POLARIZED LIGHT

WILLIAM A. SHURCLIFF

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PRODUCTION AND USE

WILLIAM A. SHURCLIFF
Research Fellow in Physics, Harvard University

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PREFACE

Because polarized light is being used increasingly—by physicists, chemists, biologists, metallurgists, mineralogists, mechanical engineers, and electronics engineers—the need for a serious book on the production and use of polarized light has become increasingly evident. Every year hundreds of additional articles dealing with polarized light appear in various scientific journals. New applications are constantly being reported. But there has been no book to delineate the central concepts, to indicate a comprehensive terminology, to compare the different types of polarizers, and to present the rules governing the combinations of polarizers and retardation plates. There has been no careful review of the hundreds of kinds of applications, and no substantial bibliography.

The most important event in the modern history of polarized light was the invention of the sheet-type polarizer, by Edwin Herbert Land in 1928. His invention of the microcrystalline species of sheet-type polarizer (J-sheet), and the later invention by Land and his associates of the molecular species (H-sheet, K-sheet, HR-sheet, and so forth), provided scientist and engineer with polarizers having almost every desirable feature. Nearly every branch of science has felt the impact of these inventions. Yet the technology of these modern polarizers has received scant mention in the scientific literature.

Four powerful tools for predicting the effects of polarizers, retardation plates, and so on have recently come into prominence, but have not been discussed in available textbooks in a serious, systematic way. The new tools are the Stokes vector, the Poincaré sphere, the Mueller calculus, and the Jones calculus. They make it possible to calculate with ease the behavior of polarizer-retarder combinations that formerly seemed almost hopelessly complicated. Here we describe the tools in detail and illustrate their use. In addition, all the commonly required matrices of the Mueller and Jones calculi are listed, for ready reference.

The author's early training in polarization phenomena was acquired in the research laboratory directed by Dr. E. H. Land. The writings by Dr. Land and his colleagues Dr. Cutler D. West and Dr. R. Clark Jones established the foundations on which this book is based.

The author's debt to Dr. R. Clark Jones, the inventor of the Jones calculus, is immeasurable. The sections dealing with the Stokes vector, the Mueller calculus, and the Jones calculus could not have been written without long and painstaking coaching by him. The help received from E. S. Emerson and A. S. Makas in various practical aspects of polarizer technology has been of great value. Many other colleagues have helped, directly or indirectly, to make this monograph possible.

W. A. Shurcliff

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1 CONVENTIONAL DESCRIPTION OF POLARIZED LIGHT

1.1. Introduction. In this book the term *light* stands for *electromagnetic radiation.* Usually we have in mind the 400–700 mμ range (visual range), but often we include also the shorter-wavelength range (ultraviolet) and the longer-wavelength range (infrared). On some occasions we include also the x-ray and gamma-ray ranges and the radio range. The total range in which polarization plays a part covers more than sixty octaves.

Polarized light is one of nature's ultimates. A slender, monochromatic, *polarized* ray cannot be subdivided into simpler components: no simpler components exist. The process of analysis can advance no further.

There is much to be gained, however, by considering how a beam of polarized light behaves and how it may be depicted. When such a beam encounters a birefringent crystal, a dichroic film, or an oblique dielectric surface, a great variety of behaviors may result. The question is: Can we find, for the polarized beam, a representation so pertinent and so versatile that, merely by examining the representation, we can predict the outcome of any given encounter?

Fortunately, several highly successful representations have been invented. Some are pictorial, others mathematical. Some are well suited to solving simple problems, others are to be preferred when the problems are complicated.

To ask whether a given representation is "true" is futile. It must suffice that the representation assists ready recollection of the behavior and permits easy solution of the various problems encountered.

The present chapter reviews the classical (pictorial and wave-train) methods of representing polarized light. Chapter 2 considers certain more modern and more powerful methods.

Polarized light, besides being of interest *per se*, serves as a tool, or probe, for evaluating the properties of matter. The tool exhibits the ultimate in speed, and perhaps the ultimate in delicacy and convenience. It has the merit of being completely convertible; that is, the polarization form can be altered at will, with no loss in power and no increase in entropy flux. In many respects, polarized light, being the simplest kind of light, is easier to deal with than ordinary light: the physical manipulations may be cleaner, and the mathematical procedures for predicting the experimental outcomes are simpler. Physicists and chemists find that polarized light has uses far beyond those of unpolarized light. Biologists, astronomers, and engineers find that polarized light solves many problems that are otherwise insoluble. If light is man's most useful tool, polarized light is the quintessence of utility.

