国外数学名著系列

（影印版）9

Pieter Wesseling

Principles of Computational Fluid Dynamics

计算流体力学原理

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《国外数学名著系列》(影印版)序

要使我国的数学事业更好地发展起来，需要数学家淡泊名利并付出更艰苦地努力。另一方面，我们也要从客观上为数学家创造更有利的发展数学事业的外部环境，这主要是加强对数学事业的支持与投资力度，使数学家有较好的工作与生活条件，其中也包括改善与加强数学的出版工作。

从出版方面来讲，除了较好较快地出版我们自己的成果外，引进国外的先进出版物无疑也是十分重要与必不可少的。从数学来说，施普林格（Springer）出版社至今仍然是世界上最具权威的出版社。科学出版社影印一批他们出版的好的新书，使我国广大数学家能以较低的价格购买，特别是在边远地区工作的数学家能普遍见到这些书，无疑是对推动我国数学的科研与教学十分有益的事。

这次科学出版社购买了版权，一次影印了23本施普林格出版社出版的数学书，就是一件好事，也是值得继续做下去的事情。大体上分一下，这23本书中，包括基础数学书5本，应用数学书6本与计算数学书12本，其中有些书也具有交叉性质。这些书都是很新的，2000年以后出版的占绝大部分，共计16本，其余的也是1990年以后出版的。这些书可以使读者较快地了解数学某方面的前沿，例如基础数学中的数论、代数与拓扑三本，都是由该领域大数学家编著的“数学百科全书”的分册。对从事这方面研究的数学家了解该领域的前沿与全貌很有帮助。按照学科的特点，基础数学类的书以“经典”为主，应用和计算数学类的书以“前沿”为主。这些书的作者多数是国际知名的大数学家，例如《拓扑学》一书的作者诺维科夫是俄罗斯科学院的院士，曾获“菲尔兹奖”和“沃尔夫数学奖”。这些大数学家的著作无疑将会对我国的科研人员起到非常好的指导作用。

当然，23本书只能涵盖数学的一部分，所以，这项工作还应该继续做下去。更进一步，有些读者面较广的好书还应该翻译成中文出版，使之有更大的读者群。

总之，我对科学出版社影印施普林格出版社的部分数学著作这一举措表示热烈的支持，并盼望这一工作取得更大的成绩。

王　元

2005年12月3日

Preface

The technological value of computational fluid dynamics has become undisputed. A capability has been established to compute flows that can be investigated experimentally only at reduced Reynolds numbers, or at greater cost, or not at all, such as the flow around a space vehicle at re-entry, or a loss-of-coolant accident in a nuclear reactor. Furthermore, modern computational fluid dynamics has become indispensable for design optimization, because many different configurations can be investigated at acceptable cost and in short time. A distinguishing feature of the present state of computational fluid dynamics is, that large commercial computational fluid dynamics computer codes have arisen, and found widespread use in industry. The days that a great majority of code users were also code developers are gone. This attests to the importance and a certain degree of maturity of computational fluid dynamics as an engineering tool. At the same time, this creates a need to go back to basics, and to disseminate the basic principles to a wider audience. It has been observed on numerous occasions, that even simple flows are not correctly predicted by advanced computational fluid dynamics codes, if used without sufficient insight in both the numerics and the physics involved. The present book aims to elucidate the principles of computational fluid dynamics. With a variation on Lamb's preface to his classic Hydrodynamics, owing to the elaborate nature of some of the methods of computational fluid dynamics, it has not always been possible to fit an adequate account of them into the frame of this book.

When technology progresses from the pre-competitive to the competitive stage, unavoidably, something like an information stop sets in. To protect investments, and because of the relatively long learning curve to be traversed in order to become familiar with a large computer code, a certain sluggishness of change makes itself felt. These consequences of the widespread distribution of large computational fluid dynamics codes needs to be counteracted by the dynamics of unencumbered scientific enquiry, not to pursue change for change's sake, but because much improvement seems feasible. Therefore I hope the book will be helpful not only to users of computational dynamics codes, but also to researchers in the field.

The book has grown out of graduate courses for doctoral students and practicing engineers, held under the auspices of the J.M. Burgers Center, the national inter-university graduate school for fluid dynamics in The Netherlands. I expect teachers of advanced courses of computational fluid dynamics courses will find this a useful book. For an introductory course the book seems too advanced, but I have found selected material from the manuscript useful in teaching an introductory undergraduate course.

