VOCATIONAL-TECHNICAL
MATHEMATICS

ROBERT D. SMITH

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NOTICE TO THE READER

Vocational-Technical Mathematics

PREFACE

Vocational-Technical Mathematics is written to provide practical vocational and technical applications of mathematical concepts. Presentation of concepts is followed by applied examples and problems which have been drawn from diverse occupational fields.

Both content and method are those used by the author in teaching related vocational-technical mathematics on both the secondary and post-secondary levels. Each unit is developed as a learning experience based on preceding units. The applied examples and problems progress from the simple to those whose solutions are relatively complex. Many problems require the student to work with illustrations such as are found in trade and technical manuals, handbooks, and drawings.

The book was written from material developed for classroom use and it is designed for classroom purposes. However, the text is also very appropriate for self-instruction use. Great care has been taken in presenting explanations clearly and in giving easy to follow procedural steps in solving examples. One or more examples are given for each mathematical concept presented. Throughout the book, practical application examples from various occupations are shown to illustrate the actual on-the-job uses of the mathematical concept. Students often ask, "Why do we have to learn this material and of what actual practical value is it?" This question was constantly kept in mind in writing the book and every effort was made to continuously provide an answer.

An understanding of mathematical concepts is emphasized in all topics. Much effort was made to avoid the mechanical *plug in* approach often found in vocational related textbooks. A practical rather than an academic approach to mathematics is taken. Derivations and formal proofs are not presented; instead, understanding of concepts followed by the application of concepts in real situations is stressed.

Section I, Fundamentals of General Mathematics contains the prerequisites for the subsequent sections. It is assumed that students using this book have previously studied the topics presented in Section I; therefore the material in Section I is structured differently than that of the rest of the text. The approach taken is that of treating the topics entirely as review material. Mathematical procedures are not presented in this section, rather an example of the concept is given followed by student exercises and problems. It is felt that the topics, exercises, and problems presented in Section I are of value to the instructor and students for diagnostic and placement testing purposes.

Student exercises and applied problems immediately follow the presentation of concept and examples. In addition, exercises and occupationally related problems

are included at the end of each unit. The book contains a sufficient number of exercises and problems to permit the instructor to selectively plan assignments.

The Solutions and Tests Booklet contains the complete solutions to many problems, the answers to all problems, and a set of section achievement reviews developed for each of the six sections. The achievement reviews are comprehensive and are designed to measure students' achievement of all of the unit objectives contained within each section.

Illustrations, examples, exercises, and practical problems expressed in metric units of measure are a basic part of the content of the entire text. Emphasis is placed on the ability of the student to think and to work with equal ease with both the English and the metric systems. Tables of equivalent units of measure are contained in the Appendix.

An analytical approach to problem solving is emphasized in the plane geometry, computed measure, and trigonometry sections. The approach is that which is used in actual on-the-job trade and technical occupations applications. Integration of algebraic and geometric principles with trigonometry by careful sequencing and treatment of material also helps the student in solving occupationally related problems.

About the Author

The author, Robert D. Smith, is presently a faculty member in the Vocational-Technical Education Department at Central Connecticut State College, New Britain, Connecticut. Mr. Smith has had experience in the manufacturing industry as tool designer, quality control engineer, and chief manufacturing engineer. He has also been active in teaching applied mathematics, physics, and industrial materials and processes on the secondary school level. He is the author of Delmar's MATHEMATICS FOR MACHINE TECHNOLOGY and APPLIED GENERAL MATHEMATICS.

Mr. Smith has been involved in several professional organizations in his field of interest, including The American Technical Education Association and The Society of Manufacturing Engineers.

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SECTION 1
Fundamentals of General Mathematics

UNIT 1
Whole Numbers

OBJECTIVE

After studying this unit you should be able to

- solve whole number problems by applying the principles of whole numbers.

ADDING WHOLE NUMBERS

Example.   Add and check. $55\,907 + 12 + 7\,785\,079 + 3\,986$

$$
\begin{array}{r}
7\,844\,984 \text{ Ck} \\
55\,907 \\
12 \\
7\,785\,079 \\
+\quad 3\,986 \\
\hline
\mathbf{7\,844\,984} \; Ans
\end{array}
$$

Exercise 1-1

Add and check.

1. $73 + 24$
2. $876 + 97$
3. $4\,397 + 8\,512$
4. $8\,677 + 12\,393 + 155\,071$
5. $64 + 178 + 9\,766$

SUBTRACTING WHOLE NUMBERS

Example.   Subtract and check. $5\,532\,006 - 3\,350\,078$

$$
\begin{array}{r}
5\,532\,006 \\
-\;3\,350\,078 \\
\hline
\mathbf{2\,181\,928} \; Ans
\end{array}
\qquad
\begin{array}{r}
2\,181\,928 \\
+\;3\,350\,078 \\
\hline
5\,532\,006 \text{ Ck}
\end{array}
$$

Exercise 1-2

Subtract and check.

