海外优秀数学类教材系列丛书
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影印版

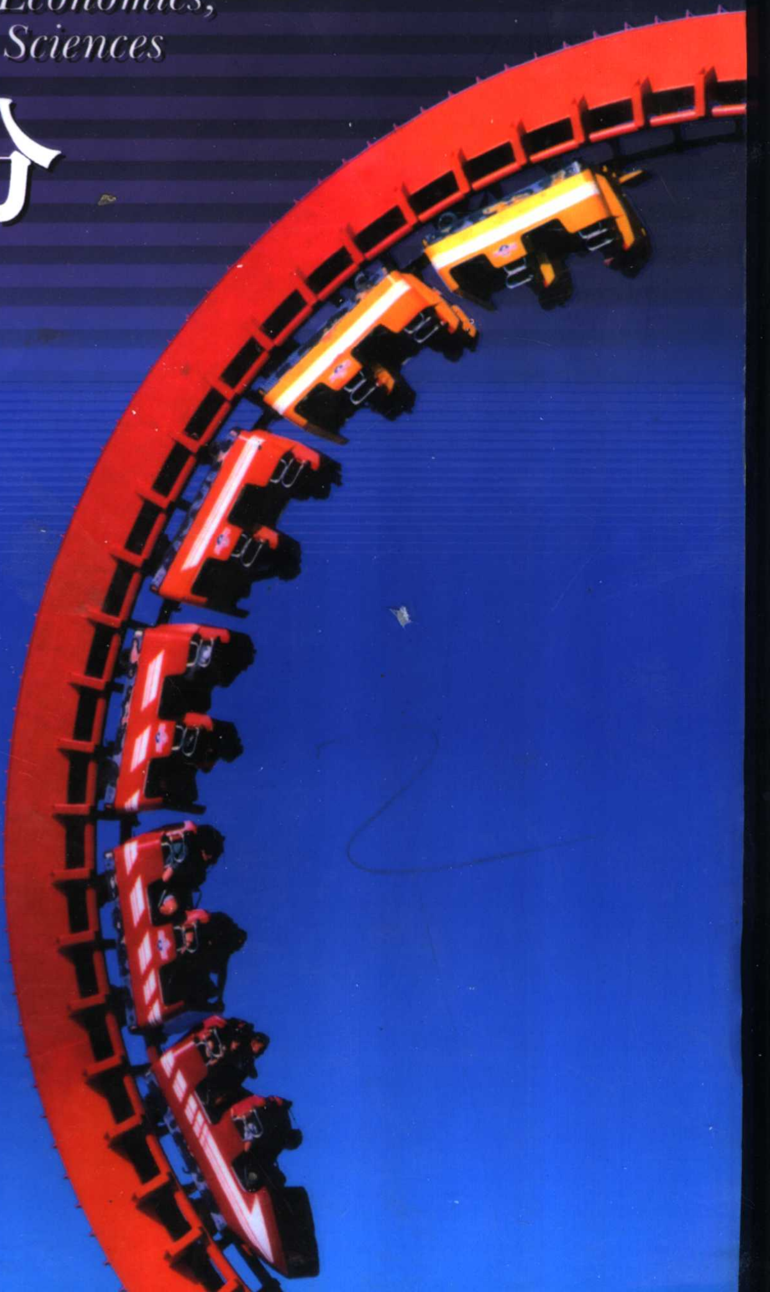
Brief Calculus (Second Edition)

Solving Problems in Business, Economics, and the Social and Behavioral Sciences

简明微积分及其应用

(第2版)

Bill Armstrong
Don Davis



高等教育出版社
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出版者的话

在我国已经加入 WTO、经济全球化的今天，为适应当前我国高校各类创新人才培养的需要，大力推进教育部倡导的双语教学，配合教育部实施的“高等学校教学质量与教学改革工程”和“精品课程”建设的需要，高等教育出版社有计划、大规模地开展了海外优秀数学类系列教材的引进工作。

高等教育出版社和 Pearson Education, John Wiley & Sons, McGraw - Hill, Thomson Learning 等国外出版公司进行了广泛接触，经国外出版公司的推荐并在国内专家的协助下，提交引进版权总数 100 余种。收到样书后，我们聘请了国内高校一线教师、专家、学者参与这些原版教材的评介工作，并参考国内相关专业的课程设置和教学实际情况，从中遴选出了这套优秀教材组织出版。

