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GMAT®
Math Foundations

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INTRODUCTION

Do positive and negative numbers make your head swim? Do you use ten steps to solve a problem when you could have used three? Are you confused about the difference between a trapezoid and a rectangle? And when was the last time you had to add a bunch of fractions without a calculator? Being able to apply math skills and concepts is the most basic foundation for doing well on the GMAT. The problem is you may not have seen all the topics tested lately. Plus, improving your skills is often a boring, tedious process. But it doesn't have to be! Allow us to introduce the Foundations Method.

Devised by the experts at Kaplan, it makes learning mathematics as painless as possible. Unlike other books on math skills review, this book covers only the principles and concepts you need to master for the GMAT—nothing more, nothing less—using a systematic routine for memorization that uses your own real-life situations as practice exercises. It's the most convenient way to learn math while going about your everyday life.

HOW TO USE THIS BOOK

All the math you need for the GMAT you probably learned by your sophomore year in high school. Yes, *high school*! In fact, the math that appears on the GMAT is almost identical to the math tested on the SAT or ACT. You don't need to know trigonometry or calculus!

No matter how much your memories of high school algebra classes have dimmed, don't panic. The GMAT tests a limited number of core math concepts in predictable ways. Certain topics come up in every test, and chances are these topics will be expressed in much the same way; even some of the words and phrases appearing in the questions are predictable. Since the test is so formulaic, we can show you the math you're bound to encounter.

The 12 chapters in this book are divided into three sections—arithmetic, geometry, and algebra—the only math topics you need for the GMAT. Each chapter contains four key components.

1. Detailed Lessons

Each chapter focuses on a specific math topic. We'll explain the concepts involved in detail, provide lots of relevant examples, and offer strategies to help you remember what you need to know. In the process, we help you review the fundamentals from previous chapters.

2. Plentiful Practice

Here's a phrase you'll hear over and over throughout this book: *Repetition is the key to mastery*. So be prepared to practice, practice, practice! You'll find everything from simple matching exercises to exercises that ask you to apply the skills you're learning to practical, real-life situations. By "learning from all sides," so to speak, you're much more likely to retain the information. And remember, don't start a new chapter if you haven't mastered the earlier material—you'll be building on a weak foundation.

3. Summary

Each chapter concludes with a bulleted summary for quick review of important key facts you should take from the lesson.

4. Chapter Test

At the end of each chapter, you'll take a test to help you practice what you've learned and assess how well you've learned it. The tests cover material in that chapter, plus key concepts from previous chapters. The chapter test will help you make sure you have mastered the material in that chapter before you move on to the next lesson.

5. Cumulative Test

The last section of this book is a test that covers all the concepts you've been learning and reviewing throughout the whole book. It's more great review and focused practice.

A strong building needs a strong foundation. To improve your skills in math and get a high GMAT score, you need to begin with a strong foundation too—a solid understanding of basic math concepts. The chapters in this book are arranged from basic skills to more advanced topics, beginning with the properties of numbers, so every skill builds upon a firm base. With a system as easy as this, good math skills are well within your reach. All you have to do is take the first step. Good luck!

CONTENTS

SECTION I

Arithmetic

CHAPTER 1

Properties of Numbers

In this first chapter, you will review the basic facts of working with numbers, including the different classifications of numbers, the order of operations when simplifying numerical expressions, and the three basic properties of numbers.

WORKING WITH NUMBERS

All the numbers that you'll see on the GMAT are **real numbers.** The real numbers, all of the numbers that you encounter each day, are classified into various sets and subsets. All real numbers are either rational numbers or irrational numbers.

You don't really have to be concerned about numbers that aren't real, but if you're wondering, numbers that aren't real are called imaginary. For example, the square root of −1 is imaginary. If that sounds complicated, don't worry; there aren't any imaginary numbers on the test.

Rational Numbers

The **rational numbers** are the numbers that can be expressed as a fraction in the form $\frac{a}{b}$, where a and b are integers and b does not equal 0. A rational number can take various forms:

- 5 can also be written as $\frac{5}{1}$.
- 1.2 can be written as $\frac{12}{10}$ or $\frac{6}{5}$.
- −7 can be written as $\frac{-7}{1}$.
- $0.\overline{3}$ can be written as $\frac{1}{3}$.

When written as a decimal, a rational number is either terminating or repeating.

TERMINATING DECIMALS

Terminating decimals are numbers such as 0.25 or 3.2897; in other words, they are decimals that end. They may extend to just one or two decimal places or for dozens, hundreds, or more.

REPEATING DECIMALS

Repeating decimals are decimals that form a pattern of digits, such as 1.282828… or 3.29712971…. To indicate that the number continues into infinity, you may see an ellipses used (…), or sometimes you'll see a bar over the repeating digit or digits. For example, 1.282828… could also be written as $1.\overline{28}$.