In preparing this book the author faced a major problem as to conventions. The crux of the problem was the large number of branches of optics that must be brought into one consistent family. Traditionally, users of saccharimeters and other polarimeters employ a certain set of sign conventions, persons dealing with dichroism employ certain conventions, and similarly for persons dealing with crystallography, wave theory, the Stokes vector, the Poincaré sphere, the Mueller calculus, and the Jones calculus. Ordinarily, the incompatibility of the various sets of conventions as to signs, handedness, etc., is unnoticed and unimportant. In this book, however, one universally self-consistent set of conventions is mandatory. Accordingly, some conflict with various lesser sets is unavoidable.

1.2. Classical Pictorial Specification of a Polarized Wave Train. The classical description of a polarized wave train is well known (see, for example, Ditchburn, D-10, and Jenkins and White, J-9), and needs only brief review here.

From the standpoint of classical physics, light consists of electromagnetic waves whose vibrations are transverse to the propagation direction. *Polarized* light is light whose vibration pattern exhibits *preference:* preference as to transverse direction, or preference as to the handedness associated therewith. Different kinds of preference are indicated in Fig. 1.1. Each drawing, called a snapshot pattern, describes the monochromatic wave train at a single instant in time; the curve may be thought of as a smooth line joining the tips of a large number of vectors that indicate the directions and magnitudes of the *electric* field at various positions along the center line of the beam. The convention with respect to right and left circular polarization is easily remembered: right circular polarization is portrayed by means of a right-handed helix, such as the thread of a typical machine screw.

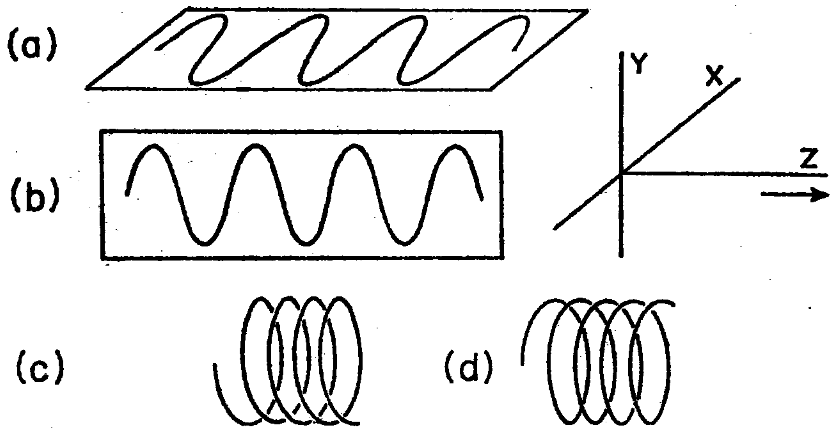

FIG. 1.1. Snapshot patterns of a horizontally traveling beam of monochromatic light that is polarized (*a*) horizontally, (*b*) vertically, (*c*) right circularly, and (*d*) left circularly.

(The reader will recall that a right-handed helix continues to appear right-handed no matter what the observer's viewpoint; consequently the present definition is free from ambiguity.) The pattern may be drawn with respect to a right-handed set of cartesian coordinates, Z being the direction of propagation, and X and Z being horizontal.

Workers in different fields (such as crystallography, theoretical physics, saccharimetry, radio technology) may employ conflicting definitions. The definitions used in this book are believed to represent the best compromise. Care has been taken to word the definitions clearly and to use them consistently.

One could, of course, deal with the magnetic, rather than the electric, vibration. When light is traveling in a vacuum or other isotropic medium, these two vibrations are orthogonal (perpendicular) and their magnitudes are always proportional to one another. To specify one is tantamount to specifying both. The decision to concentrate on the *electric* vibration is conventional, and pays tribute to the dominant role of the electric vector in the more familiar absorption processes.

