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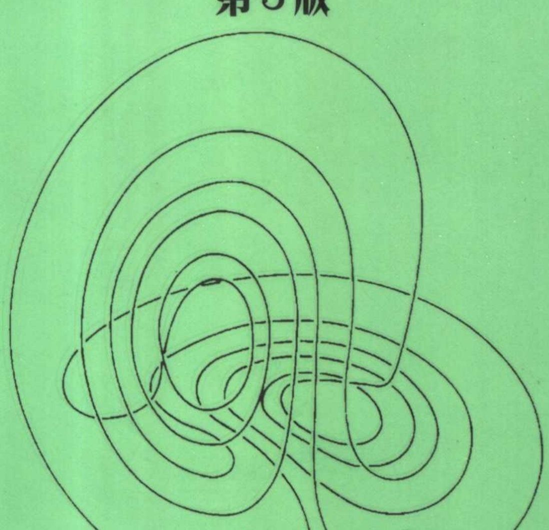
Louis H. Kauffman

KNOTS AND PHYSICS

Third Edition

纽结和物理学

第3版



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Third Edition

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本书由世界科技出版公司授权重印出版，限于中国大陆地区发行。

Preface to the First Edition

This book has its origins in two short courses given by the author in Bologna and Torino, Italy during the Fall of 1985. At that time, connections between statistical physics and the Jones polynomial were just beginning to appear, and it seemed to be a good idea to write a book of lecture notes entitled **Knots and Physics**.

The subject of knot polynomials was opening up, with the Jones polynomial as the first link polynomial able to distinguish knots from their mirror images. *We were looking at the tip of an iceberg!* The field has grown by leaps and bounds with remarkable contributions from mathematicians and physicists - a wonderful interdisciplinary interplay.

In writing this book I wanted to preserve the flavor of those old Bologna/Torino notes, and I wanted to provide a pathway into the more recent events. After a good deal of exploration, I decided, in 1989, to design a book divided into two parts. The first part would be combinatorial, elementary, devoted to the bracket polynomial as state model, partition function, vacuum-vacuum amplitude, Yang-Baxter model. The bracket also provides an entry point into the subject of quantum groups, and it is the beginning of a significant generalization of the Penrose spin-networks (see Part II, section 13^0.) Part II is an exposition of a set of related topics, and provides room for recent developments. In its first incarnation, Part II held material on the Potts model and on spin-networks.

Part I grew to include expositions of Yang-Baxter models for the Homfly and Kauffman polynomials - as discovered by Jones and Turaev, and a treatment of the Alexander polynomial based on work of Francois Jaeger, Hubert Saleur and the author. By using Yang-Baxter models, we obtain an induction-free introduction to the existence of the Jones polynomial and its generalizations. Later, Part I grew some more and picked up a chapter on the 3-manifold invariants of Reshetikhin and Turaev as reformulated by Raymond Lickorish. The Lickorish model is completely elementary, using nothing but the bracket, trickery with link diagrams, and the tangle diagrammatic interpretation of the Temperley-Lieb algebra. These 3-manifold invariants were foretold by Edward Witten in his landmark paper on quantum field theory and the Jones polynomial. Part I ends with an introduction to Witten's functional integral formalism, and shows how the knot polynomials arise in that context. An appendix computes the Yang-Baxter solutions for spin-preserving R-matrices in dimension two. From this place the Jones and Alexander polynomials appear as twins!

