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数学模型

Mathematical Modeling for Industry and Engineering

(美) Thomas Svobodny 著

机械工业出版社
CHINA MACHINE PRESS

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Original English Language title: Mathematical Modeling for Industry and Engineering by Thomas Svobodny

ISBN 0-13-260894-4

北京市版权局著作权合同登记号：图字 01-2004-4540 号

图书在版编目（CIP）数据

数学模型/（美）斯沃博德尼（Svobodny,T.）著.—北京：机械工业出版社，2005.1

（时代教育·国外高校优秀教材精选）

ISBN 7-111-15862-8

Ⅰ.数… Ⅱ.斯… Ⅲ.数学模型—高等学校—教材—英文 Ⅳ.O22

中国版本图书馆 CIP 数据核字（2004）第 135412 号

机械工业出版社（北京市百万庄大街 22 号 邮政编码 100037）

责任编辑：郑 玫

封面设计：饶 薇　　　　责任印制：石 冉

北京中兴印刷有限公司印刷·新华书店北京发行所发行

2005 年 1 月第 1 版第 1 次印刷

1000mm×1 400mm B5 · 17.25 印张 · 671 千字

定价：46.00 元

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引进国外优秀原版教材，在有条件的学校推动开展英语授课或双语教学，自然也引进了先进的教学思想和教学方法，这对提高我国自编教材的水平，加强学生的英语实际应用能力，使我国的高等教育尽快与国际接轨，必将起到积极的推动作用。

为了做好教材的引进工作，机械工业出版社特别成立了由著名专家组成的国外高校优秀教材审定委员会。这些专家对实施双语教学做了深入细致的调查研究，对引进原版教材提出了许多建设性意见，并慎重地对每一本将要引进的原版教材一审再审，精选再精选，确认教材本身的质量水平，以及权威性和先进性，以期所引进的原版教材能适应我国学生的外语水平和学习特点。在引进工作中，审定委员会还结合我国高校教学课程体系的设置和要求，对原版教材的教学思想和方法的先进性、科学性严格把关。同时尽量考虑原版教材的系统性和经济性。

这套教材出版后，我们将根据各高校的双语教学计划，举办原版教材的教师培训，及时地将其推荐给各高校选用。希望高校师生在使用教材后及时反馈意见和建议，使我们更好地为教学改革服务。

机械工业出版社

序

本书源于作者多年的数学模型课程笔记，他看到有些书有很好的模型实例，但由于数学结构不好而不受学生欢迎，但那些“易读”的数学模型教材的内容又太过容易了，所以他努力在此书中结合它们的优点。

作者在使本书宜于讲授的同时使本书适于自学，没有学过相关课程的学生也能阅读，此书特别对在课堂上安排大量讨论内容的教师有帮助。大量的例子和众多的数学方法可以开阔学生的视野，树立起建好数学模型的信心。实际上，各章大都相互独立的，又有一定的内在联系，所以一种方法会在不同的地方出现。

本书内容包括建模探索、稳定性和分岔、量纲、增长和松弛、振动、随机的思想、随机过程、复杂系统、快餐与连锁类问题、波动、扩散等。本书的起点较低，只需要微积分，一些微分方程和矩阵代数知识就能阅读。

本书的教学目的是让学生感到数学是一种了解世界的有效手段，模型本质上不是数学，而是用数学展现对不同现象间的理解。与其他大量的数学模型教材相比，本书所涉及的数学内容广而深，所涉及的实际问题宽而难，模型与实际问题结合十分紧密，能体现数学模型教材的发展趋势。

本书适合作为大学本科和研究生的数学模型课程教材。

北京师范大学

刘来福

Preface

This book grew out of notes used in a modeling course attended by upper-division undergraduate mathematics, science, engineering, and economics students that I and others have taught over several years in the mathematics department at Wright State University. From the beginning I had to rely heavily on home-made lecture notes, as I could not find a text suitable for beginners, and yet dealt with models challenging to a third or fourth year student. Some books had good modeling examples, but the students didn't like to read them as they didn't develop the mathematical structure; however, I found the "easy-reading", generalized, modeling texts too light on material. After a couple of years of hearing from colleagues that they would like the same sort of text that I envisioned, I started to write this book.

The technical prerequisites for a reading of the text are minimal: a solid calculus course, some exposure to differential equations (such as is sometimes found in calculus courses), and a little matrix algebra.

