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（影印版）4

Olav Kallenberg

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Second Edition

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《国外数学名著系列》(影印版)序

要使我国的数学事业更好地发展起来，需要数学家淡泊名利并付出更艰苦地努力。另一方面，我们也要从客观上为数学家创造更有利的发展数学事业的外部环境，这主要是加强对数学事业的支持与投资力度，使数学家有较好的工作与生活条件，其中也包括改善与加强数学的出版工作。

从出版方面来讲，除了较好较快地出版我们自己的成果外，引进国外的先进出版物无疑也是十分重要与必不可少的。从数学来说，施普林格（Springer）出版社至今仍然是世界上最具权威的出版社。科学出版社影印一批他们出版的好的新书，使我国广大数学家能以较低的价格购买，特别是在边远地区工作的数学家能普遍见到这些书，无疑是对推动我国数学的科研与教学十分有益的事。

这次科学出版社购买了版权，一次影印了 23 本施普林格出版社出版的数学书，就是一件好事，也是值得继续做下去的事情。大体上分一下，这 23 本书中，包括基础数学书 5 本，应用数学书 6 本与计算数学书 12 本，其中有些书也具有交叉性质。这些书都是很新的，2000 年以后出版的占绝大部分，共计 16 本，其余的也是 1990 年以后出版的。这些书可以使读者较快地了解数学某方面的前沿，例如基础数学中的数论、代数与拓扑三本，都是由该领域大数学家编著的“数学百科全书”的分册。对从事这方面研究的数学家了解该领域的前沿与全貌很有帮助。按照学科的特点，基础数学类的书以“经典”为主，应用和计算数学类的书以“前沿”为主。这些书的作者多数是国际知名的大数学家，例如《拓扑学》一书的作者诺维科夫是俄罗斯科学院的院士，曾获“菲尔兹奖”和“沃尔夫数学奖”。这些大数学家的著作无疑将会对我国的科研人员起到非常好的指导作用。

当然，23 本书只能涵盖数学的一部分，所以，这项工作还应该继续做下去。更进一步，有些读者面较广的好书还应该翻译成中文出版，使之有更大的读者群。

总之，我对科学出版社影印施普林格出版社的部分数学著作这一举措表示热烈的支持，并盼望这一工作取得更大的成绩。

王 元

2005 年 12 月 3 日

Praise for the First Edition

"It is truly surprising how much material the author has managed to cover in the book. ... More advanced readers are likely to regard the book as an ideal reference. Indeed, the monograph has the potential to become a (possibly even 'the') major reference book on large parts of probability theory for the next decade or more." *—M. Scheutzow (Berlin)*

"I am often asked by mathematicians ... for literature on 'a broad introduction to modern stochastics.' ... Due to this book, my task for answering is made easier. This is it! A concise, broad overview of the main results and techniques .... From the table of contents it is difficult to believe that behind all these topics a streamlined, readable text is at all possible. It is: Convince yourself. I have no doubt that this text will become a classic. Its main feature of keeping the whole area of probability together and presenting a general overview is a real success. Scores of students ... and indeed researchers will be most grateful!" *—P.A.L. Embrechts (Zürich)*

"The theory of probability has grown exponentially during the second half of the twentieth century, and the idea of writing a single volume that could serve as a general reference ... seems almost foolhardy. Yet this is precisely what Professor Kallenberg has attempted ... and he has accomplished it brilliantly. ... With regard to his primary goal, the author has been more successful than I would have imagined possible. It is astonishing that a single volume of just over five hundred pages could contain so much material presented with complete rigor, and still be at least formally self-contained. ... As a general reference for a good deal of modern probability theory [the book] is outstanding. It should have a place in the library of every probabilist. Professor Kallenberg set himself a very difficult task, and he should be congratulated for carrying it out so well."

—R.K. Getoor (La Jolla, California)

"This is a superbly written, high-level introduction to contemporary probability theory. In it, the advanced mathematics student will find basic information, presented in a uniform terminology and notation, essential to gaining access to much present-day research. ... I congratulate Professor Kallenberg on a noteworthy achievement."

—M.F. Neuts (Tucson, Arizona)

"This is a very modern, very ambitious, and very well-written book. The scope is greater than I would have thought possible in a book of this length. This is made possible by the extremely efficient treatment, particularly the proofs .... [Kallenberg] has succeeded in his mammoth task beyond all reasonable expectations. I think this book is destined to become a modern classic." *—N.H. Bingham (London)*

"Kallenberg has ably achieved [his] goal and presents all the important results and techniques that every probabilist should know. ... We do not doubt that the book ... will be widely used as material for advanced post-graduate courses and seminars on various topics in probability."

—*jste, European Math. Soc. Newsletter*

"This is a very well written book. ... Much effort must have been put into simplifying and streamlining proofs, and the results are quite impressive. ... I would highly recommend [the book] to anybody who wants a good concise reference text on several very important parts of modern probability theory. For a mathematical sciences library, such a book is a must."

