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Preface

This text is intended to provide an in-depth, self-contained, treatment of optical waveguide theory. We have attempted to emphasize the underlying physical processes, stressing conceptual aspects, and have developed the mathematical analysis to parallel the physical intuition. We also provide comprehensive supplementary sections both to augment any deficiencies in mathematical background and to provide a self-consistent and rigorous mathematical approach. To assist in understanding, each chapter concentrates principally on a single idea and is therefore comparatively short. Furthermore, over 150 problems with complete solutions are given to demonstrate applications of the theory. Accordingly, through simplicity of approach and numerous examples, this book is accessible to undergraduates. Many fundamental topics are presented here for the first time, but, more importantly, the material is brought together to give a unified treatment of basic ideas using the simplest approach possible. To achieve such a goal required a maturation of the subject, and thus the text was intentionally developed over a protracted period of the last 10 years.

Layout of material

The book is divided into three parts. Part I presents those geometrical and elementary ray methods necessary for the analysis of propagation on multimode optical waveguides. Part II provides the electromagnetic theory approach, with emphasis on waveguides that propagate only one or a few modes. For these waveguides, the methods of Part I are inaccurate. Part III offers supplementary sections on mathematical methods, mainly to augment the more physical discussion of Parts I and II. It is possible to read Part II without Part I, although this is not recommended. Furthermore, while all chapters need not be read sequentially, we recommend familiarity at least with the topics of each chapter. Assistance on this and other matters is given in the introductions to Parts I and II.

References

The references we cite are those with which we are most familiar and which have helped us understand the subject as presented here. While there has been no attempt to give credit to each contributor, we have tried to cite the original papers which brought new and important methods to the theory of optical waveguides covered in this text.

Suggestions for instruction

Because our primary goal is to produce a comprehensive treatment, we have, for completeness, intentionally developed all sections in their most logical order. Consequently, with restricted interests in mind, it is not necessary to read every chapter. In this regard, we offer advice to the reader throughout the text. In particular, for a first reading of the subject, only selected chapters need be read to grasp the essential concepts. Thus, for a short course consisting of, say, 30 1-hour lectures extending over a 10 week period, we recommend reading most of Chapters 1, 3, 5, 6 and 10 in Part I and Chapters 11, 12, 13, 14, 15, 19 and 20 in Part II. With this plan, the primary omission concerns radiation losses.

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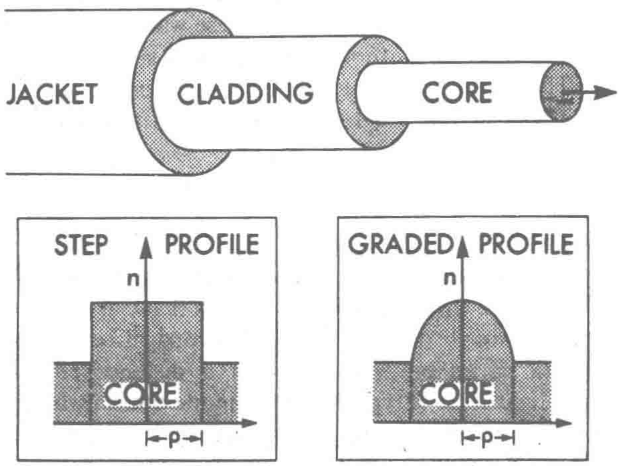
PART I

Ray Analysis of Multimode Optical Waveguides

Introduction to Part I

An optical waveguide is a dielectric structure that transports energy at wavelengths in the infrared or visible portions of the electromagnetic spectrum. In practice, waveguides used for optical communications are highly flexible fibers composed of nearly transparent dielectric materials. The cross-section of these fibers is small – comparable to a human hair – and generally is divisible into three layers as shown in Fig. I–1. The central region is the *core*, which is surrounded by the *cladding*, which in turn is surrounded by a protective jacket. Within the core, the *refractive-index profile n* can be *uniform* or *graded*, while the cladding index is typically uniform. The two situations correspond to the *step-*



MULTIMODE FIBER	SINGLE-MODE FIBER
$12.5\ \mu m < \rho < 100\ \mu m$	$2\ \mu m < \rho < 5\ \mu m$
$0.8\ \mu m < \lambda < 1.6\ \mu m$	$0.8\ \mu m < \lambda < 1.6\ \mu m$
$0.01 < \Delta < 0.03$	$0.003 < \Delta < 0.01$

Fig. I–1 Nomenclature, profiles and ranges of dimensions for typical optical fibers, where ρ is the core radius, λ is the free-space wavelength of light and $\Delta = (1 - n_{cl}^2/n_{co}^2)/2$ [1].

index and *graded-index* profiles shown in the insets in Fig. I–1. It is necessary that the core index be greater than the cladding index, at least in some region of the cross-section, if guidance is to take place. For the majority of applications, most of the light energy propagates in the core and only a small fraction travels in the cladding. The jacket is almost optically isolated from the core, so for this reason we usually ignore its effect and assume an unbounded cladding for simplicity in the analysis.

