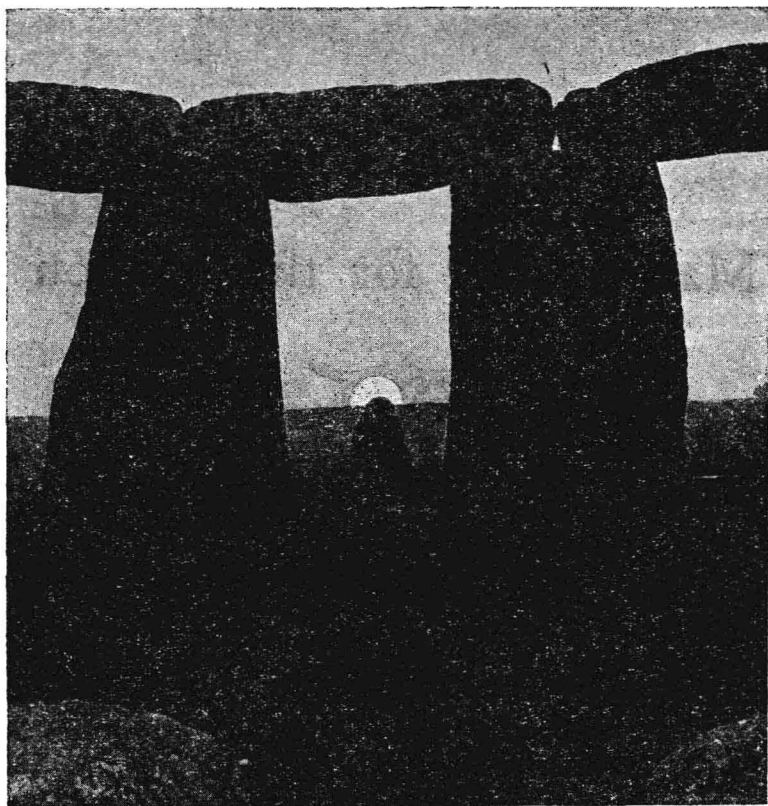


Mathematics for the Million



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The earliest geometrical problems arose from the need for a calendar to regulate the seasonal pursuits of settled agriculture. The recurrence of the seasons was recognized by erecting monuments in line with the rising, setting or transit of celestial bodies. This photograph, taken at Stonehenge, shows how the position of a stone marked the day of the summer solstice when the sun rises furthest north along the eastern boundary of the horizon.

Mathematics for the Million

by *LANCELOT HOGBEN*

Illustrated by
J. F. HORRABIN



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TO
C. K. OGDEN

“It is a remarkable fact that the mathematical inventions which have proved to be most accessible to the masses are also those which exercised the greatest influence on the development of pure mathematics.”

Tobias Dantzig in NUMBER. The
Language of Science

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Author's Excuse for Writing It

I WROTE this book in hospital during a long illness for my own fun. A few friends from among the million or so intelligent people who have been frightened by mathematics while at school persuaded me to publish it. I have agreed to do so on the understanding that they would relieve me of a task, which would interfere with my professional work, by correcting the proofs.

With no pretensions to be a specialist I want to make two things clear. The first is that I have written it in my capacity as a private citizen interested in education. The other is that, whatever objections may be raised against the approach adopted and views expressed in it, it will have fulfilled its aim if it stimulates the interest and removes the inferiority complex of some of the million or so who have given up hope of learning through the usual channels. Readers accustomed to the appearance of the heavens in the southern hemisphere will need to bear in mind that the illustrations are constructed from the standpoint of an observer living north of the Equator. The asides and soliloquies should not be taken too seriously. They are put in to sweeten the pill. Maybe many of them have as little nutritive value as saccharine.

The reader who wishes to know more about the place of mathematics in the history of culture may consult such works as the histories of Rouse Ball, Cantor, Heath, Sullivan, Dantzig, Cajori, the invaluable two volumes of Smith, and Neugebauer's recent treatise on early mathematics. Other useful books in which the relation of mathematics to the growth of scientific knowledge is indicated are Abel Rey's *La Science Orientale avant les Grecs*, Carl Snyder's *The World Machine*, Sir Norman Lockyer's *Stonehenge and other British Monuments*, and Wright's *Geographical Lore in the Time of the Crusades*. Two volumes which have appeared since this book was written are worth adding. One is Professor Farrington's *Science in Antiquity*. The other is Bell's *Search for Truth*. A good general textbook for the reader who wishes to get practice is Mann's *Practical Mathematics*.

If the book had any pretensions to be a work of scholarship, the writer would have documented it, and made it clear when, as often happens, the views expressed are not shared by others. It would then

have been less lively, and therefore less adapted to fulfil the only aim which seemed to justify its publication.

Many helpful suggestions made by readers of the first edition have been incorporated in the new one, which has been carefully revised by Dr. J. C. P. Miller, Lecturer in Applied Mathematics in the University of Liverpool. One of the appendices was added at the suggestion of Professor E. Neville.

Foreword to the Second American Edition

IN THE foreword to the first edition of this book I explained how I wrote it for my own fun while in hospital during a long illness. I insisted that it was never intended to be regarded as the work of a specialist with academic pretensions to rank as an authoritative textbook. As I blended into the writing of it the gleanings of what I had read about the history of civilization and of mathematics from authors who know more than I am entitled to assess, I amused myself by recalling difficulties which I had painfully mastered before I understood difficulties which cleverer people had surmounted more tortuously and perhaps more painfully in earlier chapters of the romance of human enlightenment. So it would not be strictly true to say that no one was more surprised by the success of the book than the author. The real truth is that it was written in the conviction that the study of mathematics could be made exciting to ordinary people, as, for instance, myself.

When I left hospital, I had no intention of publishing it in its original form, and did so only because of the importunity of my publisher, Mr. Norton who is also—paradoxically—my friend. If the fact that non-mathematical readers for whom it was intended showed their appreciation in the way which is most congenial to an author who has family responsibilities did not surprise me as much as it surprised my friends, it is fair to say that I should have taken more trouble with the preparation of the first edition, had I realized the dimensions of the public it was destined to reach. What did surprise me was the cordiality and generosity with which critics who themselves were specialists commended such merits as it has and condoned its—to them—evident defects. Many of my reviewers wrote to me personally making suggestions for the improvement of a subsequent edition, and I have since received hundreds of letters from appreciative readers who have pointed out obscurities or errors in proof correcting. The result is that the present edition is in a real sense, a co-operative work, thanks in no small measure to the many American readers who have enjoyed the earlier edition or have wished to see the book prosper. Their names are too numerous to mention and I can only thank them collectively as one of the authors of what I have now come to regard as a work of collective American effort.

Besides the various improvements indicated in the foregoing paragraph the second edition has several new features, appendices added at the suggestion of Professor Neville, of Reading University, an index, and answers to problems and examples prepared by my friend J. C. P. Miller, Lecturer in Applied Mathematics in Liverpool University.

The existence of the second edition is sufficient evidence that the book has fulfilled its function of stimulating an interest among those whom the formal approach of school and college has failed to satisfy. I need not apologize for the defects which remain, because the reader who is able to get his inferiority complex cured painlessly can continue his studies with other books by writers who know more than I do. What I specially wish to express in this foreword is my gratitude to all the nameless co-authors who have contributed to the making of what is at least a work of entertainment.

Lancelot Hogben

Mathematics for the Million

