



10TH ... EDITION

# AUTOMATIC CONTROL SYSTEMS

FARID GOLNARAGHI

BENJAMIN C. KUO

Mc  
Graw  
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Education

# Automatic Control Systems

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**FARID GOLNARAGHI**

*Simon Fraser University*

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*University of Illinois at Urbana-Champaign*

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**Automatic Control Systems, Tenth Edition**

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*To my wife, Mitra, and my daughters, Sophia and Carmen,  
the joys of my life.*

FG

## About the Authors

**Dr. Farid Golnaraghi** has been a professor and the Director of Mechatronic Systems Engineering at Simon Fraser University in Vancouver since 2006. He also holds a Burnaby Mountain Endowed Chair at SFU, and his primary research focuses on intelligent vehicle systems. Prior to joining SFU, Dr. Golnaraghi was a professor of Mechanical and Mechatronics Engineering at the University of Waterloo. His pioneering research has resulted in two textbooks, more than 150 journal and conference papers, four patents, and two start-up companies.

**Dr. Benjamin C. Kuo** was a member of the faculty of the Department of Electrical and Computer Engineering at the University of Illinois at Urbana-Champaign, which he joined in 1958 upon the completion of his Ph.D. and where he remained for 31 rewarding years. He was a true visionary pioneer in the field of automatic control, and the impact of his distinguished career as a researcher and educator cannot be overstated.

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## In Memory of Professor Benjamin C. Kuo— A Visionary Pioneer and a Brilliant Teacher

When people think of an academic institution, it is its people—their intellect, their brilliance, and their impact—that defines it in their mind. The late Professor Benjamin Kuo is one of those brilliant professors who defines the University of Illinois, and, in particular, the Department of Electrical and Computer Engineering.

Ben was both an alumnus and a member of the faculty of the Department of Electrical and Computer Engineering at the University of Illinois. Upon the completion of his Ph.D., in 1958, he joined the faculty of the department and served it for 31 years through a stellar and most-influential career.

Ben was a visionary pioneer in the field of automatic control. His book *Automatic Control Systems*, which was first published in 1962, has been translated into many languages, and through nine editions over almost 50 years became one of the most widely read books in the field.

His symposium on Incremental Motion Control Systems and Devices, which was first held in 1972, became a pole of attraction for researchers and engineers from industry and academia from all over the world and contributed to the advancement of the field in a singular and most-influential manner.

The impact of Ben's distinguished career as a researcher and an educator is best described in terms of the ways he touched the lives of people:

- The over 120 graduate students who did their research under Ben's advising and mentorship, and who launched exciting careers in industry and academia and spread the new ideas of digital control and contributed to the growth of the field.
- The hundreds of thousands of students around the world who, through Ben's book, were introduced to the subject of automatic control and got excited about its multifaceted applications. I was one of them as an undergraduate student in Greece.
- And, finally, the billions of people all over the globe whose lives are easier and safer today thanks to the ubiquitous control electronics that Professor Benjamin Kuo dreamed about in the fertile grounds of the University of Illinois several decades ago.

The contributions of a brilliant mind endure long after the person passes and is no longer there.

*Andreas C. Cangellaris*  
Dean, College of Engineering, and  
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University of Illinois at Urbana-Champaign

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# Preface

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This tenth edition is with a new publisher, McGraw-Hill Education. It represents a complete overhaul of the textbook, which delivers practical coverage designed to introduce readers to the essential concepts of automatic control systems. The new edition features significant enhancements of all chapters, a greater number of solved examples, labs using both LEGO® MINDSTORMS® and MATLAB®/SIMLab, and a valuable introduction to the concept of Control Lab. The book gives students a real-world understanding of the subject and prepares them for the challenges they will one day face.

For this edition, we increased the number of examples, added more MATLAB toolboxes, and enhanced the MATLAB GUI software, ACSYS, to allow interface with LEGO MINDSTORMS. We also added more computer-aided tools for students and teachers. The new edition has been 5 years in the making, and was reviewed by many professors to better fine-tune the new concepts. In this edition, Chaps. 1 through 3 are organized to contain all background material, while Chaps. 4 through 11 contain material directly related to the subject of control. The Control Lab material is presented in great detail in App. D.

The following appendices for this book can be found at [www.mhprofessional.com/golnaraghi](http://www.mhprofessional.com/golnaraghi):

Appendix A: Elementary Matrix Theory and Algebra

Appendix B: Mathematical Foundations

Appendix C: Laplace Transform Table

Appendix D: Control Lab

Appendix E: ACSYS 2013: Description of the Software

Appendix F: Properties and Construction of the Root Loci

Appendix G: General Nyquist Criterion

Appendix H: Discrete-Data Control Systems

Appendix I: Difference Equations

Appendix J:  $z$ -Transform Table

The following paragraphs are aimed at three groups: professors who have adopted the book or who we hope will select it as their text; practicing engineers looking for answers to solve their day-to-day design problems; and, finally, students who are going to live with the book because it has been assigned for the control-systems course they are taking.

## To Professors

The material assembled in this book is an outgrowth of junior- and senior-level control-systems courses taught by Professors Golnaraghi and Kuo at their respective universities throughout their teaching careers. The first nine editions have been adopted by hundreds of universities in the United States and around the world and have been translated into at least six languages.