The ***sectional pattern*** (Fig. 1.2) is perhaps the most familiar of all the characterizations. A horizontally polarized beam is portrayed as a

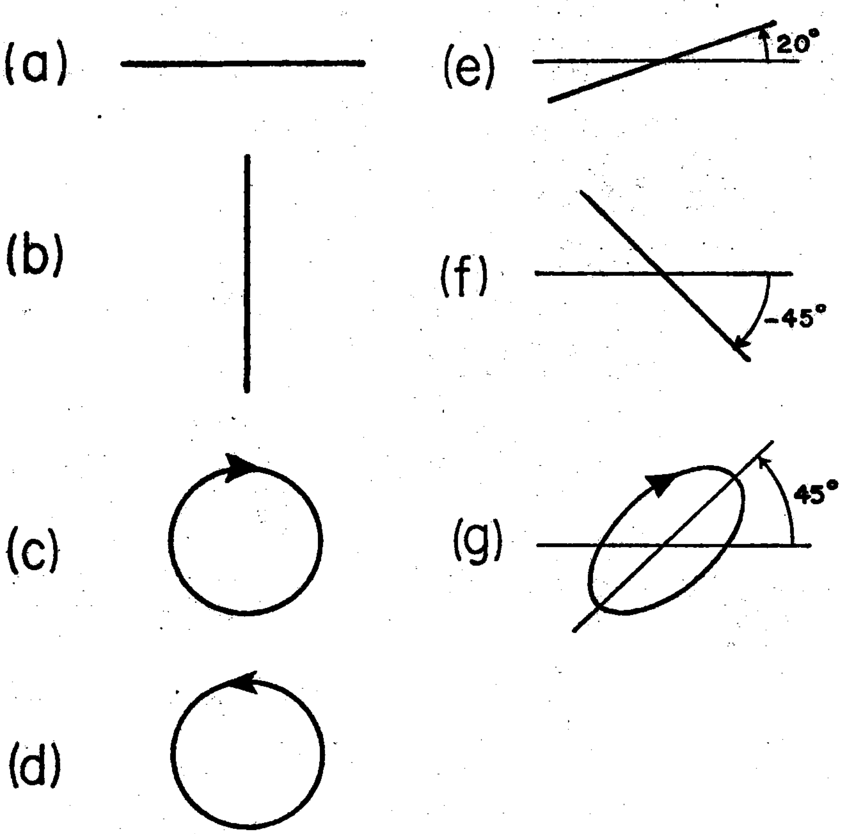

FIG. 1.2. Sectional pattern of a beam polarized (*a*) horizontally, (*b*) vertically, (*c*) right circularly, (*d*) left circularly, (*e*) linearly at 20°, (*f*) linearly at −45°, (*g*) right elliptically at 45°.

short horizontal line; vertical polarization is indicated by a vertical line. Right-circular polarization is portrayed by a circle having a clockwise sense. The sectional pattern may be thought of as an end view of the snapshot pattern, as seen by an observer who is situated in the path of the beam (specifically, far out on the *Z*-axis) and is looking toward the light source, which is at the origin of coordinates.

The clockwise sense of the circle describing right-circular polarization is consistent with the definition involving a ***right-handed helix:*** if a right-handed helix is moved bodily toward an observer (without rotation) through a fixed, transverse, reference plane, the point of intersection of helix and plane executes a *clockwise* circle.

The sectional patterns are easy to draw, even for light that is polarized elliptically. Also, they can embrace polychromatic light; however, we must then think of the scale of the pattern as changing more or less rapidly, depending on the frequency bandwidth of the beam; consequently the patterns (Fig. 1.3) cease to be of simple, closed type.

