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Nikolay V. Banichuk, Svetlana Yu. Ivanova

OPTIMAL STRUCTURAL DESIGN

CONTACT PROBLEMS AND
HIGH-SPEED PENETRATION

This monograph studies optimization problems for rigid punches in elastic media and for high-speed penetration of rigid strikers into deformed elastoplastic, concrete, and composite media using variational calculations, tools from functional analysis, and stochastic and min-max (guaranteed) optimization approaches with incomplete data. The book presents analytical and numerical results developed by the authors during the last ten years.



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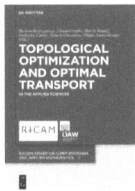
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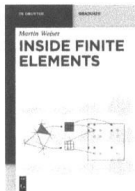


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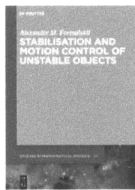


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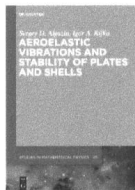


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Preface

Problems and methods of optimization of mechanical systems and structures are attracting considerable attention at the present time. Interest in research in this area has grown in connection with the rapid development of such fundamental sciences as astronautics and aeronautics, space physics, and important applied studies in the domain of military technology, civil engineering, space technologies, shipbuilding, and design of precision machinery.

Fundamental domains of these investigations are devoted to structural optimization with uncertainties (incomplete data), optimization of contact interaction of rigid and deformable bodies, high-speed penetration of strikers into solid medium, and perforation problems.

The main goal of the authors consists in the demonstration of potential possibilities and efficiency of analytical and numerical methods developed in this monograph for the solution to optimization problems of quasi-static interaction of rigid punches with elastic foundation and high-speed penetration of strikers into elastic-plastic medium.

This monograph is devoted to the exposition of new ways of formulating and solving problems of structural optimization with incomplete information. We recall some research results concerning the optimum shape and structural properties of bodies subjected to external loadings. This study is devoted to overcoming mathematical difficulties caused by nonadditive functionals and nonconvex quality criteria and also by incompleteness of information. Most of the attention of the book is devoted to the minimax approach using worst-case scenario, i.e., guaranteed approach. However, a probabilistic approach that does not guarantee the result is also described in the monograph because it gives more “optimistic” results.

The authors offer a systematic and careful development of many aspects of quasi-statics and dynamics of contact interaction of rigid, elastic, and elastic-plastic bodies. Note that the presented treatment is classical, i.e., it uses classical analysis: classical mechanics and calculus of variations, functional analysis, minimax approach of game theory and inequality analysis, and probabilistic approaches.

The present monograph consists of three parts and 15 chapters. The first part consisting of seven chapters is devoted to the problems of optimal design in the frame of quasi-statics of contact interaction of punch and elastic medium. The second part consisting of eight chapters is devoted to high-speed striker penetration into deformable medium and to shape optimization of penetrating bodies and structural optimization of protective layered plates and slabs. The third part contains four appendixes, which describe some properties and extremum conditions of nonadditive functionals, the elements of sensitivity analysis, the basic notions of multicriteria optimization, and also the computation scheme of evolutionary method (genetic algorithm [GA]), used for the numerical solution to considered optimization problems.

Chapter 1 considers the problem of optimization of contact pressure arising between the rigid punch and the elastic medium. The problem formulation and analysis are performed in the framework of quasi-statics approach using the model, which does not take into account the friction forces. The punch shape is taken as an unknown design variable. The mean square discrepancy arising between contact pressure and some given distribution is considered as an optimized (minimized) functional. It is assumed that the total forces and moments applied to the punch are given. These assumptions are reduced to the constraints imposed on the pressure distribution by equilibrium conditions. In this chapter, it was shown that the formulated optimization problem can be decomposed and reduced to two successive problems. The first problem consists in the determination of pressure distribution optimizing the quality functional. The solution to this problem is found analytically. The second problem consists in the determination of optimal punch shape, for which the obtained pressure distribution is realized. The solution to the second problem is found in the form of simple layer potential. The analytical solution to the optimization problem is performed for the punches of various shapes in plan. As an example, the rigorous solution to the punch with rectangular contact region was presented.