Other relatively recent introductions to the subject of computational fluid dynamics that the reader will find useful are Ferziger and Perić (1996), Fletcher (1988), Hirsch (1988), Hirsch (1990), Peyret and Taylor (1985), Roache (1998a), Shyy (1994), Sod (1985), Tannehill, Anderson, and Pletcher (1997), Versteeg and Malalasekera (1995), Wendt (1996). The two volumes by Hirsch give an èspecially wide coverage. The present book differs from these works in the following respects. More mathematical and numerical analysis is given, but the mathematical background of the reader is assumed not to go beyond what physicists and engineers are generally familiar with. The maximum principle for differential equations and numerical schemes gets generous attention, in order to put discussions of spurious 'wiggles', accuracy of schemes on nonuniform grids, and accuracy of numerical boundary conditions on a firm footing. Singular perturbation theory is introduced to predict qualitative features of the flow, to which numerical methods can be adapted for better accuracy and efficiency. In particular, singular perturbation theory is used with a fair amount of rigor to demonstrate convincingly how it is possible to achieve accuracy and computing cost uniform in the Reynolds number, showing that a 'numerical windtunnel' that operates at arbitrarily high Reynolds number is feasible, notwithstanding the effect of 'numerical viscosity'. Much attention is given to the principles and the application of von Neumann stability analysis, giving useful stability conditions, some of them new, for many schemes used in practice. Godunov's order barrier and how to overcome it by slope-limited schemes is discussed extensively. The theory of scalar conservation laws including the nonconvex case is treated. Distributive iteration is used as a unifying framework for describing iterative methods for the incompressible Navier-Stokes equations. The principles of Krylov subspace and multigrid methods for efficient solution of the large sparse algebraic systems that arise are introduced. Much attention is given to the complications brought about by geometric complexity of the flow domain, including an introduction to tensor analysis. A chapter on unified methods to compute incompressible and compressible flows is included. In order to help the reader along who wants to delve deeper and to quickly reach the current research frontier, references to more advanced literature are provided.

Errata and MATLAB software related to a number of examples discussed in the book my be obtained via the author's website, to be found at
`ta.twi.tudelft.nl/nw/users/wesseling`

Combining the writing of a textbook of this size with the daily tasks of a university professor was not always easy, and would have been impossible without the support of the numerical team, and in particular our secretary Tatiana Tijanova. Her dedication, love of perfection and capability to cope with repeated stress were of vital importance for keeping the manuscript organized, and finally bringing it into publishable form. I am indebted to dr. C. Vuik for advice on Chap. 7, to professor G.S. Stelling for checking up on Chap. 8, and to professor F.T.M. Nieuwstadt for casting a critical eye on what I wrote about turbulence. The enthusiasm of the students in the graduate courses on computational fluid dynamics of the J.M. Burgers Center, and the cooperation with my fellow teacher professor A.E.P. Veldman, were inspiring and stimulating. The moral support of my wife Tineke was and remains invaluable.

Delft, June 2000 P. Wesseling

Table of Contents

1. The basic equations of fluid dynamics

1.1 Introduction

Fluid dynamics is a classic discipline. The physical principles governing the flow of simple fluids and gases, such as water and air, have been understood since the times of Newton. Sect. IX of the second book of Newton's Principia starts with what came to be known as the Newtonian stress hypothesis :
"The resistance arising from the want of lubricity in the parts of a fluid, is, other things being equal, proportional to the velocity with which the parts of the fluid are separated from another". This hypothesis is followed by Proposition LI, in which the flow generated by a rotating cylinder in an unbounded medium is considered. The period of the orbit of a fluid particle is found to be proportional to the distance r from the cylinder axis. This is not correct. The source of the error is that the master balances force instead of torque; this may be of some consolation to beginning students who find mechanics difficult. The closing remark "All of this can be tested in deep stagnant water" must be taken with a grain of salt. Newton was more interested in celestial mechanics than in fluid dynamics. His aim was to test Descartes's vortex theory of planetary motion, which would gain credibility if the period of the orbit of a particle in this flow would be proportional to $r^{3/2}$; in fact, it is proportional to r^2. The mathematical formulation of the laws that govern the dynamics of fluids has been complete for a century and a half. In the nineteenth century and the beginning of the twentieth, eminent scientists and engineers were drawn to the subject, and gave it clarity, unification and elegance, as exemplified in the classic work of Lamb (1945), that first appeared in 1879. In the preface to the 1932 edition Lamb writes, that the subject has in recent years received considerable developments, classic fluid dynamics having a widening field of practical applications. This has remained true ever since, especially because in the last forty years or so classic fluid dynamics finds itself in the company of computational fluid dynamics. This new discipline still lacks the elegance and unification of its classic counterpart, and is in a state of rapid development, so that we can do no more than give a glimpse of its current status. But first, we take a look at classic fluid dynamics.