这批教材普遍具有以下特点：(1) 基本上是近 3 年出版的，在国际上被广泛使用，在同类教材中具有相当的权威性；(2) 高版次，历经多年教学实践检验，内容翔实准确、反映时代要求；(3) 各种教学资源配套整齐，为师生提供了极大的便利；(4) 插图精美、丰富，图文并茂，与正文相辅相成；(5) 语言简练、流畅、可读性强，比较适合非英语国家的学生阅读。

本系列丛书中，有 Finney、Weir 等编的《托马斯微积分》(第 10 版，Pearson)，其特色可用“呈传统特色、富革新精神”概括，本书自 20 世纪 50 年代第 1 版以来，平均每四五年就有一个新版面世，长达 50 余年始终盛行于西方教坛，作者既有相当高的学术水平，又热爱教学，长期工作在教学第一线，其中，年近 90 的 G. B. Thomas 教授长年在 MIT 工作，具有丰富的教学经验；Finney 教授也在 MIT 工作达 10 年；Weir 是美国数学建模竞赛委员会主任。Stewart 编的立体化教材《微积分》(第 5 版，Thomson Learning)配备了丰富的教学资源，是国际上最畅销的微积分原版教材，2003 年全球销量约 40 余万册，在美国，占据了约 50%～60%的微积分教材市场，其用户包括耶鲁等名牌院校及众多一般院校。本系列丛书还包括 Anton 编的经典教材《线性代数及其应用》(第 8 版，Wiley)；Jay L. Devore 编的优秀教材《概率论与数理统计》(第 5 版，Thomson Learning)等。在努力降低引进教材售价方面，高等教育出版社做了大量和细致的工作，这套引进的教材体现了一定的权威性、系统性、先进性和经济性等特点。

通过影印、翻译、编译这批优秀教材，我们一方面要不断地分析、学习、消化吸收国外优秀教材的长处，吸取国外出版公司的制作经验，提升我们自编教材的立体化配套标准，使我国高校教材建设水平上一个新的台阶；与此同时，我们还将尝试组织海外作者和国内作者合编外文版基础课数学教材，并约请国内专家改编部分国外优秀教材，以适应我国实际教学环境。

这套教材出版后，我们将结合各高校的双语教学计划，开展大规模的宣传、培训工作，及时地将本套丛书推荐给高校使用。在使用过程中，我们衷心希望广大高校教师和同学提出宝贵的意见和建议。

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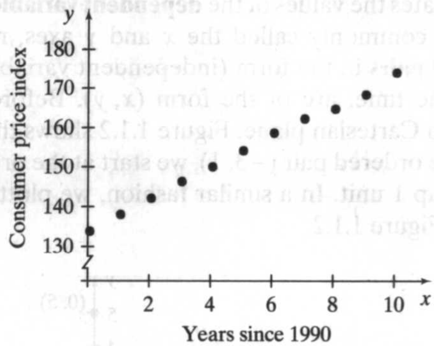
Chapter

1 Functions, Models, and Average Rate of Change



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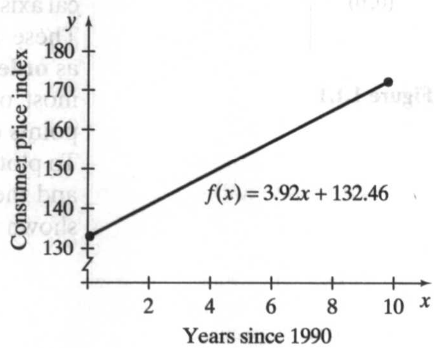

(c)

During the prosperous 1990's shopping at "the mall" proved to be a popular activity for many suburban teens. A question one might ask is whether $100 bought as many goods at the end of the decade as it did at the beginning? One source of this information is the Consumer Price Index (CPI) which is a measure of prices facing consumers. Figure b graphs the data for the CPI as a scatterplot and Figure c gives the function modeling that data. These figures show that the CPI rose steadily throughout the decade.

What We Know

We begin our study of calculus having learned the basics of algebra. These include equation solving, factoring, and the graphing of functions.

Where Do We Go

In this chapter, we will review the function concept, the properties of various types of functions, and the average rate of change of functions over an interval. We will see how data can be used to form or model functions.

Section 1.1 The Coordinate System and Functions

Plotting Points

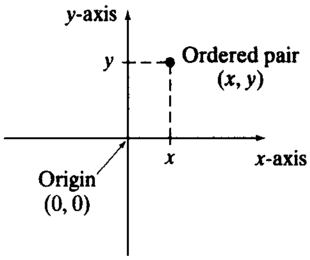

Figure 1.1.1

We start our study of functions by examining how they appear visually. The graph on which functions are plotted is called the **Cartesian plane**.