1. $87 - 23$
2. $978 - 349$
3. $7\,833 - 626$

4. $30\,092 - 28\,398$
5. $438\,078 - 29\,066$

ADDING AND SUBTRACTING WHOLE NUMBERS IN PRACTICAL APPLICATIONS

Exercise 1-3

1. A section of concrete wall and footing is shown in figure 1-1. What is the total height of the wall and footing?
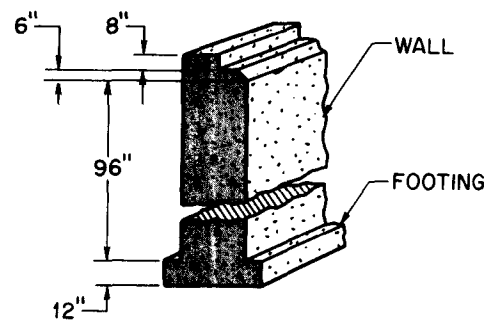
Fig. 1-1

2. During the first week of April, a print shop used the following paper stock: 5 570 sheets on Monday, 7 855 sheets on Tuesday, 7 236 sheets on Wednesday, 6 867 sheets on Thursday, and 6 643 sheets on Friday. During the following week, 4 050 more sheets are used than during the first week. Find the total sheets used during the first two weeks of April.

3. The duplicating machine operator of a manufacturing company makes copies of printed material for various departments within the company. A log is kept to record the number of copies made for each department. The monthly log is shown in figure 1-2.

DEPARTMENT	NUMBER OF COPIES			
	WEEK 1	WEEK 2	WEEK 3	WEEK 4
Production	853	712	956	1 088
Engineering and Design	1 050	936	277	732
Personnel	2 756	1 935	2 080	993
Accounting	830	0	344	130
Sales	1 202	555	3 859	2 444
Data Processing	85	53	0	187
Purchasing	1 932	1 637	767	845
Inspection	177	286	53	0
Receiving and Shipping	538	613	423	778

Fig. 1-2

  a. Find the total number of copies made in week 1.
  b. Find the total number of copies made in week 2.
  c. Find the total number of copies made in week 3.
  d. Find the total number of copies made in week 4.
  e. Find the total number of copies made for the month.

4. An automobile body shop charges $465 for repairs on a customer's car. The charges for labor are $196. Paint and materials cost $67 and replacement parts cost $110. How much profit is made on the job?

5. A machinist must know dimensions A and B in order to finish grind surfaces F and G of the casting shown in figure 1-3. All dimensions are in millimetres.

   a. Find, in millimetres, dimension A.

   b. Find, in millimetres, dimension B.
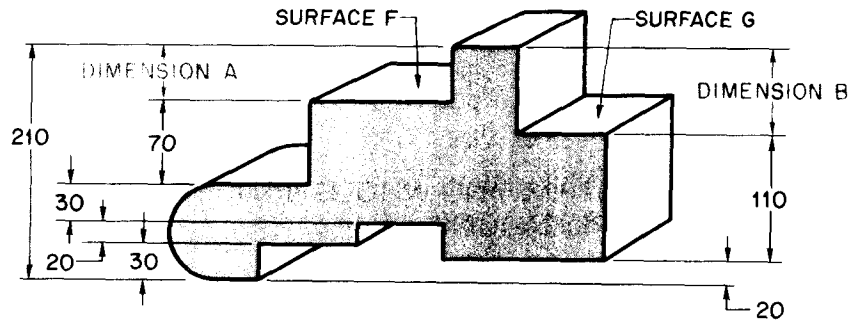
Fig. 1-3

6. A baker makes a batch of cookie mix which weighs 48 pounds. Pastry flour and other ingredients are used to make the mix. The weights of the other ingredients are 13 pounds of almond paste, 12 pounds of margarine, 5 pounds of egg whites, and 7 pounds of granulated sugar. How many pounds of pastry flour are used?

MULTIPLYING WHOLE NUMBERS

Example.   Multiply and check. 706 × 315

```
      315              706
    × 706            × 315
    1 890            3 530
  220 50             7 06
  222 390 Ans        211 8
                     222 390 Ck
```

Exercise 1-4

Multiply and check.

1. 84 × 76

2. 59 × 898

3. 305 × 8 756

4. 403 × 6 007

5. 6 959 × 27 877

DIVIDING WHOLE NUMBERS

Example.   Divide and check. 476 947 ÷ 646

```
        738 R199 Ans           738
  646)476 947                × 646
      452 2                   4 428
       24 74                 29 52
       19 38                442 8
        5 367               476 748
        5 168
          199               476 748
                          +     199
                            476 947 Ck
```

Exercise 1-5

Divide and check.

