Superstring theory

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VOLUME 2

LOOP AMPLITUDES, ANOMALIES & PHENOMENOLOGY

超弦理论

第2卷

M. B. GREEN, J. H. SCHWARZ & E. WITTEN

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Preface

Recent years have brought a revival of work on string theory, which has been a source of fascination since its origins nearly twenty years ago. There seems to be a widely perceived need for a systematic, pedagogical exposition of the present state of knowledge about string theory. We hope that this book will help to meet this need. To give a comprehensive account of such a vast topic as string theory would scarcely be possible, even in two volumes with the length to which these have grown. Indeed, we have had to omit many important subjects, while treating others only sketchily. String field theory is omitted entirely (though the subject of chapter 11 is closely related to light-cone string field theory). Conformal field theory is not developed systematically, though much of the background material needed to understand recent papers on this subject is presented in chapter 3 and elsewhere. Our discussion of string propagation in background fields is limited to the bosonic theory, and multiloop diagrams are discussed only in very general and elementary terms. The omissions reflect a combination of human frailty and an attempt to keep the combined length of the two volumes from creeping too much over 1000 pages.

We hope that these two volumes will be useful for a wide range of readers, ranging from those who are motivated mainly by curiosity to those who actually wish to do research on string theory. The first volume is supposed to be self-contained. It gives a detailed introduction to the basic ideas of string theory, requiring as background only a moderate knowledge of particle physics and quantum field theory. The second volume delves into a number of more advanced topics, including a study of one-loop amplitudes, the low-energy effective field theory, and anomalies. There is also a substantial amount of mathematical background on differential and algebraic geometry, as well as their possible application to phenomenology.

We feel that the the two volumes should be suitable for use as textbooks in an advanced graduate-level course. The amount of material is probably more than can be covered in a one-year course. This should provide the instructor the luxury of emphasizing those topics he or she finds especially important while omitting others. Despite our best efforts, it is inevitable that a substantial number of misprints, notational inconsistencies and other errors have survived. We will be grateful if they are brought to our attention so that we can correct them in future editions.

We have benefitted greatly from the assistance of several people whom we are pleased to be able to acknowledge here. Kyle Gary worked with skill and diligence in typing substantial portions of the manuscript, as well as figuring out how to implement the formatting requirements of Cambridge University Press in TeX, the type-setting system that we have used. Marc Goroff brought his wealth of knowledge about computing systems to help solve a myriad of problems that arose in the course of this work. We also received help with computing systems from Paul Kyberd and Vadim Kaplunovsky. Patricia Moyle Schwarz put together the index and made useful comments on the manuscript. Harvey Newman set up communications links that enabled us to transfer files between Pasadena, Princeton and London. Judith Wallrich helped to compile the bibliography. Useful criticisms and comments on the text were offered by Čedomir Crnković, Chiara Nappi, Ryan Rohm and Larry Romans.

We would like to dedicate this book to our parents.

1986

Michael B. Green
John H. Schwarz
Edward Witten

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8. One-Loop Diagrams in the Bosonic String Theory

Our discussions of string scattering amplitudes in the first volume of this book were limited to tree diagrams. These are the lowest-order approximations to string scattering amplitudes. In principle, quantum corrections to the tree level or classical results should be obtained by a perturbation expansion derived from string quantum field theory. Our present state of knowledge does not make this possible. Historically, loop diagrams were constructed by using unitarity to construct loop diagrams from tree diagrams. This unitarization of the tree diagrams led, in time, to the topological expansion, as sketched in chapter 1.

As has been explained in chapters 1 and 7, the tree amplitudes for on-mass-shell string states can be represented by functional integrals over Riemann surfaces that are topologically equivalent to a disk (for open strings) or a sphere (for closed strings). Higher-order corrections are identified with functional integrals over surfaces of higher genus. An important ingredient in the calculation of scattering amplitudes is the correlation function of vertex operators corresponding to the external particles emitted from the surface. The possible world-sheet topologies include surfaces with holes or 'windows' cut out (for type I theories, where the surfaces have boundaries) or 'handles' attached. For theories with oriented strings the surfaces must be orientable. Similarly, for theories containing only closed strings the surfaces must be closed.