The Cartesian plane can be thought of as two number lines that are perpendicular to one another, as shown in Figure 1.1.1. The point at which the lines cross is called the **origin**. The horizontal number line, or horizontal axis, locates the values of the **independent variable**, which is usually denoted by x. The vertical axis locates the values of the **dependent variable**, which is usually denoted by y. These are commonly called the x and y axes, respectively. Points are plotted as **ordered pairs** in the form (independent variable, dependent variable), which, most of the time, are of the form (x, y). Before we continue, let's plot some points on a Cartesian plane. Figure 1.1.2 shows the graph of some ordered pairs. To plot the ordered pair $(-3, 1)$, we start at the origin and move to the left 3 units and then up 1 unit. In a similar fashion, we plotted the remainder of the points shown in Figure 1.1.2.

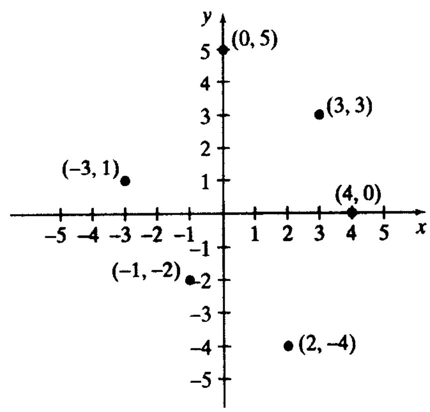

Figure 1.1.2

In the study of applied calculus, we often use tabular data as a basis for forming a mathematical model. When we plot data, we are producing what is called a **scatterplot**. Frequently, the values of the independent variable are a measure of time, primarily in years. The dependent variable then represents some phenomenon in business, life, or social science that is related to time.

Example 1 Creating a Scatterplot

The data in Table 1.1.1 represent the U.S. imports of products from France for the years 1990 to 1994. The value of the imports is in billions of U.S. dollars. Let x represent the number of years since 1990, and let y represent the value in billions of dollars of U.S. imports from France. Construct ordered pairs to represent the data, and plot the data.

Solution

To avoid working with large values of the independent variable x, it is convenient to let x represent the number of years since 1990 (see Table 1.1.2). The

Table 1.1.1

Year	Imports
1990	10.7
1991	13.3
1992	14.8
1993	15.3
1994	16.8

SOURCE: U.S. Census Bureau
www.census.gov

Table 1.1.2

Years since 1990	Imports
0	10.7
1	13.3
2	14.8
3	15.3
4	16.8

year 1990 corresponds with $x = 0$, 1991 corresponds to $x = 1$, and so on. Notice that we get these values by taking the year and subtracting 1990. This process is called **standardizing the values**. So constructing these ordered pairs gives (0, 10.7), (1, 13.3), (2, 14.8), (3, 15.3), and (4, 16.8). Using these ordered pairs, we get the scatterplot shown in Figure 1.1.3.

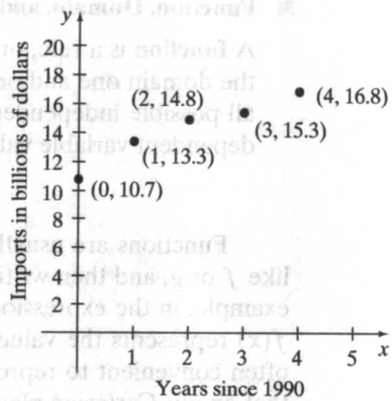

Figure 1.1.3 ■

Checkpoint 1 Now work Exercise 11.

Notice in Example 1 that the possible values of x came from the set $\{0, 1, 2, 3, 4\}$ and the possible values of y came from the set $\{10.7, 13.3, 14.8, 15.3, 16.8\}$. These sets of numbers have special names in mathematics. The set of all possible values of the independent variable, in this case x, is called the **domain**; the set of all possible values of the dependent variable, in this case y, is called the **range**. These two sets of numbers are critical in the study of functions.

Function Notation and Evaluating Functions

Many times, a dependent relationship exists between two phenomena. The price of a concert ticket may **depend** on the popularity of the band, the cost of manufacturing may depend on the quantity manufactured, and the life expectancy of a person may depend on the year in which she or he was born. These examples exhibit a direct relationship, or correspondence, between **independent variable** values (price, quantity, year) and the **dependent variable** values (popularity, cost, life length). We write these relationships between the independent and dependent variables using **functions**.