1. 365 ÷ 5

2. 4 800 ÷ 16

3. 46 877 ÷ 5

4. 56 107 ÷ 79

5. 58 218 ÷ 836

MULTIPLYING AND DIVIDING WHOLE NUMBERS IN PRACTICAL APPLICATIONS

Exercise 1-6

1. A building contractor orders 25 lengths of channel iron and 18 lengths of I beam as shown in figure 1-4. What is the total length of channel iron and I beam ordered?

CHANNEL IRON

I BEAM
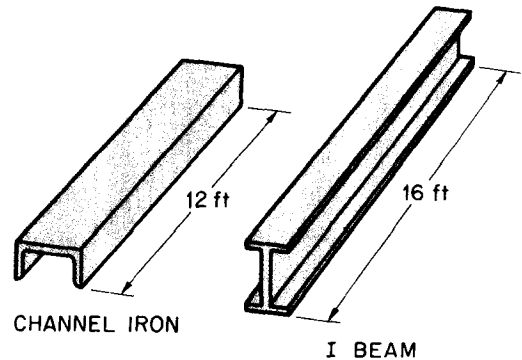
Fig. 1-4

2. The invoice shown in figure 1-5 is mailed to the Center Sports Shop by a billing clerk of the M & N Sports Equipment Manufacturing Company. (An invoice is a bill sent to a retailer by a manufacturer or wholesaler for merchandise purchased by the retailer.) The extension shown in the last column of the invoice is the product of the number (quantity) of units multiplied by the price of one unit. Find the extension amount for each item on the invoice and add the extensions to determine total cost.

	QUANTITY	UNIT	UNIT PRICE	DESCRIPTION	EXTENSION
a.	15	dozen	$ 2	Floats	$30
b.	24	each	$11	Fishing rods	
c.	1	box	$18	Spools of line	
d.	18	each	$ 7	Reels	
e.	36	each	$ 5	Baseball bats	
f.	5	box	$36	Baseballs	
g.	24	package	$ 6	Golfballs	
h.	15	each	$14	Putters	
i.				Total Cost _____	

Fig. 1-5

3. The drill jig shown in figure 1-6 is laid out by a machine drafter. All dimensions are in centimetres.

   a. Find, in centimetres, dimension A.
   b. Find, in centimetres, dimension B.
   c. Find, in centimetres, dimension C.
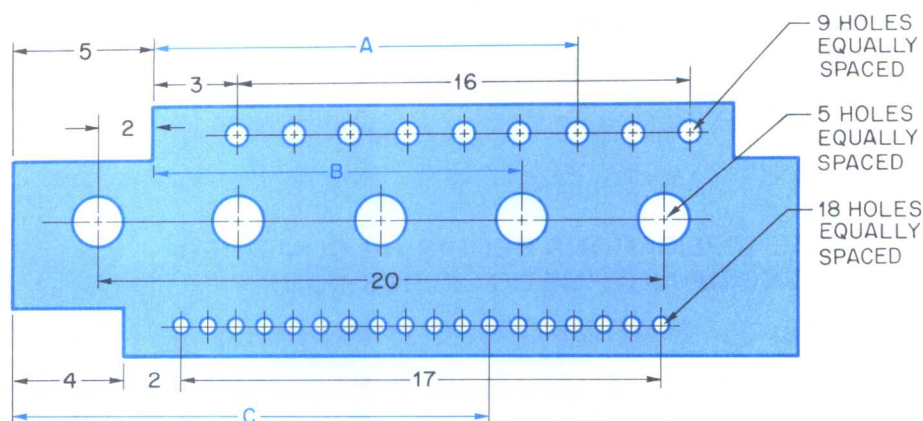
Fig. 1-6

4. Figure 1-7 shows the front view of a wooden counter that is to be built for a clothing store. All pieces of the counter except the top and back are to be made of the same thickness and width of lumber. How many total feet (1′ = 12″) of lumber should be ordered for this job? Do not include the top or back. Allow 6 feet for waste.
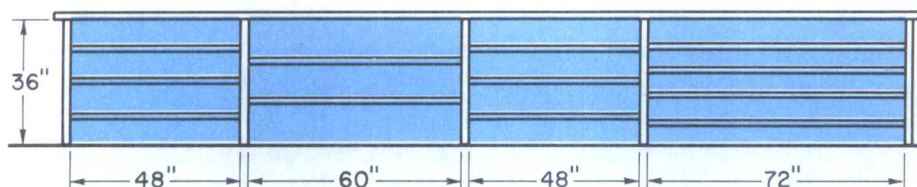
Fig. 1-7

5. An 8-pound cut of roast beef is to be medium roasted at 350°F. Total roasting time is determined by allowing 15 minutes roasting time for each pound of beef. If the roast is placed in a preheated oven at 2:00 PM, at what time should it be removed?