Multimode and single-mode waveguides

Optical waveguides can be conveniently divided into two subclasses called *multimode* waveguides (with comparatively large cores) and *single-mode* waveguides (with comparatively small cores). The demarcation between the two is discussed in Section 10–3, and in Chapters 11 and 12. Multimode waveguides obey the condition $(2\pi\rho/\lambda)\,(n_{co}^2 - n_{cl}^2)^{1/2} \gg 1$, where ρ is a linear dimension in the core, e.g. the radius of the fiber core, λ is the wavelength of light in free space, n_{co} is the maximum refractive index in the core and n_{cl} is the uniform refractive index in the cladding. These waveguides are the subject of Part I, while, in Part II, we are more concerned with single-mode and few-mode waveguides. The range of dimensions for fibers presently used in long distance communications is given in Fig. I–1 [1]. For the constituency of these fibers we refer the reader elsewhere [2, 3].

Ray tracing

Electromagnetic propagation along optical waveguides is described exactly by Maxwell's equations. However, it is well known that classical geometric optics provides an approximate description of light propagation in regions where the refractive index varies only slightly over a distance comparable to the wavelength of light. This is typical of multimode optical waveguides used for communication. Thus, the most direct and conceptually simple way to describe light propagation in multimode waveguides is by tracing rays along the core. Accordingly, *the first five chapters are based on classical geometric optics only.* Those readers interested in the reduction of the solutions of Maxwell's equations to classical geometric optics are referred to the beginning of Chapter 35.

Wave effects

By using classical geometric optics, *we ignore all wave effects.* In multimode waveguides, wave effects are usually negligible, as we show in Chapter 10, but there are exceptional situations when such effects accumulate exponentially with the distance light travels. In these cases, wave effects must be retained, since they can have a significant influence in long waveguides. Examples include power losses due to radiation, absorption by the cladding, and radiation from bends, and are discussed in Chapters 6 to 9. In each situation, we modify the classical geometric optics description by taking into account the local plane wave nature of light.

Modal description

There is an alternative approach to describing propagation in multimode waveguides, which relies on the small-wavelength limit of the electromagnetic modes of an optical waveguide. While this leads to results identical to those of classical geometric optics, it requires unnecessary algebraic manipulation, in addition to a knowledge of mode theory. This alternative approach is outlined later in Chapter 36, and can be regarded as an example which shows how Maxwell's equations lead to the results of Part I in the limit of small wavelength.

Pulse spreading

The phenomenon of greatest practical interest in fibers used for long-distance communications is *the spread of pulses as they propagate along the fiber*. For idealized multimode fibers, pulse spreading is easily described by classical geometric optics, as we show in Chapter 3. If attention is limited to pulse spreading, it is sufficient to cover Chapter 3, together with a few results from Chapter 1. However, our purpose in Part I is more ambitious, since we lay the foundation for a comprehensive geometric optics treatment of optical waveguides. In this way, we can more fully appreciate the consequences of departures from the ideal conditions desired in practice.

REFERENCES

1. Keck, D. B. (1981) 'Fiber design for cables.' Third International Conference on Integrated Optics and Optical Fiber Communications, San Francisco, April, 1981.
2. Miller, S. E. and Chynoweth, A. G. (eds.) (1979) *Optical Fiber Communications*, Academic Press, New York.
3. Midwinter, J. E. (1979) *Optical Fibers for Transmission*, Wiley, New York.

CHAPTER 1

Bound rays of planar waveguides

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We begin our ray analysis of multimode optical waveguides with the planar, or slab waveguide, which is the simplest dielectric structure for illustrating the principles involved, and has application in integrated optics. Since we can analyse its light transmission characteristics in terms of a superposition of ray paths, it is important to fully appreciate the behavior of individual rays. In this chapter we study the trajectories of rays within planar waveguides, concentrating on those rays – the *bound* rays – which propagate without loss of energy on a nonabsorbing waveguide, and can, therefore, propagate arbitrarily large distances.