Although the use of the whole book would be ideal for a one-year introduction to applied math, the form of the book reflects to some extent a couple of one-quarter courses that are taught at Wright State University: MTH 306/606 Mathematical Modeling, attended by junior-level and senior-level mathematics majors and graduate students in math education, biomedical engineering, biophysics, economics, mechanical engineering, electrical engineering, computer science, and statistics; and MTH 333/533 Partial Differential Equations, attended by junior-level and senior-level math and physics majors, as well as graduate students in electrical, mechanical, and biomechanical engineering. A student can take the courses in either order. The modeling course covers, typically, Chapters 2 through 6 with supplementary material from Chapters 7, 8, and 9. The PDE course is based on Chapters 9, 10, and 11, with supplementary material from chapters 3, and 5,6.

At the same time, I have tried to make it as suitable for self-study as I possibly could. Besides making the subject accessible to the large number of students who find themselves without a relevant course, this should be of help to a teacher who wants to devote a large amount of classtime to the discussion of projects. The wide range of examples and mathematical techniques will broaden the student's vista and help get across the idea that there is no fixed set of tools for modeling.

In fact, the chapters are largely self-contained, although there are definite connections and built-in redundancies so that the student can see the same idea used in a different context. In this way, a clear stream can be followed (see the figure at the end of this preface). As the graphic indicates, Chapter 3 is the foundation of the book. It is natural to proceed from there to Chapter 4. One can go directly to Chapter 6 if probabilistic methods are to be the main content of the course. Chapters 1 and 2 have found use in surveying and exploring with students some of the simple ideas of applied mathematics, but their quick pace may dismay the insecure student. In particular, the first half of Chapter 1 is meant as entertainment; if the student doesn't find it so, it could be skipped. The rest of chapter 1 contains some background material. The chapters themselves are written in a sort of newspaper pyramid style so that one can either study a chapter thoroughly or simply read the first part of each chapter. Sections that are not necessary for later chapters and/or require more mathematical sophistication and/or ask for much classtime are marked with an asterisk (*).

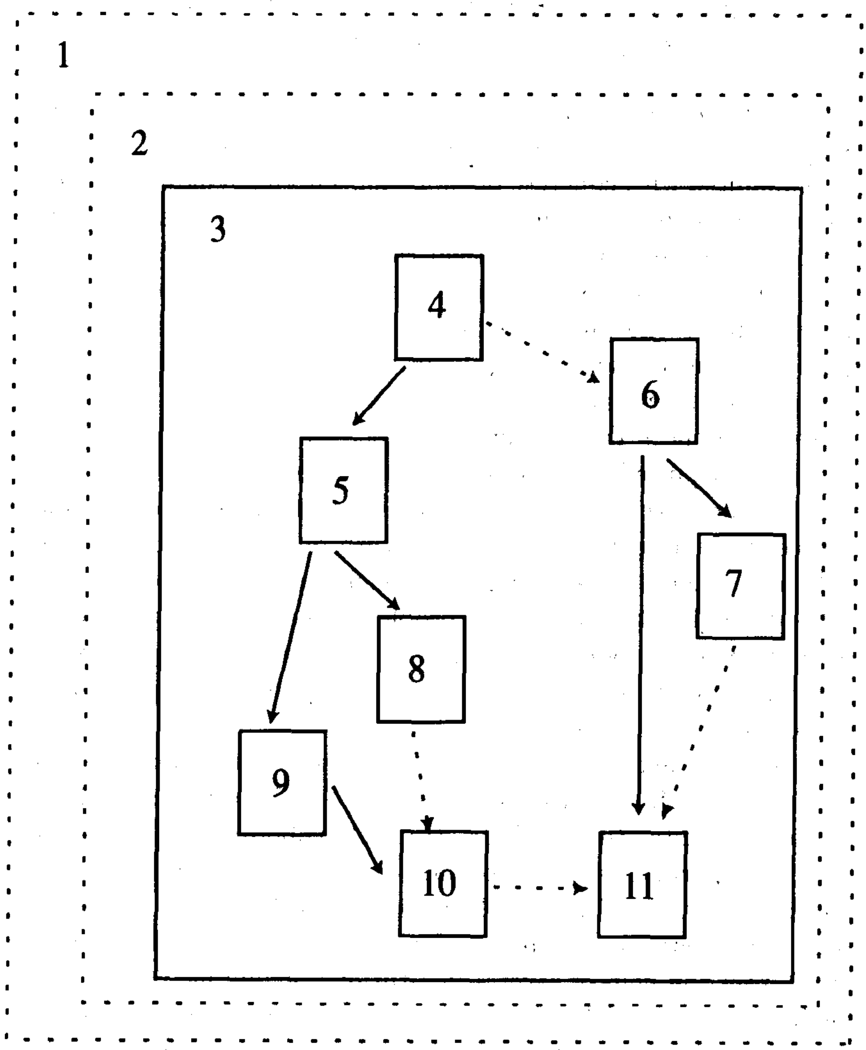
The pedagogical intent of this book is to help develop in students a feeling for the use of mathematics as a tool in the understanding of the world. A common complaint of students when beginning the modeling course is that they "lack the physical intuition" to be able to model. The book is put together with the feeling that a "modeling intution" can be nurtured in the mathematics student. It is mainly a matter of developing confidence, not just in problem solving, but in ability to approach complicated phenomena by asking a few simple questions. To this end and to encourage students to do much of the thinking on their own, exercises are built into the narrative. These exercises function as a governor: if they are trivial, the student can pick up speed; if they are not quite understood, a rereading of the text will be called for. Some require little work and may function simply to keep the student's pencil sharpened, but are designed to help the student take first responsibility for learning. Other exercises require some amount of thinking and/or search for data. There are also problem sections at the end of the chapters; these consist mainly of particular models, some of which may be suitable for a class project. Several independent trails can be followed through the problem sections. For example, chemical reactions and compartment models are introduced in the problems of Chapter 4 and reappear in problem sections in several later chapters.

