PATH INTEGRALS

in

Quantum Mechanics Statistics and Polymer Physics

2nd Edition

量子力学统计学和聚合物理学中的

路径积分

第 2 版

Hagen Kleinert

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Nature alone knows what she wants.
GOETHE

Preface

Since this book first appeared three years ago, a number of important developments have taken place calling for various extensions to the text.

Chapter 4 now contains a discussion of the features of the semiclassical quantization which are relevant for multidimensional chaotic systems.

Chapter 3 derives perturbation expansions in terms of Feynman graphs, whose use is customary in quantum field theory. Correspondence is established with Rayleigh-Schrödinger perturbation theory. Graphical expansions are used in Chapter 5 to extend the Feynman-Kleinert variational approach into a systematic *variational perturbation theory*. Analytically inaccessible path integrals can now be evaluated with arbitrary accuracy. In contrast to ordinary perturbation expansions which always diverge, the new expansions are convergent for all coupling strengths, including the strong-coupling limit.

Chapter 19 is new. It deals with relativistic path integrals, which were previously discussed only briefly in two sections at the end of Chapter 15. As an application, the path integral of the relativistic hydrogen atom is solved.

Chapter 16 is extended by a theory of particles with fractional statistics (*anyons*), from which I develop a theory of polymer entanglement. For this I introduce nonabelian Chern-Simons fields and show their relationship with various knot polynomials (Jones, HOMFLY). The successful explanation of the fractional quantum Hall effect by anyon theory is discussed—also the failure to explain high-temperature superconductivity via a Chern-Simons interaction.

Chapter 17 offers a novel variational approach to tunneling amplitudes. It extends the semiclassical range of validity from high to low barriers. As an application, I increase the range of validity of the currently used large-order perturbation theory far into the regime of low orders. This suggests a possibility of greatly improving existing resummation procedures for divergent perturbation series of quantum field theories.

The Index now also contains the names of authors cited in the text. This may help the reader searching for topics associated with these names. Due to their great number, it was impossible to cite all the authors who have made important contributions. I apologize to all those who vainly search for their names.

In writing the new sections in Chapters 4 and 16, discussions with Dr. D. Wintgen and, in particular, Dr. A. Schakel have been extremely useful. I also thank Professors G. Gerlich, P. Hänggi, H. Grabert, M. Roncadelli as well as Mr. A. Pelster and Mr. R. Karrlein for a number of relevant comments. Printing errors were corrected by my secretary Ms. S. Endrias and by my editor Ms. Lim Feng Nee of World Scientific.

Many improvements are due to my wife Annemarie.

H. Kleinert
Berlin, December 1994

Preface to the First Edition

These are extended lecture notes of a course on path integrals which I delivered at the Freie Universität Berlin during winter 1989/1990. My interest in this subject dates back to 1972 when the late R. P. Feynman drew my attention to the unsolved path integral of the hydrogen atom. I was then spending my sabbatical year at Caltech, where Feynman told me during a discussion how embarrassed he was, not being able to solve the path integral of this most fundamental quantum system. In fact, this had made him quit teaching this subject in his course on quantum mechanics as he had initially done.[1] Feynman challenged me: "Kleinert, you figured out all that group-theoretic stuff of the hydrogen atom, why don't you solve the path integral!" He was referring to my 1967 Ph.D. thesis[2] where I had demonstrated that all dynamical questions on the hydrogen atom could be answered using only operations within a *dynamical group* $O(4,2)$. Indeed, in that work, the four-dimensional oscillator played a crucial role and the missing steps to the solution of the path integral were later found to be very few. After returning to Berlin, I forgot about the problem since I was busy applying path integrals in another context, developing a field-theoretic passage from quark theories to a collective field theory of hadrons.[3] Later, I carried these techniques over into condensed matter (superconductors, superfluid ^{3}He) and nuclear physics. Path integrals have made it possible to build a unified field theory of collective phenomena in quite different physical systems.[4]

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[1]Quoting from the preface of the textbook by R.P. Feynman and A.R. Hibbs, *Quantum Mechanics and Path Integrals*, McGraw-Hill, New York, 1965: "Over the succeeding years, ... Dr. Feynman's approach to teaching the subject of quantum mechanics evolved somewhat away from the initial path integral approach."