A function f can be thought of as a process in which an independent value x in the domain is mapped to a dependent value y in the range. This is illustrated in Figure 1.1.4.

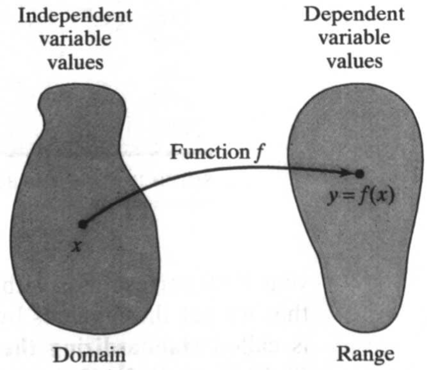

Figure 1.1.4

Function, Domain, and Range

A **function** is a rule, or a set of ordered pairs, that assigns to each element in the domain one and only one element in the range. The **domain** is the set of all possible independent variable values. The **range** is the set of all possible dependent variable values.

Functions are usually expressed by first naming the function with a letter like f or g, and then writing the independent variable letter in parentheses. For example, in the expression $f(x)$ (read "f of x"), f represents the function and $f(x)$ represents the value of f at x, where x is the independent variable. It is often convenient to represent $f(x)$ with the dependent variable y. This means that on our Cartesian plane we can now label the vertical axis with $f(x)$, $g(x)$, or any other function name that we wish to use.

Functions are not only expressed as tables and graphs, but also as mathematical expressions. For example, the function $f(x) = \frac{x+2}{5}$ means: "Take the independent value and add 2; then divide that sum by 5."

The process of "plugging" in various x-values into a function is called **evaluating** the function. To evaluate the function $f(x) = \frac{x+2}{5}$ at $x = 13$, denoted by writing $f(13)$, we substitute 13 for every x. This gives

$$f(x) = \frac{x+2}{5}$$

$$f(13) = \frac{13+2}{5} = \frac{15}{5} = 3$$

Example 2 Evaluating Functions

For the function $g(x) = -2x + 6$, evaluate $g(-2)$, $g(0)$, and $g(3)$, and write the results as ordered pairs.

Solution

Evaluating the function, we get $g(-2) = -2(-2) + 6 = 4 + 6 = 10$, which produces the ordered pair $(-2, 10)$.

Now $g(0) = -2(0) + 6 = 0 + 6 = 6$. This gives us the ordered pair $(0, 6)$.

And $g(3) = -2(3) + 6 = -6 + 6 = 0$, which produces the third ordered pair $(3, 0)$. ■

Now let's take a look at a relationship that is not a function. Figure 1.1.5 shows the graph of relationship R. Since $(11, 3)$ and $(11, -3)$ are on the graph, R assigns both 3 and -3 to the value 11. Applying our definition of function, we see that R is not a function. A visual way to tell whether a graph represents a function is to use the **vertical line test**.

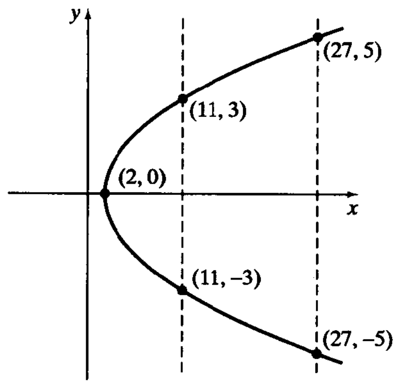

Figure 1.1.5

■ **Vertical Line Test**

If every vertical line drawn through a graph intersects the graph at only one point, then the graph represents a function.

Example 3 Determining if a Graph Represents a Function

Use the vertical line test to determine which of the graphs in Figures 1.1.6a, b, and c do not represent functions.