Chapter 2 is devoted to the problem of shape and contact pressure optimization for rigid punch interacted with elastic half-space, considered in extended formulation. In this chapter, the contact interaction is described taking into consideration not only the forces applied to the punch but also the loads acted outside of the contact region. The punch shape is considered as a design variable. The optimized (minimized) integral functional characterizes the deviation of contact pressure, arising from the punch penetration, and some given pressure distribution. The method of decomposition described in the previous chapter is generalized and used in the considered case of combined simultaneous action of the external forces applied to the punch and outside of contact region. As an example, some analytical solutions are obtained and presented in the case of rectangular (in plan) punch and application of various outside pointed forces.

In chapter 3, we study the contact interaction of punch and elastic medium in the frame of optimization formulation. It is assumed that the punch is given not only under moments and reaction forces but also under friction forces acting at the interface surface [79, 81, 88]. The movement is in quasi-static consideration. The punch shape is taken as an unknown design variable, and the minimized functional is characterized by the difference between contact pressure distribution and given purpose function of pressure distribution. The effective determination of optimal shape is achieved by solving the following two problems: determination of the optimal pressure distribution and successive boundary value problem solution for elastic half-space under the action of obtained normal pressure and friction forces [21, 22]. As an example, we present the finding of optimal shape of punch rectangular in plan.

Chapter 4 is devoted to the solution to contact problems under incomplete data. Here we present the methods of shape optimization of rigid punch, penetrating into

elastic medium in quasi-static formulation. It is supposed that the applied forces are not described beforehand but belong to the given set of external forces [19, 20]. Here we use the minimax optimization problem formulation, including the worst-case scenario for the external forces realization, belonging to the given set of admissible forces, and finding the best solution minimizing the quality functional under formulated constraints.

Probabilistic information for the shape optimization problem of rigid punch that interacted with elastic medium is investigated in chapter 5. External forces applied to the elastic medium are considered as random values having given probabilistic characteristics [14, 29, 41]. Probabilistic approach is applied as for the formulation of optimization problem as for the solution to this problem. The optimal shapes have been found for the punches having circular form in plan.

Chapter 6 is devoted to the minimization of material wear and energy dissipation. Here we consider the multicriteria contact optimization problem under integral constraint on total contact pressure force. The punch movement is investigated in quasi-static statement taking into account the arising friction forces and material wear [2, 3, 40, 79–81, 84]. All presented studies are based on the concept of the Pareto optimality [14, 42, 103] and use vector notations for vector functionals. As a multicriteria optimization problem in this chapter, we minimize the energy dissipation power, induced by friction, and the velocity of material wear. Pareto optimal solutions are constructed and presented for various weight coefficients.

Chapter 7 is devoted to the multiobjective optimization of contact pressure, wear of material, and energy dissipation. In this chapter, the problem of a moving rigid punch and elastic half-space interaction is investigated, taking into account friction of contacted surfaces, wear of materials, and arising pressure distribution. The relative punch movement is considered in the frame of quasi-static approach. The punch shape is accepted as a desired design variable. The wear volume rate, the friction dissipation power, and the discrepancy functional (between optimized contact pressure and some given pressure distribution) are taken as components of the minimized vector functional. A necessary condition for the optimality of the vector functional is derived and studied, taking into account the quasi-steady state of wear process. The solution to the optimization problem results from transformation into scalar substitute problem by means of preference functional. Formulated optimization problem is investigated analytically, using the developed decomposition approach, and exact solutions are obtained for the punch, which has a rectangular contact region and moves translationally with a constant velocity.

Chapter 8 considers the analysis and optimization problems of high-speed contact interaction of rigid body (striker) and deformed medium. These problems are very important in theoretical as in applied aspects. A two-stage mechanical model of penetration process presented in this chapter is used for the estimation of resistance forces. Theoretical and applied problems presented here are related with the problems of optimization. In this context, considerable interest is devoted to the investigation

of the problems of minimization of resistance force for rigid impactor and maximization of the depth of penetration (DOP).