[2]H. Kleinert, Fortschr. Phys. *6*, 1, (1968), and *Group Dynamics of the Hydrogen Atom*, Lectures presented at the 1967 Boulder Summer School, published in *Lectures in Theoretical Physics*, Vol. X B, pp. 427–482, ed. by A.O. Barut and W.E. Brittin, Gordon and Breach, New York, 1968.

[3]See my 1976 Erice lectures, *Hadronization of Quark Theories*, published in *Understanding the Fundamental Constituents of Matter*, Plenum press, New York, 1978, p. 289, ed. by A. Zichichi.

[4]H. Kleinert, Phys. Lett. B *69*, 9 (1977); Fortschr. Phys. *26*, 565 (1978); *30*, 187, 351 (1982).

The hydrogen problem came up again in 1978 as I was teaching a course on quantum mechanics. To explain the concept of quantum fluctuations, I gave an introduction to path integrals. At the same time, a postdoc from Turkey, I. H. Duru, joined my group as a Humboldt fellow. Since he was familiar with quantum mechanics, I suggested that we should try solving the path integral of the hydrogen atom. He quickly acquired the basic techniques, and soon we found the most important ingredient to the solution: The transformation of time in the path integral to a new path-dependent pseudotime, combined with a transformation of the coordinates to "square root coordinates" (to be explained in Chapters 13 and 14).[5] These transformations led to the correct result, however, only due to good fortune. In fact, our procedure was immediately criticized for its sloppy treatment of the time slicing.[6] A proper treatment could, in principle, have rendered unwanted extra terms which our treatment would have missed. Other authors went through the detailed time-slicing procedure,[7] but the correct result emerged only by transforming the measure of path integration inconsistently. When I calculated the extra terms according to the standard rules I found them to be zero only in two space dimensions.[8] The same treatment in three dimensions gave nonzero "corrections" which spoiled the beautiful result, leaving me puzzled.

Only recently I happened to locate the place where the three-dimensional treatment went wrong. I had just finished a book on the use of gauge fields in condensed matter physics.[9] The second volume deals with ensembles of defects which are defined and classified by means of operational cutting and pasting procedures on an ideal crystal. Mathematically, these procedures correspond to nonholonomic mappings. Geometrically, they lead from a flat space to a space with curvature and torsion. While proofreading that book, it suddenly occurred to me that the transformation by which the path integral of the hydrogen atom is solved also produces torsion. Moreover, this happens only in three dimensions. In two dimensions, where the time-sliced path integral had been solved without problems, torsion is absent. Thus I realized that the transformation of the time-sliced measure had a hitherto unknown sensitivity to torsion.

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[5]I.H. Duru and H. Kleinert, Phys. Lett. B *84*, 30 (1979), Fortschr. Phys. *30*, 401 (1982).

[6]G.A. Ringwood and J.T. Devreese, J. Math. Phys. *21*, 1390 (1980).

[7]R. Ho and A. Inomata, Phys. Rev. Lett. *48*, 231 (1982); A. Inomata, Phys. Lett. A *87*, 387 (1981).

[8]H. Kleinert, Phys. Lett. B *189*, 187 (1987); contains also a criticism of Ref. 7.

[9]H. Kleinert, *Gauge Fields in Condensed Matter*, World Scientific, Singapore, 1989, Vol. I, pp. 1–744, *Superflow and Vortex Lines*, and Vol. II, pp. 745–1456, *Stresses and Defects*.

It was therefore essential to find a correct path integral for a particle in a space with curvature and torsion. This was a nontrivial task since the literature was ambiguous already for a purely curved space, offering several prescriptions to choose from. The corresponding equivalent Schrödinger equations differ by multiples of the curvature scalar.[10] The ambiguities are path integral analogs of the so-called *operator-ordering problem* in quantum mechanics. When trying to apply the existing prescriptions to spaces with torsion, I always ran into a disaster, some even yielding noncovariant answers. So, something had to be wrong with all of them. Guided by the idea that in spaces with constant curvature the path integral should produce the same result as an operator quantum mechanics based on a quantization of angular momenta, I was eventually able to find a consistent *quantum equivalence principle* for path integrals,[11] thus offering also a unique solution to the operator-ordering problem. This was the key to the leftover problem in the Coulomb path integral in three dimensions—the proof of the absence of the extra time slicing contributions presented in Chapter 13.

Chapter 14 solves a variety of one-dimensional systems by the new techniques.

