



# Robust and Adaptive Model Predictive Control of Nonlinear Systems

Martin Guay, Veronica Adetola and Darryl DeHaan

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### Chapter 1

#### Introduction

Most physical systems possess parametric uncertainties or unmeasurable disturbances. Examples in chemical engineering include reaction rates, activation energies, fouling factors, and microbial growth rates. Since parametric uncertainty may degrade the performance of model predictive control (MPC), mechanisms to update the unknown or uncertain parameters are desirable in application. One possibility would be to use state measurements to update the model parameters offline. A more attractive possibility is to apply adaptive extensions of MPC in which parameter estimation and control are performed online.

The literature contains very few results on the design of adaptive nonlinear MPC (NMPC) [1, 125]. Existing design techniques are restricted to systems that are linear in the unknown (constant) parameters and do not involve state constraints. Although MPC exhibits some degree of robustness to uncertainties, in reality, the degree of robustness provided by nominal models or certainty equivalent models may not be sufficient in practical applications. Parameter estimation error must be accounted for in the computation of the control law.

This book attempts to bridge the gap in adaptive robust NMPC. It proposes a design methodology for adaptive robust NMPC systems in the presence of disturbances and parametric uncertainties. One of the key concepts pursued is set-based adaptive parameter estimation. Set-based techniques provide a mechanism to estimate the unknown parameters as well as an estimate of the parameter uncertainty set. The main difference with established set-based techniques that are commonly used in optimization is that the proposed approach focusses on real-time uncertainty set estimation. In this work, the knowledge of uncertain set estimates are exploited in the design of robust adaptive NMPC algorithms that guarantee robustness of the NMPC system to parameter uncertainty. Moreover, the adaptive NMPC system is shown to recover nominal NMPC performance when parameters have been shown to converge to their true values.

The book provides a comprehensive introduction to NMPC and nonlinear adaptive control. In the first part of the book, a framework for the study, design, and analysis of NMPC systems is presented. The framework highlights various mechanisms that can be used to improve computational requirements of standard NMPC systems. The robustness of NMPC is presented in the context of this framework.

The second part of the book presents an introduction to adaptive NMPC. Starting with a basic introduction to the problems associated with adaptive MPC, a robust