As the genus of a surface increases, the power of the coupling constant that accompanies it also increases. For example, adding a handle to a surface is equivalent to adding a loop of closed strings (as in fig. 8.1) and increases the order of a diagram by a factor of κ^2, where κ is the gravitational coupling constant. Cutting a window out of a surface (which is only possible in theories that contain open strings) adds a boundary and hence it increases the number of internal open strings (fig. 8.2a). The order of the diagram is increased by $g^2 \sim \kappa$ for each window, where g is the Yang–Mills coupling constant. However, the presence of a window does not always correspond to adding a loop of open strings. For example,

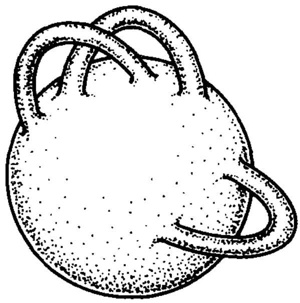
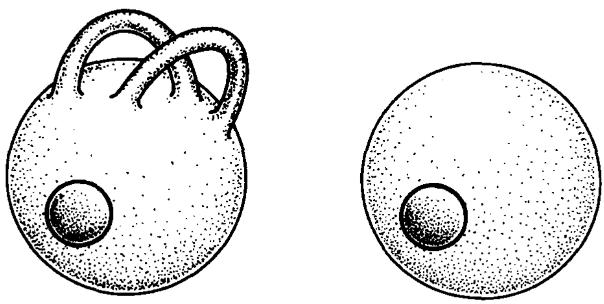


Figure 8.1. A handle added to a world sheet of arbitrary topology.



(a) (b)

Figure 8.2. Cutting a window out of a world sheet adds a boundary. This increases the number of internal open-string propagators as seen in (a). Cutting a window out of a spherical world sheet results in a diagram that is topologically a disk, as shown in (b).

cutting a window out of a sphere is a modification of the (type I) closed-string tree amplitude, which gives a world sheet that is topologically a disk with external closed-string particles attached at interior points of the surface (fig. 8.2b). Type I superstring theory is based on unoriented open and closed strings and therefore also includes nonorientable surfaces.

This topological classification of diagrams in string theories is certainly strikingly different from the classification of Feynman diagrams in point-particle field theory. In string theories there are far fewer diagrams to consider at each order in perturbation theory, and there is no meaningful separation of diagrams into tadpoles, mass insertions, vertex corrections, etc. At the one-loop level, the analysis of world-sheet path integrals is tractable. In fact, one-loop diagrams can be generated by the same operator methods that we used for tree diagrams in chapter 7. Beyond the one-loop level, the analysis of world-sheet path integrals involves somewhat esoteric mathematics, which we will not explore in this book.

In the bosonic theory calculations based on the covariant operator formalism require the same mathematical manipulations as those that arise in light-cone gauge, at least when the external on-shell states are taken to have vanishing + components of momentum. Given Lorentz invariance, amplitudes for external particles with momenta restricted in this way completely determine the amplitudes provided that there are not too many external states. Although we use the covariant method in most of this chapter, very similar techniques also apply to the light-cone gauge method in this special frame.