Special emphasis is given in Chapter 8 to instability (*path collapse*) problems in the euclidean version of Feynman's time-sliced path integral. These arise for actions containing bottomless potentials. A general stabilization procedure is developed in Chapter 12. It must be applied whenever centrifugal barriers, angular barriers, or Coulomb potentials are present.[12]

Another project suggested to me by Feynman—the improvement of a variational approach to path integrals explained in his book on statistical mechanics[13]—found a faster solution. We started work during my sabbatical stay at the University of California at Santa Barbara in 1982. After a few meetings and discussions, the problem was solved and the preprint drafted. Unfortunately, Feynman's illness prevented him from reading the final proof of the paper. He was able to do this only three years later when I came to the University of California at San Diego for another sabbatical leave. Only then could the paper be submitted.[14]

Due to recent interest in lattice theories, I have found it useful to exhibit the solution of several path integrals for a finite number of time slices, without

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[10]B.S. DeWitt, Rev. Mod. Phys. *29*, 337 (1957); K.S. Cheng, J. Math. Phys. *13*, 1723 (1972), H. Kamo and T. Kawai, Prog. Theor. Phys. *50*, 680, (1973); T. Kawai, Found. Phys. *5*, 143 (1975), H. Dekker, Physica A *103*, 586 (1980), G.M. Gavazzi, Nuovo Cimento *101*A, 241 (1981); M.S. Marinov, Physics Reports *60*, 1 (1980).

[11]H. Kleinert, Mod. Phys. Lett. A *4*, 2329 (1989); Phys. Lett. B *236*, 315 (1990).

[12]H. Kleinert, Phys. Lett. B *224*, 313 (1989).

[13]R.P. Feynman, *Statistical Mechanics*, Benjamin, Reading, 1972, Section 3.5.

[14]R.P. Feynman and H. Kleinert, Phys. Rev. A *34*, 5080, (1986).

going immediately to the continuum limit. This should help identify typical lattice effects seen in the Monte Carlo simulation data of various systems.

The path integral description of polymers is introduced in Chapter 15 where stiffness as well as the famous excluded-volume problem are discussed. Parallels are drawn to path integrals of relativistic particle orbits. This chapter is a preparation for ongoing research in the theory of fluctuating surfaces with extrinsic curvature stiffness, and their application to world sheets of strings in particle physics.[15] I have also introduced the field-theoretic description of a polymer to account for its increasing relevance to the understanding of various phase transitions driven by fluctuating line-like excitations (vortex lines in superfluids and superconductors, defect lines in crystals and liquid crystals).[16] Special attention has been devoted in Chapter 16 to simple topological questions of polymers and particle orbits, the latter arising by the presence of magnetic flux tubes (Aharonov-Bohm effect). Their relationship to Bose and Fermi statistics of particles is pointed out and the recently popular topic of fractional statistics is introduced. A survey of entanglement phenomena of single orbits and pairs of them (ribbons) is given and their application to biophysics is indicated.

Finally, Chapter 18 contains a brief introduction to the path integral approach of nonequilibrium quantum-statistical mechanics, deriving from it the standard Langevin and Fokker-Planck equations.

I want to thank several students in my class, my graduate students, and my postdocs for many useful discussions. In particular, T. Eris, F. Langhammer, B. Meller, I. Mustapic, J. Nieschk, T. Sauer, L. Semig, J. Zaun, and Drs. G. Germán, D. Johnston, and C. Holm, have all contributed with constructive criticism. Dr. U. Eckern from Karlsruhe University clarified some points in the path integral derivation of the Fokker-Planck equation in Chapter 18. Useful comments are due to Dr. P.A. Horvathy and to my colleague Prof. W. Theis. Their careful reading uncovered many shortcomings in the first draft of the manuscript. Special thanks go to Dr. W. Janke with whom I had a fertile collaboration over the years and many discussions on various aspects of path integration.

Thanks go also to my secretary S. Endrias for her help in preparing the manuscript in LaTeX, thus making it readable at an early stage, and to U. Grimm for drawing the figures.

[15]A.M. Polyakov, Nucl. Phys. B *268*, 406 (1986), H. Kleinert, Phys. Lett. B *174*, 335 (1986).

[16]See Ref. 9.

Finally, and most importantly, I am grateful to my wife Dr. Annemarie Kleinert for her inexhaustable patience and constant encouragement.

H. Kleinert
Berlin, January 1990

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