It is clear to its teachers that modeling is not mathematics per se, but certainly the point of it is to use mathematics to show the underlying links between apparently disparate phenomena. Indeed, in many mathematics books, one often comes across a footnote remarking that the subject presently under discussion can also be clothed a different way, and in such and such a context. In this book these footnotes have been collected and expanded. I'd like to make the seemingly paradoxical statement that an engineer or scientist will want to take a modeling course, not to learn abstractions, but rather the opposite: to become better acquainted with concrete phenomena. Often in engineering texts one sees a formula derived and then some magical mathematics applied to it, and the result is a "theoretical rule of thumb" that appears in a box on the page. The student engineer accepts the boxed result as a substitution for the phenomenon and while becoming a practicing engineer will continue to do so. If he or shethe engineer ends up doing something more than paper work, he or she will notice a disparity and the boxed formula is thrown into the trash bin of "theory" which is disparagingly regarded as being unrelated to the "real world". On the other hand, the development of mathematical skills is necessary for a development of modeling skills. It may turn out that the mathematical technique needed for a particular model is yet to be found. Many problems started out as modeling problems and turned into areas of (pure) mathematics. Some of the great mathematicians spent an extraordinary amount of their time on modeling (Archimedes, Newton, Euler, Bernoulli, and others).

It is not possible to acknowledge everyone who aided in the development of this book. Special thanks must go to Jim Vance, Gloria Sickles, Masahiro Yamashita, Zdenek Kalva, Gabriel Svobodny, who carried out some of the experiments, and especially Anne-Marie Svobodny, who is responsible for much of the final art-work. Early partial versions of the text were class-room tested by David Miller and Larry Turyn. I benefited from the help of several institutions, including Wright State University, Center for Theoretical Study in Prague, and Dayton Museum of Natural History. I would like to thank the reviewers, Lester

Caudill, Ann Morlet, Walter Pranger, and Allan Struthers, and the editorial and production staff at Prentice Hall, especially George Lobell and Bob Walters. Of course I owe a debt to the authors of the many books that I have enjoyed reading and have found especially valuable in writing this one; the reader will find them in the recommended reading sections at the end of every chapter.

Thomas Svobodny
Dayton, Ohio



Contents

Chapter 1

The Modeling Adventure

1 What Is Modeling?

A first point that should be made is that we reflexively draw upon models to aid in our understanding of and our dealings with the world around us. Scientists, for example, must construct models of one sort or another to make sense of their findings, to communicate these findings to others, and to make comparisons with the work of their colleagues. Models are an indispensable part of that thrilling aspect of scientific endeavor: prediction making.