8.1 Open-String One-Loop Amplitudes

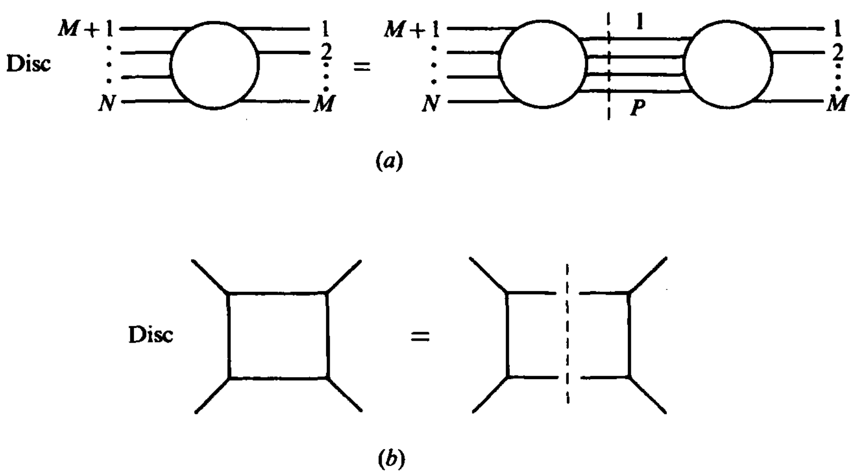


Figure 8.3. (*a*) Unitarity equates the discontinuity of a scattering amplitude (with M incoming and N outgoing particles) across a threshold cut (due to P intermediate particles) to the product of $M \rightarrow P$ and $P \rightarrow N$ scattering amplitudes integrated over intermediate state phase space. (*b*) At one loop, unitarity relates the discontinuity of a loop diagram to the integral of the product of two tree diagrams over the phase space for the intermediate on-shell two-particle states.

In point-particle theories the one-loop diagrams can be determined by unitarity in terms of tree diagrams without using the apparatus of second-quantized field theory. Unitarity requires that scattering amplitudes should have suitable branch cuts as a function of the Lorentz-invariant quantities formed out of the external momenta. These cuts arise from the regions of momentum space in which intermediate states are on their mass shells. For example, fig. 8.3*a* depicts the unitarity equation for an amplitude with M incoming and N outgoing particles. A given set of P intermediate on-shell physical states contributes to the discontinuity across the branch cut an amount that is proportional to the product of the amplitude for $M \to P$ particles multiplied by the amplitude for $P \to N$ particles integrated over the accessible phase space for the intermediate particles.

When expanded as a power series in the coupling constant this nonlinear equation relates the discontinuity of a one-loop amplitude to the product of two tree amplitudes. In this case, illustrated in fig. 8.3*b*, the number of intermediate states, P, is two. In particular, the form of the one-loop amplitude, including its normalization, is determined in terms of the tree diagrams up to an arbitrary entire function of these invariants. In the case of ordinary field theory, the arbitrary entire function corresponds to the arbitrariness associated with the renormalization procedure. In gauge invariant field theories, it is also necessary to avoid including in loop diagrams the contributions of timelike or longitudinally polarized gauge mesons. These contributions can be removed by going to a light cone or unitary gauge, or can be canceled by correctly including the Faddeev-Popov ghosts.

Similar considerations apply to the construction of the one-loop amplitudes in string theories from the tree diagrams. In this case the requirement of Regge behavior at high energies eliminates the ambiguity that exists in field theory. Regge behavior forces amplitudes to vanish in certain asymptotic regimes; addition of an entire function of the momenta to one-loop diagrams would inevitably spoil this property.

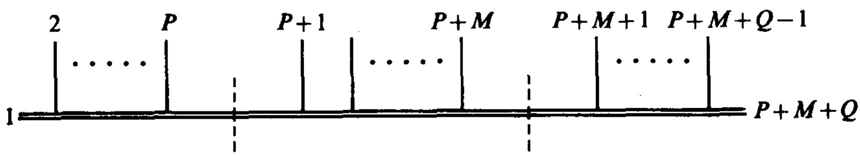


Figure 8.4. A general tree diagram with $P+M+Q$ ground-state particles factorized to give a tree with two arbitrary excited states and M ground states.

For example, the tree diagram of fig. 8.4 illustrates the interaction of $P+M+Q$ on-shell open-string states. It can be factorized as shown in the figure to obtain the amplitude for an arbitrary pair of 'excited' states to couple to M on-shell states. Ignoring the presence of unphysical states for the moment the one-loop amplitude is obtained by *sewing* the excited states together, *i.e.*, by inserting a propagator between the initial and the final excited states and summing over all possible states as well as integrating over their momenta. In the complete amplitude it is necessary to sum over loop diagrams with twists inserted in all possible ways in the internal propagators of the loop.