Chapter 9 is devoted to the problems of high-speed penetration of rigid pointed and truncated impactors into deformed medium. The DOP of non-axisymmetric impactors is evaluated and is taken as an optimized functional. Formulated problems of shape optimization with constraints on the length and volume of the impactor are investigated for the case of maximum DOP. In this chapter, solid and hollow strikers are considered. As a design variable, the number of faces of pyramidal body (striker) is taken into consideration. Results of DOP computations are presented for different impactor shapes and for different problem parameters. The influence of the pyramidal impactor truncation on the value of the optimized functional is investigated.

Chapter 10 is devoted to the problem of finding the shape of axisymmetric rigid shell, moving in the deformable media with constant velocity, which is considered as a multicriteria (multipurpose) optimization problem with vector quality criteria. The total resistance force and the mass of material are taken as components of the minimized vector functional. The formulated multiobjective optimization problem for the vector functional is investigated analytically, and the Pareto optimal set of optimal shapes and the Pareto front are found for the rigid axisymmetric shells.

Chapter 11 is devoted to the modeling of high-speed penetration of a rigid axisymmetric body (impactor with a flat bluntness) into an elastic-plastic media with account for its rotation about the axis of symmetry. The body has an arbitrary convex shape of the meridian. The resistance to the motion is represented as the sum of the body drag and the contribution of friction. The dynamic system governing the body motion is derived in this chapter, and the qualitative and numerical analyses of the projectile movement and the perforation of a slab are performed. The problem of shape optimization of impactor with a flat bluntness is studied using evolutionary algorithm.

Chapter 12 concerns the problem of movement of rigid axisymmetric shell in deformable medium. The model of thin-walled shell of revolution is formulated, and the two-term quadratic expression is used for the estimation of the resistance force. In this chapter, the general analytical representations are derived for shell acceleration and arising membrane stresses. Dynamical strength analysis is performed and presented in particular cases of axisymmetric shells of different shapes.

In chapter 13, we apply global approach to finding nonlocal optimal layered structures. We consider a multicomponent monolithic armor consisting of a system of solid homogeneous plates. The impactor is modeled in this chapter as a short cylindrical rod. The goal of this chapter is to determine the layered structure that provides the maximum of the ballistic limit velocity.

The structural optimization of layered slab is considered in chapter 14, and the process of maximization of ballistic limit velocity of high-speed penetration of the striker is evaluated. Theoretical investigations of the impactor shape are presented.

Chapter 15 is devoted to the shape optimization problem for axisymmetric rigid striker penetrating into the deformed slab of given thickness. Taking into account the ballistic limit velocity as a minimized functional criterion, the problem of shape optimization is performed under constraints imposed on mass, length, and base radius of striker.

In Appendixes A–D, we presented some topics devoted to modern aspects of optimization theory, which are not included in classical textbooks and are used in our monograph. Appendix A devoted to nonadditive functionals and their extremum conditions describes the integrodifferential equations arising in this context. These equations are very new and required future investigations.

Appendix B is devoted to sensitivity analysis of the functionals and to multivariate evaluations. This appendix helps us reduce the volume of computations required in multipurpose and multivariate problems [14]. It is especially effective in the problems with incomplete data, when finding the “worst” action and optimal guaranteed solution is required to consider infinite number of possible realizations of external actions.

In Appendix C, we shortly describe the basic notions of multicriteria optimization. Here we present general optimization formulations using the Pareto criteria. The ways of solving of multicriteria optimization problems are also discussed.

Appendix D is devoted to evolutionary optimization algorithm of global optimization. In this appendix, we use conventional terminology to describe the computational procedure based on stochastic approaches for improving solutions (successively generated populations). The presented approach of evolutionary optimization is known as genetic algorithm (GA) [76].

The book is intended for graduate and postgraduate students, specialists in mechanics and applied mathematics, engineers, university professors, etc. It is partially based on courses of lectures delivered by N.V. Banichuk at the Faculty of Information Technology of the University of Jyväskylä, at the Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences, and at the Department of Mathematics and Informatics of the University of Cagliari.

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