Most undergraduate control courses have labs dealing with time response and control of dc motors—namely, speed response, speed control, position response, and position control. In many cases, because of the high cost of control lab equipment, student exposure



to test equipment is limited, and as a result, many students do not gain a practical insight into the subject. In this tenth edition, recognizing these limitations, we introduce the concept of **Control Lab**, which includes two classes of experiments: **SIMLab (model-based simulation)** and **LEGOLab (physical experiments)**. These experiments are intended to supplement, or replace, the experimental exposure of the students in a traditional undergraduate control course.

In this edition, we have created a series of inexpensive control experiments for the **LEGO MINDSTORMS NXT** dc motor that will allow students to do their work within the **MATLAB** and **Simulink**<sup>®</sup> environment—even at home. See App. D for more details. This cost-effective approach may allow educational institutions to equip their labs with a number of LEGO test beds and maximize student access to the equipment at a fraction of the cost of currently available control-systems experiments. Alternatively, as a supplemental learning tool, students can take the equipment home after leaving a deposit and learn at their own pace. This concept has proven to be extremely successful at Simon Fraser University, Professor Golnaraghi's home university in Vancouver, Canada.

The labs include experiments on **speed and position control** of dc motors, followed by a **controller design project** involving control of a simple robotic system conducting a pick-and-place operation and position control of an elevator system. Two other projects also appear in Chaps. 6 and 7. The specific goals of these new experiments are

- To provide an in-depth, practical discussion of the dc motor speed response, speed control, and position control concepts.
- To provide examples on how to identify the parameters of a physical system, experimentally.
- To give a better feel for controller design through realistic examples.

This text contains not only conventional MATLAB toolboxes, where students can learn MATLAB and utilize their programming skills, but also a graphical MATLAB-based software, **ACSYS**. The **ACSYS** software added to this edition is very different from the software accompanying any other control book. Here, through extensive use of MATLAB GUI programming, we have created software that is easy to use. As a result, students need only to focus on learning control problems, not programming!

### To Practicing Engineers

This book was written with the readers in mind and is very suitable for self-study. Our objective was to treat subjects clearly and thoroughly. The book does not use the theorem-proof-Q.E.D. style and is without heavy mathematics. We have consulted extensively for wide sectors of the industry for many years and have participated in solving numerous control-systems problems, from aerospace systems to industrial controls, automotive controls, and control of computer peripherals. Although it is difficult to adopt all the details and realism of practical problems in a textbook at this level, some examples and problems reflect simplified versions of real-life systems.

### To Students

You have had it now that you have signed up for this course and your professor has assigned this book! You had no say about the choice, though you can form and express your opinion on the book after reading it. Worse yet, one of the reasons that your professor made the selection is because he or she intends to make you work hard. But please don't misunderstand us: What we really mean is that, though this is an easy book to study (in our opinion), it is a no-nonsense book. It doesn't have cartoons or nice-looking photographs to amuse you. From here on, it is all business and hard work. You should have had the prerequisites

of subjects found in a typical linear-systems course, such as how to solve linear ordinary differential equations, Laplace transforms and applications, and time-response and frequency-domain analysis of linear systems. In this book, you will not find too much new mathematics to which you have not been exposed before. What is interesting and challenging is that you are going to learn how to apply some of the mathematics that you have acquired during the past 2 or 3 years of study in college. In case you need to review some of the mathematical foundations, you can find them in the appendices, at [www.mhprofessional.com/golnaraghi](http://www.mhprofessional.com/golnaraghi). You will also find the Simulink-based SIMLab and LEGOLab, which will help you to gain understanding of real-world control systems.

This book has numerous illustrative examples. Some of these are deliberately simple for the purpose of showing new ideas and subject matter. Some examples are more elaborate, in order to bring the practical world closer to you. Furthermore, the objective of this book is to present a complex subject in a clear and thorough way. One of the important learning strategies for you as a student is not to rely strictly on the textbook assigned. When studying a certain subject, go to the library and check out a few similar texts to see how other authors treat the same subject. You may gain new perspectives on the subject and discover that one author treats the material with more care and thoroughness than the others. Do not be distracted by written-down coverage with oversimplified examples. The minute you step into the real world, you will face the design of control systems with nonlinearities and/or time-varying elements as well as orders that can boggle your mind. You may find it discouraging to be told now that strictly linear and first-order systems do not exist in the real world.

## Special Acknowledgments

Farid Golnaraghi wishes to thank Robert L. Argentieri, Editorial Director, Engineering, International & Professional Group, McGraw-Hill Education, for his efforts and support in publishing this new edition. Special thanks go to the reviewers for their invaluable comments and suggestions; the prepublication reviews have had a great impact on this revision. More special thanks go to the many School of Mechatronic Systems Engineering at Simon Fraser University faculty, students, and research associates, and all contributing undergraduate co-op students.

Professor Golnaraghi is grateful to and recognizes the Kuo estate, particularly Lori Dillon, for their support of this project.

Finally, he wishes to thank the late Professor Benjamin Kuo for sharing the pleasure of writing this wonderful book, and for his teachings, patience, and support throughout this experience.

*Farid Golnaraghi*  
*Vancouver, British Columbia, Canada*

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