Continuum hypothesis

The dynamics of fluids is governed by the conservation laws of classical physics, namely conservation of mass, momentum and energy. From these laws partial differential equations are derived and, under appropriate circumstances, simplified. It is customary to formulate the conservation laws under the assumption that the fluid is a continuous medium (*continuum hypothesis*). Physical properties of the flow, such as density and velocity can then be described as time-dependent scalar or vector fields on $\mathbb{R}^3$, for example $\rho(t, \boldsymbol{x})$ and $\boldsymbol{u}(t, \boldsymbol{x})$.

For the flowing medium we restrict ourselves here to gases and liquids. More general media, such as mixtures of gases and liquids (*multiphase flows*), will not be considered. For a liquid the continuum hypothesis is always satisfied in practice. A gas satisfies the continuum hypothesis (to a sufficient degree) if $\mathrm{K} \ll 1$, with K the *Knudsen number*, defined as $\mathrm{K} = \lambda/L$, with λ the mean free path and L the length scale of the flow phenomenon under study. Consider for example flow over a flat plate with free-stream velocity V (Fig. 1.1). It is known that at the plate a boundary layer is generated, in

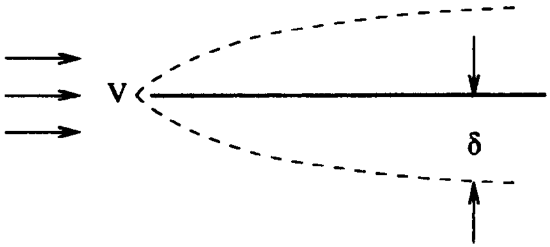

Fig. 1.1. Flow over a flat plate

which the velocity changes from zero to V. The thickness δ of this boundary layer is the relevant length scale. If the fluid is air at room temperature then $\lambda = 0.4\ \mu\mathrm{m}$. With $V = 1$ m/s, experiment and boundary layer theory tell us that $\delta \cong 2.5$ cm at 0.5 m downstream of the leading edge, so that here $\mathrm{K} = 0.16 * 10^{-4}$. Hence, at this location momentum exchange due to friction takes place over a length scale of about 60,000 mean free paths, and the continuum hypothesis is very well satisfied. Perhaps this would not be so quite near the leading edge of an extremely sharp flat plate, but that need not concern us here. We will throughout assume the continuum hypothesis, and the flowing medium, be it gas or liquid, will often be called the fluid. The most common situation in technology where the continuum hypothesis has to be abandoned is the flow of very rarefied gases. Such a flow regime occurs at a certain stage of atmospheric re-entry of space vehicles. Unexpectedly perhaps, for flows of the interstellar medium often $\mathrm{K} \ll 1$, because of the size of the galactic length scale, so that the continuum hypothesis can be safely applied there.

Lagrangean and Eulerian formulation

The continuum hypothesis enables us to speak of the properties of a flow at a point in space, and of the physical properties of an infinitesimally small volume element of the fluid, to which we shall refer for brevity as a *material particle* . A flow can be described exhaustively by specification of the physical properties of each material particle as a function of time. This kind of specification of a flow is called the *Lagrangean formulation.* Alternatively, a flow may be described by specification of the time history of the flow properties at every fixed point of the domain. This is called the *Eulerian formulation.* The second formulation is usually more accessible for analysis and computation than the first, but sometimes the Lagrangean formulation may be preferable, for example when fluid interfaces have to be tracked. In most cases we ask for flow properties at fixed locations, such as the pressure at a wall, and this information is provided directly by the Eulerian formulation, to which we will adhere throughout this book. The Eulerian and Lagrangean points of view meet in the transport theorem, to be discussed in Sect. 1.3.

Selection of topics

The reader is assumed to be familiar with the principles of fluid dynamics, of vector analysis, of numerical linear algebra and of the numerical analysis of partial differential equations.

Fluid dynamics is a vast discipline, utilizing many different mathematical models. As the Mach number M (to be introduced later) varies, we encounter incompressible ($M \ll 1$), subsonic ($0 < M < 1$), transonic ($M \cong 1$), supersonic ($M > 1$) and hypersonic ($M \gg 1$) flow. In hypersonic flow, chemical processes taking place in the fluid have to be accounted for, giving rise to the discipline of aerothermochemistry. Multiphase flows play a large role in chemical engineering and reservoir engineering. Flows in porous media are governed by the Darcy equations. In hydraulic engineering the shallow-water equations are predominant. In ship hydrodynamics the free surface (water-air interface) often has to be accounted for. Capillary forces may be important. As the Reynolds number (to be introduced shortly) increases, transition from laminar to turbulent flow occurs, giving rise to a plethora of more or less semi-empirical turbulence models. Rotation causes special effects, important in oceanography, and in atmospheric and planetary fluid dynamics. We have made a selection of topics. The book is focussed on the incompressible and compressible Navier-Stokes equations, restricting ourselves to $M \lesssim 2$, thus catering mainly to the needs of industrial and environmental fluid dynamics and aeronautics. We will also pay attention to the shallow-water equations. Fortunately, many of the underlying principles carry over to cases not treated. In particular, a thorough understanding of the analytical and computational aspects of the comparatively simple convection-diffusion equation gives valuable insight in more complex models. Therefore this equation will receive much attention.

