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Paul Busch Marian Grabowski Pekka J. Lahti

Operational Quantum Physics

运算量子物理学



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Operational Quantum Physics

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In memory of Eugene P. Wigner

Preface

Operational Quantum Physics offers a systematic presentation of quantum mechanics which makes exhaustive use of the full probabilistic structure of this theory. Accordingly the notion of an observable as a positive operator valued (POV) measure is explained in great detail, and the ensuing quantum measurement theory is developed and applied both to a resolution of long-standing conceptual and interpretational puzzles in the foundations of quantum mechanics, and to an analysis of various recent fundamental experiments. Fundamental to the present approach is the distinction between sharp and unsharp observables: the former correspond to the commonly used self-adjoint operators or their spectral measures, while the latter are given by POV measures that are not built by projection operators.

The book, or different parts of it, will be of interest to advanced students or researchers in quantum physics, to philosophers of physics, as well as to mathematicians working on operator valued measures. The first two chapters provide the motivations behind and a systematic development of the physical concepts and mathematical language. It is here where the measurement theoretic and operational foundations are laid for a realistic interpretation of quantum mechanics as a theory for individual systems. This interpretation, which has been the authors' guide in their research work in quantum physics, seems to come closest to physicists' practice in devising theoretical models and conceiving new experiments. It is illustrated in the phase space measurement model of Chapter VI, where Heisenberg's interpretation of the indeterminacy relation in terms of individual, irreducible quantum inaccuracies is demonstrated. Apart from this instance, most of the other issues treated do not presuppose adherence to such an interpretation of quantum mechanics.

Chapter III illustrates the use of POV measures in carrying out the covariance point of view for the operational definition of an observable. The underlying space-time symmetry is that of the Galilei group. While an analogous programme can be carried out for the Einstein-relativistic case, it has not been included here since the corresponding measurement theoretic foundation does not exist. The aspect of covariance offers a physically satisfactory unifying approach and at the same time opens up a variety of mathematical questions.

The foundations of quantum mechanics are addressed in Chapters IV-VI. For example, the possibility of measuring jointly noncommuting observables is spelled out conceptually and in terms of realisable models, some of them proposed recently in the field of quantum optics from which the majority of the experimental examples provided in Chapter VII are drawn.

Although this book applies quantum measurement theory as a tool in foundational investigations and analyses of experiments, it does not address the perennial measurement problem. It does, however, provide evidence that unsharp observables may be important in developing a coherent quantum picture of classical physical phenomena and thus of the occurrence of definite events. In fact POV measures allow for a novel concept of coarse-graining, as reviewed in Chapter V and applied in Chapter VI in the characterisation of a quasi-classical phase space measurement situation.

This book has grown out of research work that the authors have enjoyed carrying out in collaborations over more than the past decade. Support in the form of Fellowships, grants and exchange programs has been extended to us by the Academy of Finland, the Alexander von Humboldt-Foundation, the BMFT in Bonn, the Polish Academy of Sciences, the Research Institute for Theoretical Physics in Helsinki, and the University of Turku Foundation. In the final stage of this work one of us (PB) enjoyed hospitality and support from the Department of Physics, Harvard University.

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Prologue

The theory of quantum mechanics on Hilbert space has been the basis of fruitful and deep research into virtually all branches of physics for nearly seventy years. There seems to be no instance of a conflict between theoretical predictions and experimental results. In view of this success it is remarkable that a few conceptual problems have resisted any attempted resolution even until now. Some of them became tractable once the probabilistic structure of quantum mechanics was properly appreciated in its full generality. Gleason's theorem and the introduction of the notion of an observable as a *positive operator valued (POV) measure* were the crucial steps in this development. Interestingly, the latter discovery was made independently in a variety of rather disparate areas of quantum physics, motivations ranging from foundational interests to fairly practical needs. This wide scope of the concept of POV measure already demonstrates its status as an integral part of the basic structure of quantum mechanics. The traditional notion of observables as self-adjoint operators constitutes a special case, represented by projection operator valued (PV, or spectral) measures.

The incorporation of POV measures into the quantum vocabulary has not only opened up new ways of approaching longstanding theoretical puzzles, it also gave rise to an elaboration of quantum measurement theory into a conceptually sound and mathematically rigorous, powerful tool for analysing physical experiments. The ensuing research activities have led to a variety of reviews and monographs dealing with such diverse topics as probabilistic and statistical aspects of quantum theory, quantum estimation and detection, quantum theory of open systems, photon counting theory, or quantum mechanics on phase space. Much of this work is done on a high technical level and has thus contributed to setting new standards of rigour for investigations in the foundations of quantum physics. At the same time the POV measure approach to quantum observables has gradually induced a probabilistic reformulation of quantum theory that is conceptually simpler and closer to experimental practice than the traditional approach.