1–1 Planar waveguides

The planar, or slab, waveguide is illustrated in Fig. 1–1. It consists typically of a *core* layer of thickness 2ρ sandwiched between two layers which form the *cladding*. As explained in the Introduction, we assume, for simplicity, that the cladding is unbounded. The planes $x = \pm\rho$ are the *core–cladding interfaces*. Since the waveguide extends indefinitely in all directions orthogonal to the x-axis, the problem is two dimensional. The z-axis is located along the axis of the waveguide midway between the interfaces. The refractive-index profile $n(x)$ in Fig. 1–1 can be uniform or graded across the core, and assumes a uniform value n_{cl} in the cladding. It is necessary that the core refractive index take some values greater than n_{cl} for the waveguide to have guidance properties. Furthermore, we assume in this chapter that the profile does not vary with z, so that the waveguide is *translationally invariant*, or *cylindrically symmetric*.

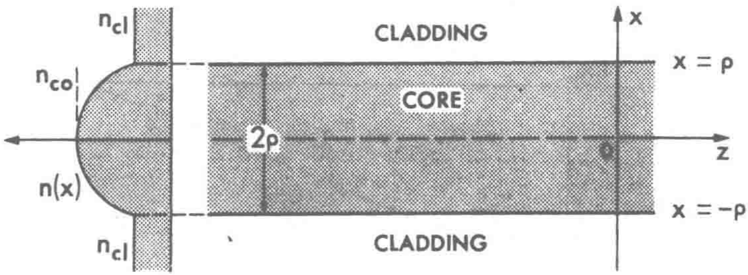

Fig. 1–1 Nomenclature and coordinates for describing planar waveguides. A representative graded profile varies over the core and is uniform over the cladding, assumed unbounded.

The parameters defined in Fig. 1–1 can be combined with the free-space wavelength λ of the light propagating along the waveguide to form a single dimensionless parameter V, known as the *waveguide parameter*, or *waveguide frequency*. If n_{co} is the maximum value of $n(x)$, which need not concur with the on-axis value $n(0)$, then we define

$$V = \frac{2\pi\rho}{\lambda}(n_{co}^2 - n_{cl}^2)^{1/2}. \qquad (1\text{–}1)$$

Alternative forms for V are given inside the front cover. The ray theory presented here is restricted to *multimode waveguides*, i.e. waveguides satisfying $V \gg 1$, for reasons discussed in Chapters 10 and 36.

STEP-PROFILE PLANAR WAVEGUIDES

With reference to Fig. 1–2, the step-index planar waveguide has the refractive-index profile defined by

$$n(x) = n_{co}, \quad -\rho < x < \rho; \qquad n(x) = n_{cl}, \quad |x| > \rho, \tag{1–2}$$

where n_{co} and n_{cl} are constants and $n_{co} > n_{cl}$. We now show how to construct ray paths within the core using ray tracing and Snell's laws. One of the most important problems is to determine the conditions necessary for a ray to be *bound*, i.e. the ray propagates along the nonabsorbing waveguide without loss of power.

1–2 Construction of ray paths

Propagation within the uniform core of the step-index waveguide of Fig. 1–2 is along straight lines. If a ray originates at P on one interface and makes angle θ_z with the waveguide axis, it will meet the opposite interface at Q as shown. The

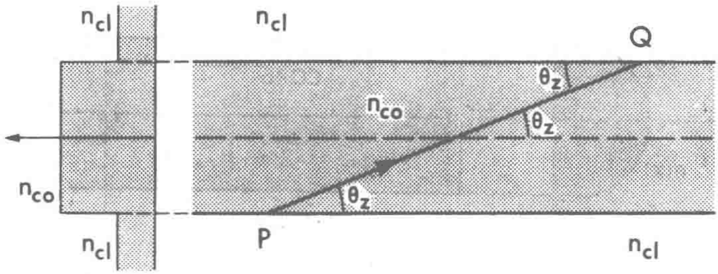

Fig. 1–2 Propagation along a straight line between interfaces in the core of a step-profile planar waveguide.

situation at Q is equivalent to incidence at an interface between two half-spaces of refractive indices n_{co} and n_{cl} as shown in Fig. 1–3. Reflection in this situation is governed by Snell's law [1, 2]. While these laws are usually expressed in terms of angles relative to the normal QN, as in Section 35–2, we prefer to retain the complementary angle θ_z. The reason will become apparent in Chapter 2 when classifying ray paths within optical fibers. Thus, in terms of complementary angles, the incident ray at Q is *totally internally reflected* if $0 \leqslant \theta_z < \theta_c$, and is partly reflected and partly *refracted* if $\theta_c < \theta_z \leqslant \pi/2$, where θ_c is the *complement of the critical angle*, defined by

$$\theta_c = \cos^{-1}\left\{\frac{n_{cl}}{n_{co}}\right\} = \sin^{-1}\left\{1 - \frac{n_{cl}^2}{n_{co}^2}\right\}^{1/2}. \tag{1–3}$$