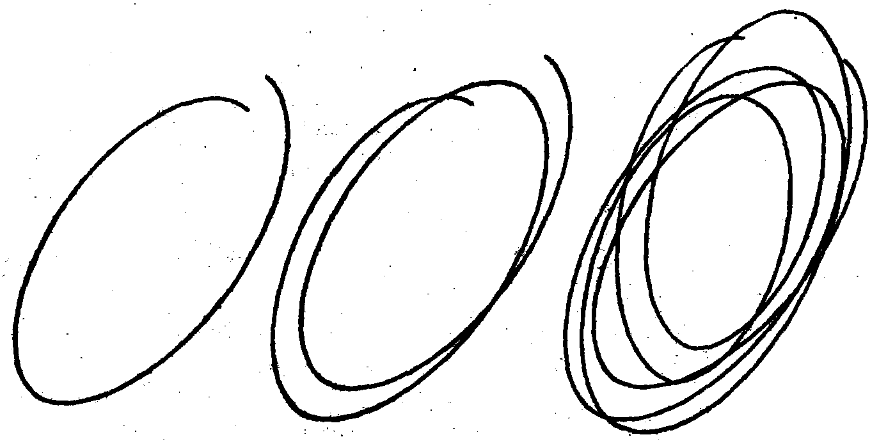

FIG. 1.3. Appearance of sectional pattern of an elliptically polarized beam having small but appreciable bandwidth, assuming observation times of (*a*) about 1 cycle, (*b*) about 2 cycles, (*c*) many cycles.

The general sectional pattern of a monochromatic beam — an ellipse — may be described with the aid of the terms defined in Fig. 1.4. The angle α (between the major semiaxis and the X-axis) is called the ***azimuth*** of the sectional pattern; $90° \geq \alpha \geq -90°$. The ratio b/a of the semiaxes is called the ***ellipticity;*** the symbol β may be used to represent arc tan b/a; $90° \geq \beta \geq -90°$. Ellipticity is used in preference to the eccentricity, which is defined as $(a^2 - b^2)^{1/2}/a$.

In some instances the ratio A_y/A_x is of interest; A_y is the maximum value of the Y-component of the electric vector, and A_x is the maximum value of the X-component. The angle $|\text{arc tan } (A_y/A_x)|$ will be

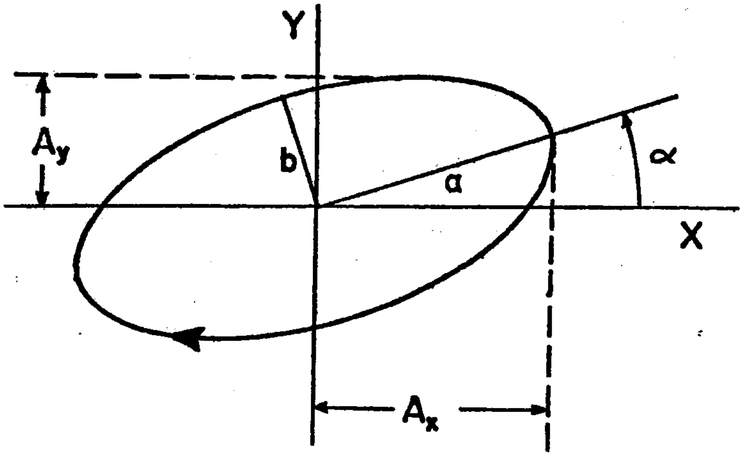

FIG. 1.4. Elliptically polarized light. In this example, $\alpha = 20°$, the ellipticity $b/a = 0.4$, and the handedness is clockwise, as judged by an observer situated far out on the Z-axis and looking backward toward the source, which is at the origin.

called R. When α is in the neighborhood of $\pm 45°$, or when the ellipse is very slender, the angles $|\alpha|$ and R are not very different, but under other circumstances they differ greatly.

Polarization Types and Forms. Linear polarization, circular polarization, and elliptical polarization may be referred to as the three *polarization types*. Obviously, the elliptical type includes the others as special cases; ellipticities of 0 and 1 correspond to linear and circular polarization respectively.

The linear type of polarization includes an infinite number of *polarization forms*, differing as to azimuth α. Circular polarization includes two forms, differing as to handedness. Elliptical polarization includes an infinite number of forms, differing as to azimuth, ellipticity, and handedness.

Orthogonal Forms. Two forms of linear polarization that differ by exactly 90° in azimuth are said to be orthogonal, assuming the directions of propagation to be the same (Fig. 1.5). Right- and left-circularly polarized beams are orthogonal. Two elliptically polarized beams are orthogonal if the azimuths of the major axes differ by 90°, the handednesses are opposite, and the ellipticities are identical.

