Graduate Texts in Mathematics

Rainer Kress

Numerical Analysis

数值分析

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Preface

No applied mathematician can be properly trained without some basic understanding of numerical methods, i.e., *numerical analysis.* And no scientist and engineer should be using a package program for numerical computations without understanding the program's purpose and its limitations. This book is an attempt to provide some of the required knowledge and understanding. It is written in a spirit that considers numerical analysis not merely as a tool for solving applied problems but also as a challenging and rewarding part of mathematics. The main goal is to provide insight into numerical analysis rather than merely to provide numerical recipes.

The book evolved from the courses on numerical analysis I have taught since 1971 at the University of Göttingen and may be viewed as a successor of an earlier version jointly written with Bruno Brosowski [10] in 1974. It aims at presenting the basic ideas of numerical analysis in a style as concise as possible. Its volume is scaled to a one-year course, i.e., a two-semester course, addressing second-year students at a German university or advanced undergraduate or first-year graduate students at an American university.

In order to make the book accessible not only to mathematicians but also to scientists and engineers, I have planned it to be as self-contained as possible. As prerequisites it requires only a solid foundation in differential and integral calculus and in linear algebra as well as an enthusiasm to see these fundamental and powerful tools in action for solving applied problems. A short presentation of some basic functional analysis is provided in the book to the extent required for a modern presentation of numerical analysis and a deeper understanding of the subject.

An introductory book of a few hundred pages cannot completely cover all classical aspects of numerical analysis and all of the more recent developments. I am willing to admit that the choice of some of the topics in the present volume is biased by my own preferences and that some important subjects are omitted.

I was taught numerical analysis in the mid sixties by my thesis adviser, Professor Erich Martensen, at the Technische Hochschule in Darmstadt. Martensen's perspective on teaching mathematics in general and numerical analysis in particular had a great and long-lasting impact on my own teaching. Therefore, this book is dedicated to Erich Martensen on the occasion of his seventieth birthday.

I would like to thank Thomas Gerlach and Peter Otte for carefully reading the book, for checking the solutions to the problems, and for a number of suggestions for improvements. Special thanks are given to my friend David Colton for reading over the book for correct use of the English language. Part of the book was written while I was on sabbatical leave at the Department of Mathematical Sciences at the University of Delaware and the Department of Mathematics at the University of New South Wales. I gratefully acknowledge the hospitality of these institutions. I also am grateful to Springer-Verlag for being willing to take the economic risk of adding yet another volume to the already huge number of existing introductions to numerical analysis.

Göttingen, September 1997 Rainer Kress

Preface

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Glossary of Symbols

Sets and Spaces

$\mathbb{N}$	set of natural numbers
$\mathbb{Z}$	set of integers
$\mathbb{R}$	set of real numbers
$\mathbb{C}$	set of complex numbers
$\lvert x \rvert$	absolute value of a real or complex number x
(a, b)	open interval $(a, b) := \{x \in \mathbb{R} : a < x < b\}$
$[a, b]$	closed interval $[a, b] := \{x \in \mathbb{R} : a \leq x \leq b\}$
$\overline{x}$	conjugate of a complex number x
$\mathbb{R}^n$	n-dimensional real Euclidean space
$\mathbb{C}^n$	n-dimensional complex Euclidean space
$C[a, b]$	space of real- or complex-valued continuous functions on the interval $[a, b]$
$C^m[a, b]$	space of m-times continuously differentiable functions
$L^2[a, b]$	space of real- or complex-valued square-integrable functions
$\{a_1, \ldots, a_m\}$	set of m elements $a_1, \ldots, a_m$
$U \times V$	product $U \times V := \{(x, y) : x \in U,\ y \in V\}$ of two sets U and V
$U \setminus V$	difference set $U \setminus V := \{x \in U : x \notin V\}$ for two sets U and V
$\overline{U}$	closure of a set U
$F : X \to Y$	a mapping with domain X and range in Y

Vectors and Matrices

$x = (x_1, \ldots, x_n)$	row vector in $\mathbb{R}^n$ or $\mathbb{C}^n$ with components $x_1, \ldots, x_n$
$x^T = (x_1, \ldots, x_n)^T$	the transpose of x, i.e., a column vector
$x^* = (\bar{x}_1, \ldots, \bar{x}_n)^T$	the adjoint of x
$A = (a_{jk})$	$m \times n$ matrix with elements a_{jk}
A^T	the transpose of A
A^*	the adjoint of A
$A^\dagger$	the pseudo-inverse of A
A^{-1}	the inverse of an $n \times n$ matrix A
$\det A$	the determinant of an $n \times n$ matrix A
$\mathrm{cond}(A)$	the condition number of an $n \times n$ matrix A
$\rho(A)$	the spectral radius of an $n \times n$ matrix A
I	the $n \times n$ identity matrix
$\mathrm{diag}(a_1, \ldots, a_n)$	diagonal matrix with diagonal elements $a_1, \ldots, a_n$

Norms

$\Vert \cdot \Vert$	norm on a linear space
$\Vert \cdot \Vert_1$	ℓ_1 norm of a vector, L_1 norm of a function
$\Vert \cdot \Vert_2$	ℓ_2 norm of a vector, L_2 norm of a function
$\Vert \cdot \Vert_\infty$	maximum norm of a vector or a function
$(\cdot, \cdot)$	scalar product on a linear space

Miscellaneous

$\in$	element inclusion
$\subset$	set inclusion
$\cup, \cap$	union and intersection of sets
$\emptyset$	empty set
$O(m)$	a quantity of order m
$\square$	end of proof

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Contents

1
Introduction

Numerical analysis is concerned with the development and investigation of *constructive methods* for the numerical solution of mathematical problems. This objective differs from a pure-mathematical approach as illustrated by the following three examples.

By the fundamental theorem of algebra, a polynomial of degree n has n complex zeros. The various proofs of this result, in general, are nonconstructive and give no procedure for the explicit computation of these zeros. Numerical analysis provides constructive methods for the actual computation of the zeros of a polynomial.

The solution of a system of n linear equations for n unknowns can be given explicitly by Cramer's rule. However, Cramer's rule is only of theoretical importance, since for actual computations it is completely useless for linear systems with more than three unknowns. An important task in numerical analysis consists in describing and developing more practical methods for the solution of systems of linear equations.

By the Picard–Lindelöf theorem, the initial value problem for an ordinary differential equation has a unique solution (under appropriate regularity assumptions). Despite the fact that the existence proof in the Picard–Lindelöf theorem actually is constructive through the use of successive iterations, in applied mathematics there is need for more effective procedures to numerically solve the initial value problem.

In general, we may say that for the basic problems in numerical analysis existence and uniqueness of a solution are guaranteed through the results of pure mathematics. The main topic of numerical analysis is to provide efficient numerical methods for the actual computation of the solution. In