion	
		9: Eigenfunction expansion
10: Similarity solution		
		11: Superposition of solutions
	12: Diffusion-driven boundaries	
	13: Lubrication theory	

Advection transport

Supporting	Core material	Supplementary
Chapters 1–6, 8		
	14: Inviscid flow	
	15: Catalog of ideal plane flows	
		16: Complex variable methods
17: MacCormack integration		
	18: Open channel flow	
		19: Open channel flow with friction
	20: Compressible flow	
		21: Quasi-one-dimensional compressible flows
		22: Two-dimensional compressible flows

Convection transport

Supporting	Core material	Supplementary
Chapters 1–6, 8, 10		
23: Runge-Kutta integration		
	24: Boundary layer convection	
	25: Convection into developing laminar flows	
		26: Natural convection
	27: Internal flow	
	28: Fully developed transport in internal flows	
		29: Influence of temperature-dependent properties
	30: Turbulence	
	31: Fully developed turbulent flow	
	32: Turbulent heat and species transfer	
	33: Fully developed transport in turbulent flows	
		34: Turbulence over rough surfaces
		35: Turbulent boundary layer
	36: The k-epsilon model of turbulence	
		37: The k-epsilon model applied to fully developed flows

Brief Contents

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