—*K. Borovkov (Melbourne)*

"[This] is an unusual book about a wide range of probability and stochastic processes, written by a single excellent mathematician. ... The graduate student will definitely enjoy reading it, and for the researcher it will become a useful reference book and necessary tool for his or her work."

—*T. Mikosch (Groningen)*

"The author has succeeded in writing a text containing—in the spirit of Loève's *Probability Theory*—all the essential results that any probabilist needs to know. Like Loève's classic, this book will become a standard source of study and reference for students and researchers in probability theory."

—*R. Kiesel (London)*

"Kallenberg's present book would have to qualify as the assimilation of probability par excellence. It is a great edifice of material, clearly and ingeniously presented, without any nonmathematical distractions. Readers wishing to venture into it may do so with confidence that they are in very capable hands."

—*F.B. Knight (Urbana, Illinois)*

"The presentation of the material is characterized by a surprising clarity and precision. The author's overview over the various subfields of probability theory and his detailed knowledge are impressive. Through an activity over many years as a researcher, academic teacher, and editor, he has acquired a deep competence in many areas. Wherever one reads, all chapters are carefully worked through and brought in streamlined form. One can imagine what an enormous effort it has cost the author to reach this final state, though no signs of this are visible. His goal, as set forth in the preface, of giving clear and economical proofs of the included theorems has been achieved admirably. ... I can't recall that in recent times I have held in my hands a mathematics book so thoroughly worked through."

—*H. Rost (Heidelberg)*

Preface to the Second Edition

For this new edition the entire text has been carefully revised, and some portions are totally rewritten. More importantly, I have inserted more than a hundred pages of new material, in chapters on general measure and ergodic theory, the asymptotics of Markov processes, and large deviations. The expanded size has made it possible to give a self-contained treatment of the underlying measure theory and to include topics like multivariate and ratio ergodic theorems, shift coupling, Palm distributions, entropy and information, Harris recurrence, invariant measures, strong and weak ergodicity, Strassen's law of the iterated logarithm, and the basic large deviation results of Cramér, Sanov, Schilder, and Freidlin and Ventzel.

Unfortunately, the body of knowledge in probability theory keeps growing at an ever increasing rate, and I am painfully aware that I will never catch up in my efforts to survey the entire subject. Many areas are still totally beyond reach, and a comprehensive treatment of the more recent developments would require another volume or two. I am asking for the reader's patience and understanding.

Many colleagues have pointed out errors or provided helpful information. I am especially grateful for some valuable comments from Wlodzimierz Kuperberg, Michael Scheutzow, Josef Teichmann, and Hermann Thorisson. Some of the new material was presented in our probability seminar at Auburn, where I benefited from stimulating discussions with Bill Hudson, Ming Liao, Lisa Peterson, and Hussain Talibi. My greatest thanks are due, as always, to my wife Jinsoo, whose constant love and support have sustained and inspired me throughout many months of hard work.

Olav Kallenberg *March 2001*

Preface to the First Edition

Some thirty years ago it was still possible, as Loève so ably demonstrated, to write a single book in probability theory containing practically everything worth knowing in the subject. The subsequent development has been explosive, and today a corresponding comprehensive coverage would require a whole library. Researchers and graduate students alike seem compelled to a rather extreme degree of specialization. As a result, the subject is threatened by disintegration into dozens or hundreds of subfields.

At the same time the interaction between the areas is livelier than ever, and there is a steadily growing core of key results and techniques that every probabilist needs to know, if only to read the literature in his or her own field. Thus, it seems essential that we all have at least a general overview of the whole area, and we should do what we can to keep the subject together. The present volume is an earnest attempt in that direction.

My original aim was to write a book about "everything." Various space and time constraints forced me to accept more modest and realistic goals for the project. Thus, "foundations" had to be understood in the narrower sense of the early 1970s, and there was no room for some of the more recent developments. I especially regret the omission of topics such as large deviations, Gibbs and Palm measures, interacting particle systems, stochastic differential geometry, Malliavin calculus, SPDEs, measure-valued diffusions, and branching and superprocesses. Clearly plenty of fundamental and intriguing material remains for a possible second volume.

Even with my more limited, revised ambitions, I had to be extremely selective in the choice of material. More importantly, it was necessary to look for the most economical approach to every result I did decide to include. In the latter respect, I was surprised to see how much could actually be done to simplify and streamline proofs, often handed down through generations of textbook writers. My general preference has been for results conveying some new idea or relationship, whereas many propositions of a more technical nature have been omitted. In the same vein, I have avoided technical or computational proofs that give little insight into the proven results. This conforms with my conviction that the logical structure is what matters most in mathematics, even when applications is the ultimate goal.