The models that we call upon may be

1. Pictorial
2. Analogical
3. Mathematical

Our concern in this book is, of course, with the overtly mathematical, but let us first see how the ontogeny of a mathematical model sometimes replicates, at least in spirit, a phylogeny of scientific discovery.

We will turn to the example of the evolution of models for the resting potential in nerve cells.

Information is sent through nerves and muscles as signals that are electrical in nature. These signals (nerve pulses) are a time record of changes in a homeostatically permanent electric potential—a voltage—that exists between the inside and the outside of the cells. It was realized that in order to understand and model more complicated aspects of nerve-cell firing, one had to have a model of this resting potential. The existence of the resting potential follows from a straightforward observation. If one electrode of a voltmeter is placed inside the cell and the other electrode is grounded outside the cell, the needle of the voltmeter will be deflected. Let us denote the measurement of the voltmeter by V.

If the voltage reads −80 mV (a typical value), then we know the size of the electrical field, and the negative sign tells us that it is directed into the cell. An electrical field manifests a separation of charge: here, negative charge inside the cell and positive outside. How is this possible in the biological cell?

Biological fluids are *electrolytes*. That is, they are solutions of salts, whose molecules dissociate into charged atoms, or *ions*.[1] For example,

$$\begin{aligned} \mathrm{NaCl} &\longrightarrow \mathrm{Na}^+, \mathrm{Cl}^- \\ \mathrm{KCl} &\longrightarrow \mathrm{K}^+, \mathrm{Cl}^- \end{aligned}$$

Here is a first model for the existence of V: when the cell is first formed, it is filled with an overabundance of negative ions that are sealed inside, while the leftover positive ions are in the surrounding extracellular fluid. This model was quickly laid low by experimental observation. First, to any measurable accuracy, both the fluid inside and the fluid outside the cell seemed to be electroneutral. In every spatial region of the fluid large enough to be measured, there are equal numbers of positive and negative ions. Second, the cellular membrane, the wall separating inside from outside, is known to be permeable to some ions, in particular the positive potassium ions, K^+. When a substance is dissolved in another, if all outside forces are equal, the dissolved substance, or *solute*, tends to a uniform or homogeneous concentration. High concentrations of the solute disperse of their own accord. The time rate of change of this dispersal is proportional to the spatial gradient of the concentration. The constant of proportionality is negative, to mean that the motion is down a gradient. (This is called ***diffusion*** and forms the subject matter for Chapter 11.) If the membrane were permeable to all ions, then any differences in concentration across the membrane would soon even out, and then

$$V \to 0,$$

in equilibrium.

[1]Negative ions are called *anions* because they are attracted to the positive electrode, which is called the anode. Positive ions are called *cations* because they are attracted to the negative electrode, which is called the cathode.

On the other hand, if the membrane could allow only the positive potassium ions to pass through and not any corresponding negative ions, then a voltage could be set up that just balanced the diffusive force of the positive ions. To see this, suppose that there were an excess of K^+ ions inside the cell. In other words, we assume a spatial gradient exists between inside and outside. The K^+ ions would diffuse with a net outward motion. However, because overall electroneutrality would thereby be disturbed and the inside would become negative relative to the outside, an electrical field would be set up that would oppose the emigration of positive ions. At first this voltage would be small, increasing as time went on to exactly balance the diffusive force. A static equilibrium would be reached (Figure 1.1):

$$F_{\text{electric}} + F_{\text{diffusion}} = 0.$$

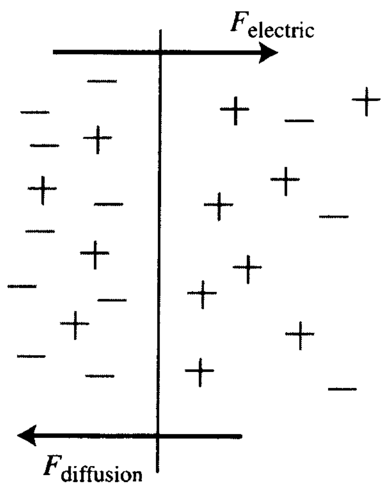


Figure 1.1 The balance between the electric and diffusive forces across the membrane

The situation is completely analogous to a common battery.[2] In a battery, the energy supplied by chemical reactions produces a separation of charge and so an electrical potential. In our case, diffusion is the driving force of the voltage. When we measure −80 mV, it is as though

[2]That chemical batteries are typically referred to as "cell" batteries is beside the point.