Fractions and decimals will be covered in depth in chapter 3.

Irrational Numbers

Irrational numbers are called "irrational" because they cannot be written as the ratio of two whole numbers and cannot be expressed as fractions. It's easier to think of irrational numbers as nonrepeating, nonterminating decimals. Irrational numbers are decimals that extend into infinity without ever forming a pattern. Some examples are $\sqrt{2}, \sqrt{3}, \sqrt{5}$, and π.

The decimal equivalent of π, as you may know, stretches on and on: 3.141592653… etc. Even when mathematicians look at millions and millions of its digits, no pattern forms. The number π is by far not the only irrational number; many square roots (such as the square root of 2) and some other kinds of numbers are also irrational.

REMEMBER THIS!

The **rational numbers** can be classified into subsets:

The **natural numbers**, also known as the counting numbers:
{1, 2, 3, 4, 5, 6, 7…}.

The **whole numbers** are the natural numbers, plus 0:
{0, 1, 2, 3, 4, 5, 6, 7…}.

The **integers** are the whole numbers and their opposites:
{…–3, –2, –1, 0, 1, 2, 3…}.

Here are some things you should know about integers:

- All numbers greater than 0 are positive numbers.
- All numbers less than 0 are negative numbers.
- Zero is neither positive nor negative.
- The even integers are divisible by 2 and include the number 0.
- The odd integers are not divisible by 2.

The integers will be covered in more depth in the next chapter.

PRACTICE 1

For each set of numbers in the first column, find the letter that corresponds with the correct set classification name. Choose the most descriptive name for the set. Answers and explanations are located at the end of the chapter.

1. $\{\sqrt{7}, \sqrt{11}, \pi, 5.1101001000\ldots\}$	A. Rational numbers
2. $\{\ldots -3, -1, 1, 3\ldots\}$	B. Whole numbers
3. $\{0, 1, 2, 3, 4\ldots\}$	C. Natural numbers
4. $\{-3, .4, -1.5, 0, 6.3\}$	D. Irrational numbers
5. $\{1, 2, 3\ldots\}$	E. Odd integers

The Order of Operations

You encounter formulas in everyday life. When traveling abroad, you may need to convert a temperature from degrees Celsius to degrees Fahrenheit to know whether to wear a coat or not. You many need to calculate the perimeter or area of your back-yard to build a fence. Formulas exist for all kinds of situations.

When simplifying a mathematical expression after you have plugged values into your formula, you do not simply work from left to right, as you do when you read a book. Just as there are rules for driving an automobile, there are rules for order when performing arithmetic operations.

For example, here's a fairly straightforward math problem: $4 + 3 \times 2$.

If someone simply performed the steps from left to right, the result would be

$$4 + 3 \times 2$$
$$= 7 \times 2$$
$$= 14$$

But someone else who chose to do the multiplication first would get a different result:

$$4 + 3 \times 2$$
$$= 4 + 6$$
$$= 10$$

PEMDAS

Clearly, the order in which we perform math steps can make a difference. That order is a predetermined **order of operations** used to evaluate expressions. Perhaps you remember the mnemonic for remembering the order of operations: PEMDAS. Or some of you may have the used memory tool "Please Excuse My Dear Aunt Sally" to recall the correct order.

REMEMBER THIS!

The order of operations is:

P **P**arentheses (grouping symbols)

E **E**xponents

MD **M**ultiply and **D**ivide from left to right

AS **A**dd and **S**ubtract from left to right

- First perform any operations within **P**arentheses. (If the expression has parentheses within parentheses, work from the inside out.)
- Next, work out any **E**xponents.
- Then do all **M**ultiplication and **D**ivision in order from left to right.
- Lastly, do all **A**ddition and **S**ubtraction in order from left to right.

Example:

Evaluate $10 \div 2 - 4 + 3 + 4 \times 7 - 18 \div 9 - 7$.

There aren't any parentheses or exponents, so go straight to multiplication and division.

$$(10 \div 2) - 4 + 3 + (4 \times 7) - (18 \div 9) - 7$$
$$= 5 - 4 + 3 + 28 - 2 - 7$$

Then do the addition and subtraction:

$$= 5 - 4 + 3 + 28 - 2 - 7$$
$$= 1 + 3 + 28 - 2 - 7$$
$$= 4 + 28 - 2 - 7$$
$$= 32 - 2 - 7$$
$$= 30 - 7$$
$$= 23$$

Example:

Evaluate $[4 + (3 \times 2 + 7)] - (5 - 2)^2 + 6 \times 4$.