Though the book is primarily intended as a general reference, it should also be useful for graduate and seminar courses on different levels, ranging from elementary to advanced. Thus, a first-year graduate course in measure-theoretic probability could be based on the first ten or so chapters, while the rest of the book will readily provide material for more advanced courses on various topics. Though the treatment is formally self-contained, as far as measure theory and probability are concerned, the text is intended for a rather sophisticated reader with at least some rudimentary knowledge of subjects like topology, functional analysis, and complex variables.

My exposition is based on experiences from the numerous graduate and seminar courses I have been privileged to teach in Sweden and in the United States, ever since I was a graduate student myself. Over the years I have developed a personal approach to almost every topic, and even experts might find something of interest. Thus, many proofs may be new, and every chapter contains results that are not available in the standard textbook literature. It is my sincere hope that the book will convey some of the excitement I still feel for the subject, which is without a doubt (even apart from its utter usefulness) one of the richest and most beautiful areas of modern mathematics.

Notes and Acknowledgments: My first thanks are due to my numerous Swedish teachers, and especially to Peter Jagers, whose 1971 seminar opened my eyes to modern probability. The idea of this book was raised a few years later when the analysts at Gothenburg asked me to give a short lecture course on "probability for mathematicians." Although I objected to the title, the lectures were promptly delivered, and I became convinced of the project's feasibility. For many years afterward I had a faithful and enthusiastic audience in numerous courses on stochastic calculus, SDEs, and Markov processes. I am grateful for that learning opportunity and for the feedback and encouragement I received from colleagues and graduate students.

Inevitably I have benefited immensely from the heritage of countless authors, many of whom are not even listed in the bibliography. I have further been fortunate to know many prominent probabilists of our time, who have often inspired me through their scholarship and personal example. Two people, Klaus Matthes and Gopi Kallianpur, stand out as particularly important influences in connection with my numerous visits to Berlin and Chapel Hill, respectively.

The great Kai Lai Chung, my mentor and friend from recent years, offered penetrating comments on all aspects of the work: linguistic, historical, and mathematical. My colleague Ming Liao, always a stimulating partner for discussions, was kind enough to check my material on potential theory. Early versions of the manuscript were tested on several groups of graduate students, and Kamesh Casukhela, Davorin Dujmovic, and Hussain Talibi in particular were helpful in spotting misprints. Ulrich Albrecht and Ed Slaminka offered generous help with software problems. I am further grateful to John Kimmel, Karina Mikhli, and the Springer production team for their patience with my last-minute revisions and their truly professional handling of the project.

My greatest thanks go to my family, who is my constant source of happiness and inspiration. Without their love, encouragement, and understanding, this work would not have been possible.

Olav Kallenberg *May 1997*

Words of Wisdom and Folly

- ♣ "A mathematician who argues from probabilities in geometry is not worth an ace" — *Socrates* (on the demands of rigor in mathematics)
- ♣ "[We will travel a road] full of interest of its own. It familiarizes us with the measurement of variability, and with curious laws of chance that apply to a vast diversity of social subjects" — *Francis Galton* (on the wondrous world of probability)
- ♣ "God doesn't play dice" [i.e., there is no randomness in the universe] — *Albert Einstein* (on quantum mechanics and causality)
- ♣ "It might be possible to prove certain theorems, but they would not be of any interest, since, in practice, one could not verify whether the assumptions are fulfilled" — *Émile Borel* (on why bothering with probability)
- ♣ "[The stated result] is a special case of a very general theorem [the strong Markov property]. The measure [theoretic] ideas involved are somewhat glossed over in the proof, in order to avoid complexities out of keeping with the rest of this paper" — *Joseph L. Doob* (on why bothering with generality or mathematical rigor)
- ♣ "Probability theory [has two hands]: On the right is the rigorous [technical work]; the left hand ... reduces problems to gambling situations, coin-tossing, motions of a physical particle" — *Leo Breiman* (on probabilistic thinking)
- ♣ "There are good taste and bad taste in mathematics just as in music, literature, or cuisine, and one who dabbles in it must stand judged thereby" — *Kai Lai Chung* (on the art of writing mathematics)
- ♣ "The traveler often has the choice between climbing a peak or using a cable car" — *William Feller* (on the art of reading mathematics)
- ♣ "A Catalogue Aria of triumphs is of less benefit [to the student] than an indication of the techniques by which such results are achieved" — *David Williams* (on seduction and the art of discovery)
- ♣ "One needs [for stochastic integration] a six months course [to cover only] the definitions. What is there to do?" — *Paul-André Meyer* (on the dilemma of modern math education)
- ♣ "There were very many [bones] in the open valley; and lo, they were very dry. And [God] said unto me, 'Son of man, can these bones live?' And I answered, 'O Lord, thou knowest.' " — *Ezekiel 37:2–3* (on the ultimate reward of hard studies, as quoted by *Chris Rogers* and *David Williams*)

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