The present book is a result of two intimately related lines of research efforts that took place in the past decade. On the one hand quantum measurement theory has found manifold successful applications leading to new insights in the analysis of fundamental experiments. On the other hand considerable progress has been made in working out the operational conditions needed for associating POV measures with properties of physical systems. Both developments have contributed to fully appreciating the relevance of quantum mechanics as a theory of *individual* objects. The need for such a *realistic* interpretation of quantum theory is strikingly evident

in these days where one is witnessing worldwide activities in carrying out exciting experiments with single microsystems such as atoms, ions, neutrons, or photons – experiments which formerly could only be conceived as thought experiments.

Our primary concern is twofold. First we wish to demonstrate the amazing capabilities of the quantum formalism if applied in its full-fledged probabilistic form. The advantages of POV measures and of measurement theory, taken as tools of investigation into the quantum world, will be illustrated in several steps and on different levels of sophistication. Yet the notion of a POV measure must itself be subjected to a measurement theoretical analysis in order to elucidate its physical meaning. Reference to this double role of the measurement theory is the ultimate purpose of the term ‘operational’ appearing in the book title.

The understanding of POV measures as representing observable properties of a physical system will be based on a realistic, individual interpretation of quantum theory. According to this interpretation quantum mechanics describes physical systems existing independently once they have been prepared or identified by observation. Evidence for the presence of a system may be ascertained by means of determining its real (or actual) properties; and using ideal measurements this can be achieved without thereby changing the system in any way. In general, however, a property will be nonobjective (or potential) and may be actualised through measurement. Such repeatable measurements can thus be used to prepare systems with actual properties. It is an important result of quantum measurement theory that the formalism is rich enough to ensure the existence of measurement operations serving these purposes demanded by the realistic interpretation. As a consequence the famous reality criterion by Einstein, Podolsky and Rosen, which states certain predictability on the basis of non-disturbing observations as a sufficient condition for ascertaining actual properties, is naturally incorporated into quantum mechanics, along with establishing the lattice of Hilbert space projection operators as representing the totality of properties of a physical system. Ordinary observables, described as PV measures, refer thus to collections of properties associated with the values of the measured quantities.

This interpretation can be extended to observables represented as POV measures provided that the criterion of reality is relaxed appropriately so as to be applicable in situations to be characterised in terms of approximately real properties. The positive operators in the range of a POV measure, also called *effects*, represent the occurrence of particular outcomes of measurements. The expectation values of the effects are interpreted as the probabilities for these events. Instead of probabilities 1 or 0 one should in general require only probabilities close to these values in order to be able to ascertain that some property is *approximately* real or absent. This leads to a generalised notion of *properties*, which comprises both, the projection operators referred to as *sharp properties*, and certain genuine effects, the so-called *unsharp properties*. Again the non-disturbing and repeatable meas-

urement operations required by such an interpretation will be shown to exist in the quantum formalism.

Unsharp observables arise naturally in the theoretical analysis of experimental procedures. What kind of properties they represent can often be determined by making reference to some known sharp observable. Indeed many POV measures derive from some PV measures by a *coarse-graining* procedure. For example, a function of the position observable Q can be considered as a coarse-grained version of Q ; this gives rise, among others, to the discretised versions of Q which are still represented as PV measures. A POV measure associated with Q arises if instead one performs a convolution of the spectral measure with some confidence function. This will be one of our prominent and prototypical examples. We shall refer to such an unsharp observable as a smeared position observable.

In order to avoid misunderstandings regarding our terminology, it is important to be aware of the new, non-classical meaning of the terms 'inaccuracy' and 'unsharpness' in the context of quantum measurements. An inaccurate measurement refers to a situation where instead of a given observable a coarse-grained version of it is measured. By contrast, the term 'unsharp measurement' shall refer to the measurement of an unsharp observable. One should bear in mind the conceptual difference between the *relation* of coarse-graining that can exist between pairs of observables, and the *property* of being an unsharp observable that pertains to some POV measures. Some, but not all, unsharp observables arise as smeared versions of a sharp observable. The unsharpness in question should not in general be taken as an imperfect perception of an underlying more sharply determined property. On the contrary, this term is intended to describe possible elements of reality whose preparation and determination are subject to inherent limitations.

The unsharpness brought about by coarse-graining may or may not admit the kind of ignorance interpretation familiar from classical physical experimentation. In general, however, the unsharpness is a reflection of a genuine quantum indeterminacy. This turns out to be the case, e.g., with the individual measurement interpretation of the uncertainty relation. The question of how to interpret the source of unsharpness and inaccuracy will be investigated in numerous examples.

The realistic interpretation of quantum theory outlined here must be regarded as tentative in one important respect. According to this interpretation physical reality is described as it emerges when investigated by measuring processes, which are themselves physical processes. Accordingly the self-consistency of the realistic interpretation would require a solution of the so-called measurement problem, which has not been achieved yet. We shall have to leave open whether a new theory of macrosystems is needed in order to explain the occurrence of objective pointer readings, or whether the universality of quantum mechanics can be held up in this respect. A discussion of these problems is offered in a monograph on the quantum theory of measurement coauthored by two of the present writers [1.1]. That book