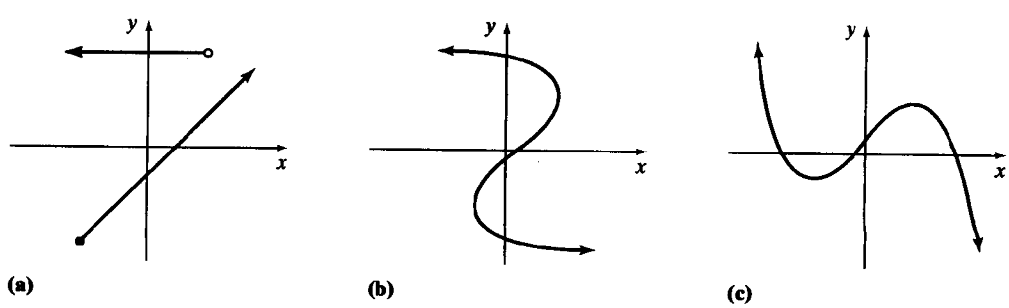

Figure 1.1.6

Solution

We see in Figure 1.1.6c that vertical lines will intersect the graph at only one place, but in Figures 1.1.6a and b they will intersect at more than one point. So the first two graphs do not represent functions, while the third graph is a function. ■

✓ Checkpoint 2

Now work Exercise 21.

Many times, we need to express the domain and range of a function as some portion of the real number line. To write this in a convenient form, we use **interval notation.** Let's say that we have the inequality notation $2 \leq x < 5$; that is, all the x-values from 2 to 5, including 2. We can conveniently write this section of the real number line using interval notation as $[2, 5)$. Intervals with two endpoints are called **finite intervals**. If the endpoints are included, the interval is **closed**. If the endpoints are not included, the interval is **open**.

Finite Intervals

Number Line	Interval Notation	Inequality Notation	Interval Type
a b	$[a, b]$	$a \leq x \leq b$	Closed
a b	(a, b)	$a < x < b$	Open
a b	$(a, b]$	$a < x \leq b$	Half-open
a b	$[a, b)$	$a \leq x < b$	Half-open

Intervals that extend indefinitely in at least one direction are called **infinite intervals**. We use the **infinity** symbol ∞ to indicate that the numbers extend positively (or to the right on the number line) without bound. We use $-\infty$ to represent **negative infinity** to show that the numbers extend negatively (or to the left on the number line) without bound. Since ∞ and $-\infty$ are only **concepts** and not real numbers themselves, we always write these endpoints as open, using parentheses.

Infinite Intervals

Number Line	Interval Notation	Inequality Notation	Interval Type
a	$[a, \infty)$	$x \geq a$	Unbounded closed
a	(a, ∞)	$x > a$	Unbounded open
b	$(-\infty, b]$	$x \leq b$	Unbounded closed
b	$(-\infty, b)$	$x < b$	Unbounded open
a	$(-\infty, \infty)$	All x in $\mathbb{R}$	Number line

Now that we have reviewed interval notation, let's return to the analysis of functions, domains, and ranges. We may think of the domain as the set of all

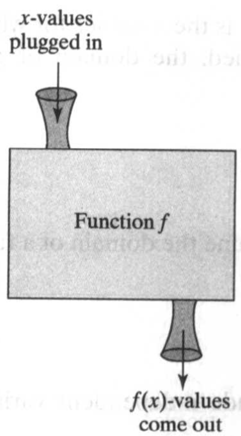

Figure 1.1.7

possible values that can be *plugged in* for the independent variable of the function. The range is the set of numbers that *come out* of the function. An illustration of this *plugged-in* and *come out* process is given in Figure 1.1.7.

To find numbers that cannot be in the domain, think of the number properties learned in algebra. We cannot divide a number by zero; for example, $\frac{3}{0}$ is not defined. And we cannot take the square root of a negative number (that is, $\sqrt{-9}$ does not have a *real* number solution). (You might have heard of complex numbers, but in applied calculus we consider only real numbers.) So, generally, when we look for domain values of functions, we exclude values that can make the denominator of a fraction zero or make the expression under a square root (or any even index root) function negative. (Another restriction will arise in Section 1.7, when we study logarithmic functions.)

Example 4 Finding the Domain of a Function Algebraically

Determine the domain of the following functions. Show the domain on a real number line and by writing in interval notation.

(a) $f(x) = \sqrt{5x-2}$ **(b)** $g(x) = \dfrac{\sqrt{x-2}}{x^2-4x}$

Solution

(a) To find the domain for f, we need the x-values so that the radicand (the expression under the square root symbol) is greater than or equal to zero. As an inequality, this means that we need

$$5x - 2 \geq 0$$
$$5x \geq 2$$
$$x \geq \frac{2}{5}$$

Figure 1.1.8

This domain is shown on the number line in Figure 1.1.8. In interval notation, the domain is $\left[\frac{2}{5}, \infty\right)$.