6. A designer fashions a dress which is 46 inches long from the neck to the hem. The first button is placed one inch from the top; the last button is placed 3 inches from the hem. The buttons are placed 2 inches apart. How many buttons are required?

COMBINED OPERATIONS IN ARITHMETIC EXPRESSIONS

Order of Operations

• First, do all operations within grouping symbols. Grouping symbols are parentheses ( ), brackets [ ], and braces { }.

  Next, do multiplication and division operations in order from left to right.

• Last, do addition and subtraction operations in order from left to right.

Example . Find the value of $(15 + 6) \times 3 - 28 \div 7$.

$$(15 + 6) \times 3 - 28 \div 7$$
$$21 \times 3 - 28 \div 7$$
$$63 - 4$$
$$59 \ Ans$$

Exercise 1-7

Perform the indicated operations.

1. $9 + 12 - 5$

2. $35 + 30 \div 5$

3. $(35 + 30) \div 5$

4. $(10 \times 8) \div (5 \times 4)$

5. $(240 - 80) \times 15 \div 3$

PRACTICAL APPLICATIONS REQUIRING COMBINED OPERATIONS OF ARITHMETIC EXPRESSIONS

Exercise 1-8

1. An engine is used to lift heavy crates on the truck loading platform shown in figure 1-8. The horsepower needed to lift crates can be found using this formula.

    $hp = (f \times d \div t) \div 550$ where hp = horsepower

    f = force in pounds

    d = distance in feet

    t = time in seconds

Fig. 1-8

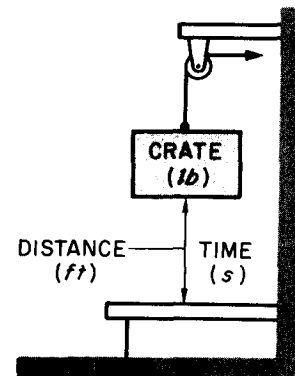The force is equal to the weight of a crate. The distance is the number of feet a crate is lifted. The time is the number of seconds needed to lift the crate a certain distance. Find the number of horsepower needed to lift each of the crates listed in figure 1-9.

CRATE	WEIGHT OF CRATE	DISTANCE LIFTED	TIME	HORSE-POWER
A	660 lb	10 ft	4 s	
B	1 100 lb	12 ft	6 s	
C	1 650 lb	14 ft	7 s	
D	3 300 lb	8 ft	16 s	
E	2 640 lb	10 ft	8 s	

Fig. 1-9

2. The accountant for a small manufacturing firm computes the annual depreciation of each piece of tooling, equipment, and machinery in the company. From a detailed itemized list, the accountant groups all items together that have the same life expectancy (number of years of usefulness) as shown in figure 1-10. Find the annual depreciation for each group and the total annual depreciation of all tooling, equipment and machinery, using the straight line formula.

    Annual Depreciation = (cost − final value) ÷ number of years of usefulness.

	GROUP	COST	FINAL VALUE	NUMBER OF YEARS OF USEFULNESS	ANNUAL DEPRECIATION
a.	Tooling	$14 500	$1 200	5 years	
b.	Equipment	$28 350	$3 750	6 years	
c.	Equipment	$17 900	$2 040	10 years	
d.	Machinery	$67 700	$7 940	8 years	
e.	Machinery	$80 300	$10 600	10 years	
f.				Total Annual Depreciation	

Fig. 1-10

3. A landscaper contracts to provide topsoil and to seed and lime the parcel of land shown in figure 1-11. In order to determine labor and material costs, the landscaper must first know the total area of the land. Find the total area in square feet.

Total Area (square feet) $= l \times w + (a \times b) \div 2$

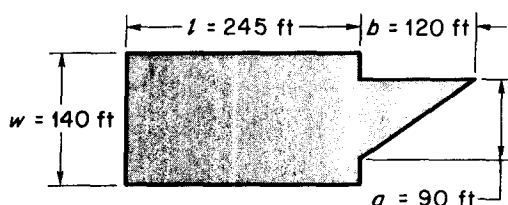**Fig. 1-11**

4. The formula called Young's Rule is used in the health field to determine a child's dose of medicine.

Child's dose $= (Age\ of\ child) \div (Age\ of\ child + 12) \times average\ adult\ dose$

What dose (number of milligrams) of morphine sulfate should be given to a 3-year-old child if the adult dose is 10 milligrams?