Part II begins with Bayman's theory of hitches – how to prove that your horse can not get away if you tie him with a well constructed clove hitch. Then follows a discussion of the experiments available with a rubber band. In sections 3^0 and 4^0 we discuss attempts to burrow beneath the usual Reidemeister moves for link diagrams. There are undiscovered realms beneath our feet. Then comes a discussion of the Penrose chromatic recursion (for colorations of planar three-valent graphs) and its generalizations. This provides an elementary entrance into spin networks, coloring and recoupling theory. Sections 7^0 and 8^0 on coloring and the Potts model are taken directly from the course in Torino. They show how the bracket model encompasses the dichromatic polynomial, the Jones polynomial and the Potts model. Section 9^0 is a notebook on spin, quantum mechanics, and special relativity. Sections 10^0 and 11^0 play with the quaternions, Cayley numbers and the Dirac string trick. Section 11^0 gives instructions for building your very own topological/mechanical quaternion demonstrator with nothing but scissors, paper and tape. Sections 12^0 and 13^0 discuss spin networks and q-deformed spin networks. The end of section 13^0 outlines work of the author and Sostenes Lins, constructing 3-manifold invariants of Turaev-Viro type via q-spin networks. Part II ends with three essays: Strings, DNA, Lorenz Attractor. These parting essays contain numerous suggestions for further play.

Much is left unsaid. I would particularly like to have had the space to discuss Louis Crane's approach to defining Witten's invariants using conformal field theory, and Lee Smolin's approach to quantum gravity – where the states of the theory are functionals on knots and links. This will have to wait for the next time!

It gives me great pleasure to thank the following people for good conversation and intellectual sustenance over the years of this project.

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Chicago, Illinois, January 1991

Preface to the Second Edition

This second edition of Knots and Physics contains corrections to misprints that appeared in the first edition, plus an appendix that contains a discussion of graph invariants and Vassiliev invariants followed by a reprinting of four papers by the author. This discussion and the included papers constitute an update and extension of the material in the original edition of the book.

Chicago, Illinois
September 1993

Preface to the Third Edition

This third edition of Knots and Physics contains corrections to misprints in the second edition plus a new article in the appendix entitled "Knot Theory and Functional Integration". This article provides an introduction to the relationships between Vassiliev invariants, Knotsevich integrals and Witten's approach to link invariants via quantum field theory.

Chicago, Illinois and
Galais, France
November 15, 2000

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Appendix

PART I

PART I. A SHORT COURSE OF KNOTS AND PHYSICS.

This first part of the book is a short course on knots and physics. It is a rapid penetration into key ideas and examples. The second half of the book consists in a series of excursions into special topics (such as the Potts Model, map coloring, spin-networks, topology of 3-manifolds, and special examples). These topics can be read by themselves, but they are informed by Part I.

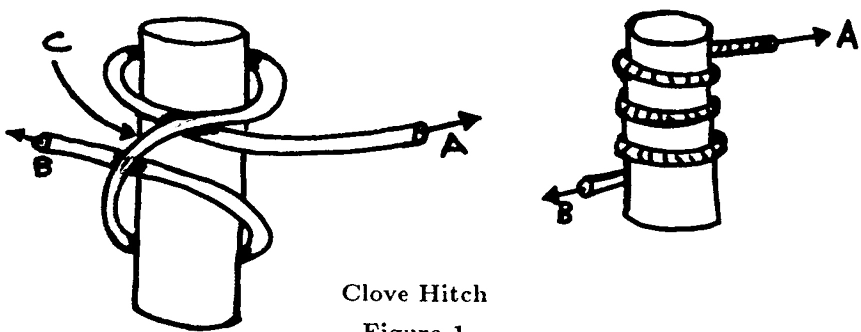
Acts of abstraction, changes of mathematical mood, shifts of viewpoint are the rule rather than the exception in this short course. The course is a rapid guided ascent straight up the side of a cliff! Later, we shall use this perspective - both for the planning of more leisurely walks, and for the plotting of more complex ascents.