Plane of Polarization. The expression *plane of polarization*, used by many authors, may be ambiguous. To some authors it means the plane containing the directions of propagation and of the electric

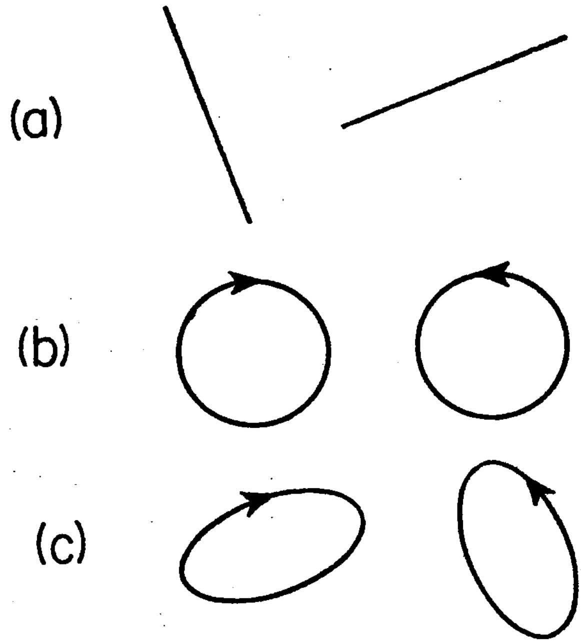

FIG. 1.5. Orthogonal pairs of beams polarized (*a*) linearly, (*b*) circularly, (*c*) elliptically.

vibration, while to others it means the plane containing the directions of propagation and of the *magnetic* vibration. Another drawback to the expression is that an experimenter can easily produce a number of beams that have the *same* plane of polarization yet *different* directions of electric vibration (Fig. 1.6). Likewise he can produce beams having *different* planes of polarization and the *same* direction of vibration.

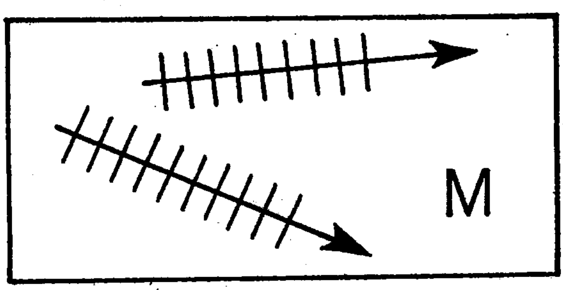

FIG. 1.6. Two beams having the same plane of polarization (plane *M*) yet different directions of electric vibration (indicated by hatch marks, all of which lie in plane *M*).

In this book these difficulties are avoided by the expedient of using terms descriptive of the key quantity, namely, the direction of the electric vibration. We refer to *linearly* polarized light and to the *direction of vibration*. We avoid expressions such as *plane polarized light*, and *plane of polarization*.

1.3. Mathematical Specification of a Polarized Wave Train. The mathematical specification of a wave train is explained in the standard textbooks on electromagnetic theory (D-10; A-1). Simple, monochromatic, linearly polarized trains of plane waves are propagated by means of transverse displacements varying sinusoidally with time and with position along the propagation direction. The magnitude ξ of the electric displacement **E** may be described by an expression such as

$$\xi = \sin(\omega t - 2\pi Z/\lambda),$$

where Z is the position along the axis of propagation, λ is the wavelength, ω is the angular frequency (2π times the ordinary frequency), and t is time.

To facilitate computations of certain sorts, one may introduce complex notation. To make the expressions more versatile, one may include a constant A, called the magnitude of the peak amplitude, and a quantity ϵ, called the epoch. The expression may take any of the following forms:

$$\begin{aligned}\xi &= Ae^{i\epsilon}e^{i\omega t}e^{-i2\pi Z/\lambda}\\ &= Ae^{i\epsilon}e^{i(\omega t-2\pi Z/\lambda)}\\ &= Ae^{i(\epsilon+\omega t-2\pi Z/\lambda)}\\ &= Ae^{i\phi},\end{aligned}$$

where $\phi = \epsilon + \omega t - 2\pi Z/\lambda$. The real part of ξ represents the instantaneous magnitude of the electric vector **E** (at time t and position Z). The quantity $Ae^{i\phi}$ is called the complex amplitude. The quantity ϕ $[= \epsilon + \omega t - 2\pi Z/\lambda]$, is the phase angle at time t and position Z. The intensity of the beam depends on A, and is, of course, proportional to A^2.

The term *intensity* is used in this book in a number of ways. Sometimes it means the total power of the beam. On other occasions it means power per unit solid angle, or power per unit solid angle and per unit cross-sectional area. The intended meaning is usually made clear by the context. Formal definitions of intensity are presented by Chandrasekhar (C-8).