Just as in ordinary field theory, covariant string-theory formulas describe states of unphysical polarization circling in the loop. Care must be taken to somehow suppress their contribution. In early calculations of string loop diagrams, the propagation of unphysical states was avoided by inserting a rather complicated physical-state projection operator in the propagators. This ensured that the circulating particles corresponded only to physical states; the procedure was analogous to some early approaches to Yang–Mills theory. A more modern approach incorporates the Faddeev–Popov ghost modes in the calculations instead. This approach is far simpler, and is the approach that we will use in performing covariant calculations.

In the bosonic theory the inclusion of ghost modes is quite easy. The vertex operators, such as the tachyon vertex operator $e^{ik\cdot X}$, are constructed from X^μ only, without ghosts, where $X^\mu(\sigma,\tau)$ is the string coordinate defined in chapter 2. When ghosts are included in the formalism, these vertex operators are understood to include a unit operator in the ghost sector of the Fock space. The ghosts circulating around the loops can then cancel the contributions of unphysical states. This is their only role.

How can we be certain that the ghosts are really correctly canceling the contributions of the unphysical states? It is particularly important to address this question, since – pending a completely satisfactory derivation of loop amplitudes from a logically satisfying starting point – there is an element of guesswork in formulating the Feynman rules including the ghosts. To gain some insight into this important question, it is possible to do the calculations in light-cone gauge. In this case, there are no unphysical states propagating in the loop – neither states that violate the Virasoro conditions, nor null states, nor ghosts. All the states in the light-cone Fock space correspond to physical propagating degrees of freedom. The light-cone amplitudes are thus manifestly unitary – or at any rate, singularities that appear are due to physical intermediate states. It will be rather clear in our discussion that – at least for processes that are easily discussed in both formalisms – the light-cone approach gives the same answers that one obtains in the covariant treatment with ghosts. Ultimately, the rules involving Faddeev–Popov modes should be derived from a logically sound starting point, perhaps a gauge-invariant nonlinear field theory of strings.

A curious feature of string theories is that new singularities can arise due to divergences of sums over intermediate states. This feature already appeared in tree amplitudes, where we saw in chapters 1 and 7 that t-channel poles arise due to divergences in the sum over s-channel poles. In the case of loop diagrams even more remarkable things can happen. For instance, an open-string loop with suitable twists can actually give rise to closed-string poles. It was by trying to reconcile these singularities with unitarity that the significance of the critical dimension first became apparent; in the critical dimension, these singularities correspond to graviton poles, and (as we discussed in §1.5.6), they are the reason that a consistent string theory without gravity does not seem possible, at the quantum level.

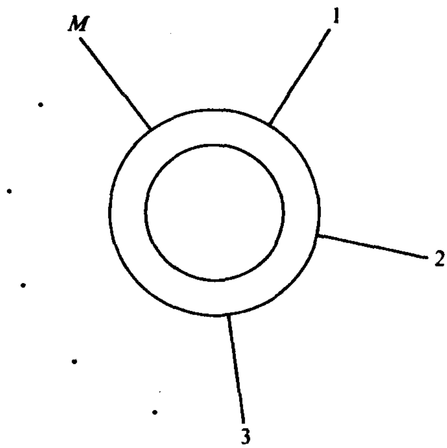


Figure 8.5. The planar loop diagram with M ground-state particles

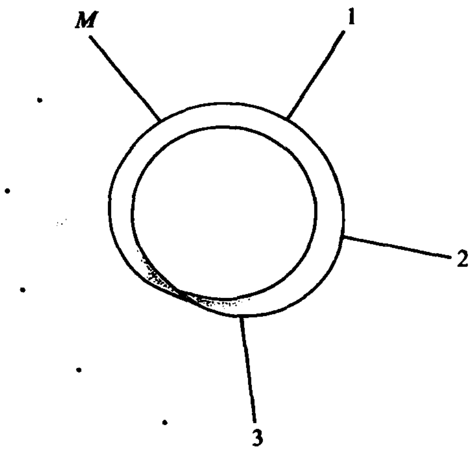


Figure 8.6. A nonorientable one-loop diagram with M external particles has a world sheet that is a Möbius strip.