Here is the plan of the short course. First we discuss the diagrammatic model for knot theory. Then we construct the bracket polynomial [LK4], and obtain from it the original Jones polynomial. We then back-track to the diagrams and uncover abstract tensors, the Yang-Baxter equation and the concept of the quantum group. With state and tensor models for the bracket in hand, we then introduce other (generalized) link polynomials. Then comes a sketch of the Witten invariants, first via combinatorial models, then using functional integrals and gauge field theory. The ideas and techniques spiral outward toward field theory and three-dimensional topology. Statistical mechanics models occur in the middle, and continue to weave in and out throughout the play. Underneath all this structure of topology and physical ideas there sounds a theme of combinatorics - graphs, matroids, coloring problems, chromatic polynomials, combinatorial structures, enumerations. And finally, throughout there is a deep underlying movement in the relation between space and sign, geometry and symbol, the source common to mathematics, physics, language and all.

And so we begin. It is customary to begin physics texts either with mathematical background, or with the results of experiments. But this is a book of knots. Could we then begin with the physics of knots? Frictional properties? The design of the clove-hitch and the bow-line? Here is the palpable and practical physics of the knots.

1^0. Physical Knots.

The **clove-hitch** is used to secure a line to a post or tree-trunk. The illustration in Figure 1 shows how it is made.

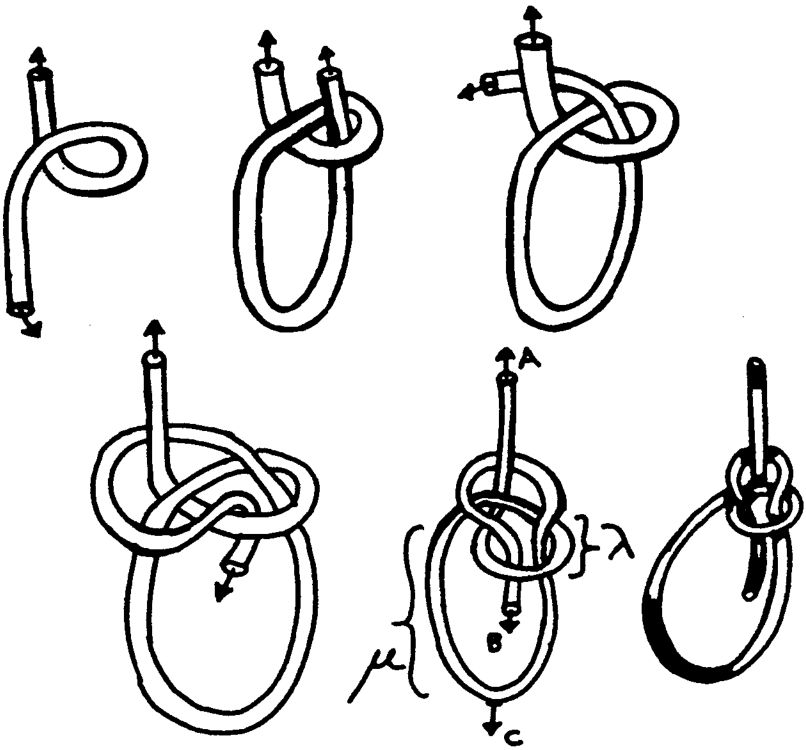


Clove Hitch
Figure 1

A little experimentation with the clove hitch shows that it holds very well indeed, with tension at A causing the line at C to grab B - and keep the assemblage from slipping.

Some physical mathematics can be put in back of these remarks - a project that we defer to Part II. At this stage, I wish to encourage experimentation. For example, just wind the rope around the post (as also in Figure 1). You will discover easily that the amount of tension needed at A to make the rope slide on the post increases exponentially with the number of windings. (The rope is held lightly at B with a fixed tension.) With the introduction of weaving - as in the clove hitch - interlocking tensions and frictions can produce an excellent bind.

In Figure 2 we find a sequence of drawings that show the tying of the **bowline**.



The Bowline
Figure 2

In the bowline we have an excellent knot for securing a line. The concentrated part of the knot allows the loop μ to be of arbitrary size. This loop is secure under tension at A. Pulling at A causes the loop λ to shrink and grab tightly the two lines passing through it. This secures μ. Relax tension at A and the knot is easily undone, making loop adjustment easy.