The direction of the electric vector does not appear in the equations, but may be specified separately — verbally or pictorially; or it may be specified by including unit vectors **i** and **j**, parallel to the X- and Y-axes respectively.

A circularly polarized beam of monochromatic light may be represented by a combination of two expressions, each having a complex magnitude of the form $Ae^{i\phi}$. One expression describes the vertical component (Y-component), and is written $A_y e^{i\phi_y}$; the other describes the horizontal component (X-component) and is written $A_x e^{i\phi_x}$. The amplitudes A_y and A_x are equal, and the phase angles ϕ_y and ϕ_x differ by 90°. If the difference $(\phi_y - \phi_x)$, called γ, is positive (90°), the light is *right*-circularly polarized. If $\gamma = -90°$, the light is left-circularly polarized.

In the general case, the Y- and X-components differ in amplitude, and γ may have any value; the general sectional pattern is, of course, an ellipse. If $180° > \gamma > 0°$, the handedness is right; if $-180° < \gamma < 0°$, the handedness is left. When $\gamma = 0°$, the pattern consists of a straight line (linear polarization), and when $|\gamma| = 90°$ the pattern is a circle.

To predict the outcome of adding two monochromatic polarized beams, one adds their instantaneous electric vectors. In general, the two vectors have different directions in real, three-dimensional space, different frequencies, and unrelated phases; hence the result of the addition is a complicated and not very useful expression. In the simple case in which both beams are linearly polarized and have the same frequency and same phase, the procedure is simply to add the two vectors representing the root-mean-square electric vibrations of the two beams (Ref. W-1, p. 27).

If the two linearly polarized beams differ in phase (by some constant amount), the procedure is more complicated; if the phase difference is 180°, and if the two beams have equal intensity, the combined beam will have zero intensity. When two *coherent* beams intersect at a slight angle, the phase relation varies, of course, from one point to another in the region of intersection (as explained in Refs. D-10 and B-43, coherent beams are beams whose phases have a fixed, or virtually fixed, relation to one another); consequently the combined beams will have high intensity at some points and low intensity at others, so that an *interference pattern* results.

If the beams are completely *incoherent*, a short-cut procedure is available: merely add the intensities of the beams. The sum of the intensities is the intensity of the combined beam.

Later chapters make it clear that the combining of beams can usually be handled more simply by certain modern methods than by the classical equations presented above. The Stokes vector provides an ideal basis for treating the combining of incoherent beams; the Jones vector is eminently applicable to coherent beams. These vectors are discussed in Chapter 2.

1.4. Unpolarized Light. Defined operationally, an unpolarized beam is a beam that, when operated on by any elementary kind of energy-conserving device that divides the beam into two completely polarized subbeams, yields subbeams that have *equal power* (in a time interval long enough to permit the powers to be measured). Thus if a beam is to qualify as unpolarized, it must exhibit no long-term preference as to lateral direction of vibration or as to handedness.

Can a perfectly monochromatic beam qualify as unpolarized? Obviously it cannot. Such a beam necessarily has a perfectly regular wave train, and consequently has a very definite and steady sectional pattern. Thus it exhibits polarization. Almost perfectly monochromatic radio waves are a common occurrence, and are found, of course, to exhibit a high degree of polarization.

Visible light, however, always possesses an appreciable bandwidth. Accordingly, such a beam may include many different forms of polarization simultaneously. If an experimenter is unable to detect any preponderant azimuth or handedness, he will perforce regard the beam as being unpolarized. (This subject has been explored by Langsdorf and DuBridge, L-14, and by Birge and DuBridge, B-29.) In this book the expression "monochromatic light" often appears; usually it means light that is *roughly* monochromatic and has sufficient bandwidth that unpolarized behavior is not precluded.

At most moments, a beam of unpolarized light has, of course, a sectional pattern that is elliptical. Hurwitz (H-41) has computed the average value of ellipticity, which turns out to be tan 15° or 0.268.

No satisfactory way of describing unpolarized light pictorially has been found. To portray unpolarized light as a many-pointed star or asterisk is conventional, but without scientific merit; the portrayal fails to suggest the most prominent features of unpolarized light: its constantly changing, predominantly elliptical, character.