(b) This function requires us to examine the restrictions on both the numerator and denominator. In the numerator, we see that we need values of x such that

$$x - 2 \geq 0$$
$$x \geq 2$$

For the denominator, we can factor x from each term to get

$$x^2 - 4x = x(x-4)$$

If we set the factored form equal to zero and solve, we get

$$x(x-4) = 0$$
$$x = 0 \quad \text{or} \quad x = 4$$

So we must also exclude the values of 0 and 4 from the domain, since they make the denominator of g zero. The resulting domain is shown on the

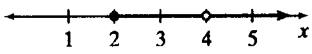

Figure 1.1.9

number line in Figure 1.1.9. Since the domain of g is the x-values for which both the numerator and denominator are defined, the domain of g is $[2, 4) \cup (4, \infty)$. ■

✓ Checkpoint 3

Now work Exercise 43.

Before continuing, let's summarize how to determine the domain of a function algebraically.

■ **Determining the Domain of a Function**

To determine the domain of a function, we exclude independent variable values that

1. Make the denominator of a fraction zero.
2. Produce a negative result under an even-indexed radical.

Using purely algebraic techniques, finding the range of a function can be difficult. Many functions require techniques learned in calculus to determine the range, but the range is relatively easy to determine when we see the graph of the function. We illustrate how in Example 5.

Example 5 Determining Domains and Ranges From a Graph

Determine visually the domain and range for the function in Figure 1.1.10, and write your answer using interval notation.

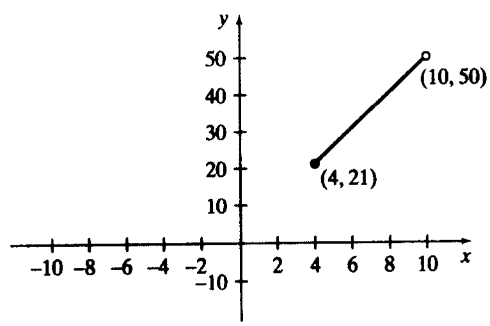

Figure 1.1.10

Solution

First, notice that the left endpoint of the function is a solid dot and the right endpoint is a hollow dot. This means that the domain is the half-open interval $[4, 10)$. Notice that the lowest point on the graph is at the ordered pair (4, 21) and the highest point is near (10, 50). So we see that the range of the function is $[21, 50)$. ■

Now let's introduce a function that we will see frequently in this text, the **price–demand function**.

■ Price–Demand Function

The **price–demand function** p gives us the price at which people buy exactly x units of product.

Generally, as the price of a product decreases, more and more people will buy the product. So for the price–demand function as the x-values increase, the p-values decrease. This is shown in Figure 1.1.11.

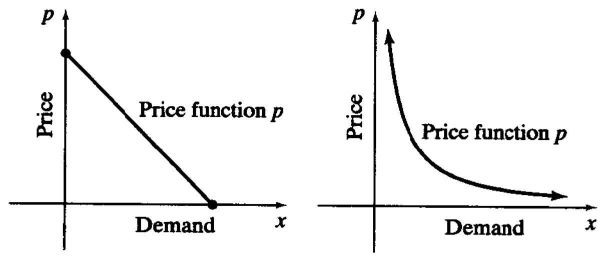

Figure 1.1.11

Example 6 Determining the Domain and Range of a Price–Demand Function

The price–demand function for electronic organs sold at Red River Mall is given by

$$p(x) = 16{,}000 - 320x$$

The graph of p is given in Figure 1.1.12. Here, x represents the number of electronic organs that people buy at Red River Mall weekly and $p(x)$ represents the price per organ in dollars.

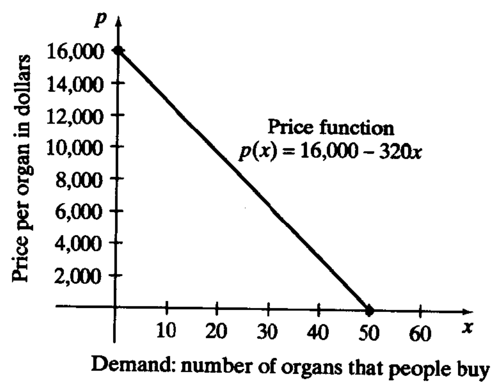

Figure 1.1.12

(a) Determine the domain and the range of the function.

(b) Evaluate $p(15)$ and interpret.

Solution

(a) Notice that the function applies only for x-values up to 50 units. For other values, $p(x)$ is not defined. We may express this restriction on x using interval notation by writing

$$p(x) = 16{,}000 - 320x \qquad [0, 50]$$