Although most practical flows are turbulent, we restrict ourselves here to laminar flow, because this book is on numerics only. Turbulence modeling is a vast subject in itself, that is briefly discussed in Sect. 1.13, where pointers to the literature are given for further study. The numerical principles uncovered for the laminar case carry over to the turbulent case. To facilitate this, viscosity is usually assumed variable.

Fluid dynamics is governed by partial differential equations. These may be solved numerically by finite difference, finite volume, finite element and spectral methods. In engineering applications, finite difference and finite volume methods are predominant. In order to limit the scope of this work, we will confine ourselves to finite difference and finite volume methods.

Since in computational fluid dynamics mathematical modeling aspects invariably play an important role, we devote the remainder of this chapter to a thorough derivation of the basic equations of fluid dynamics and their main simplifications. Of course, the subject cannot be adequately reviewed in a single chapter. For a more extensive treatment, see Batchelor (1967), Chorin and Marsden (1979), Kreiss and Lorenz (1989), Lamb (1945), Landau and Lifshitz (1959), Sedov (1971), Zucrow and Hoffman (1976), Zucrow and Hoffman (1977). A brief introduction to the history of the subject, with references to further literature, is given by Eberle, Rizzi, and Hirschel (1992).

Good starting points for exploration of the Internet for material related to computational fluid dynamics are the following websites:

`www.cfd-online.com/`

`www.princeton.edu/~gasdyn/fluids.html`

and the ERCOFTAC (European Research Community on Flow, Turbulence and Combustion) site:

`imhefwww.epfl.ch/ERCOFTAC/`

Readers well-versed in fluid dynamics may skip the remainder of this chapter, perhaps after taking note of the notation introduced in the next section. But those less familiar with this discipline will find it useful to continue with the present chapter.

1.2 Vector analysis

Cartesian tensor notation

The basic equations will be derived in a right-handed Cartesian coordinate system $(x_1, x_2, ..., x_d)$ with d the number of space dimensions. Boldfaced lower case Latin letters denote vectors, for example, $\boldsymbol{x} = (x_1, x_2, ..., x_d)$. Greek letters denote scalars. In *Cartesian tensor notation*, which we shall often use, differentiation is denoted as follows:

$$\phi_{,\alpha} = \partial\phi/\partial x_\alpha \, .$$

Greek subscripts refer to coordinate directions, and the *summation convention* is used: summation takes place over Greek indices that occur twice in a term or product, for example:

$$u_\alpha v_\alpha = \sum_{\alpha=1}^{d} u_\alpha v_\alpha \, , \quad \phi_{,\alpha\alpha} = \sum_{\alpha=1}^{d} \partial^2\phi/\partial x_\alpha^2 \, .$$

We will also use *vector notation*, instead of the *subscript notation* just explained, and may write div$\boldsymbol{u}$, if this is more elegant or convenient than the tensor equivalent $u_{\alpha,\alpha}$; and sometimes we write grad ϕ for the vector $(\phi_{,1}, \phi_{,2}, \phi_{,3})$.

Divergence theorem

We need the following fundamental theorem:

Theorem 1.2.1. *For any volume $V \subset \mathbb{R}^d$ with piecewise smooth closed surface S and any differentiable scalar field ϕ we have*

$$\int_V \phi_{,\alpha} dV = \int_S \phi n_\alpha dS \, ,$$

where $\boldsymbol{n}$ is the outward unit normal on S.

For a proof, see for example Aris (1962).

A direct consequence of this theorem is:

Theorem 1.2.2. *(Divergence theorem).*
For any volume $V \subset \mathbb{R}^d$ with piecewise smooth closed surface S and any differentiable vector field $\boldsymbol{u}$ we have

$$\int_V \text{div}\boldsymbol{u} dV = \int_S \boldsymbol{u} \cdot \boldsymbol{n} dS \, ,$$

where $\boldsymbol{n}$ is the outward unit normal on S.