With nested parentheses or brackets, begin on the inside and work your way outwards.

$$[4 + (3 \times 2 + 7)] - (5 - 2)^2 + 6 \times 4$$
$$= [4 + (6 + 7)] - (5 - 2)^2 + 6 \times 4$$
$$= [4 + 13] - (3)^2 + 6 \times 4$$
$$= 17 - 3^2 + 6 \times 4$$

Now evaluate the exponent:	$17 - 9 + 6 \times 4$
Multiply (there is no division here):	$17 - 9 + 24$
Then add and subtract as needed:	$8 + 24 = 32$

PRACTICE 2

For questions 6–9, answer TRUE or FALSE for each statement. Answers and explanations are located at the end of the chapter.

6. **T** **F** In a numeric expression, addition is always performed before subtraction.

7. **T** **F** Parentheses are evaluated before exponents.

8. **T** **F** In the expression $57 - 32 + 8$, subtraction is the last operation performed.

9. **T** **F** In the expression $700 \div 14 \times 2$, you would first multiply 14 by 2.

10. Simplify: $4^2 - (20 \div 4 \times 2)$.

11. Simplify: $(15 - 12)^3 \div 9 \times 3$.

12. Simplify: $94 - 2(4)^2 + 14$.

13. Simplify: $1{,}200 \div 10^2 - 5 \times 2$.

Other Grouping Symbols

The *P* in PEMDAS stands for parentheses, or grouping symbols. Grouping symbols include parentheses, brackets, the absolute value symbol, and a fraction bar. So to simplify $\frac{18 + 10^2 - 4 \times 2}{20 - 27 \div 3}$, treat the fraction bar as a grouping symbol and first evaluate the top (the numerator) then the bottom (the denominator). Then divide as the final step.

To simplify the numerator, first simplify your exponent: $10^2 = 100$. Second, multiply 4 times 2 to get 8. The top is now $18 + 100 - 8$. Evaluate from left to right: $118 - 8 = 110$. To simplify the denominator, first divide 27 by 3 to get 9. Then subtract: $20 - 9 = 11$. Finally, divide 110 by 11 to get 10.

In the order of operations, a radical sign is evaluated on the same level of priority as an exponent. To simplify $550 - \sqrt{9 \times 4} \times 3$, you would evaluate the radical first. Under the radical sign, multiply 9 times 4 to get 36. The square root of 36 is 6. Now the problem reads $550 - 6 \times 3$. Multiply 6 by 3 next to get $550 - 18$ for a final value of 532.

When plugging numbers into formulas, a working knowledge of the order of operations is essential. For example, to convert a temperature from degrees Fahrenheit to degrees Celsius, you use the formula $C = \frac{5}{9}(F - 32)$, where F is the degrees in Fahrenheit and C is the degrees in Celsius. If you have a temperature of 77 degrees Fahrenheit and you want to know the equivalent degrees in Celsius, substitute 77 for F in the formula: $C = \frac{5}{9}(77 - 32)$. First subtract 32 from 77, because parentheses are evaluated first: $C = \frac{5}{9}(45)$. Now multiply $\frac{5}{9}$ by 45 to get 25 degrees Celsius.

NUMBER PROPERTIES

Certain common properties of numbers are frequently used to make adding and multiplying easier. You most likely use these properties without even realizing it when you do mental arithmetic or when you add a column of numbers. These properties give you the license to change the order of operations in certain situations. In addition to making addition and multiplication of number terms easier to calculate, these three properties are frequently used in solving algebraic equations, as explained in chapter 12.

The Commutative Property of Addition

The **commutative property of addition** states that changing the order of the addends in a sum does not change the sum.

$a + b = b + a$, where a and b are any real numbers.

For example: $12.3 + 6.9 + 7.7 = 12.3 + 7.7 + 6.9$.

The order of operations dictates that 12.3 first be added to 6.9. But the addition is easier if you first add 12.3 to 7.7 because the sum will equal a whole number. The commutative property of addition gives you this freedom.

The Commutative Property of Multiplication

The **commutative property of multiplication** states that changing the order of the factors in a product does not change the product.

$a \times b = b \times a$, where a and b are any real numbers

For example: $2 \times 8 \times 5 \times 7 = 2 \times 5 \times 8 \times 7$.

If you scan a group of factors to find subproducts that equal 10, 100, or 1,000, it is easiest to multiply these factors first. The commutative property allows you to make these changes to the order of operations.

When you are combining addition and subtraction, or combining multiplication and division, the key issue is the sign (+, −, ×, or ÷) that precedes the number. For example, if a number is preceded by a subtraction sign, you must make sure that the number is subtracted. However, you can do the subtraction in any convenient order. Understanding this may help you do your arithmetic work.

It might be a bit frustrating or annoying to work through something like this:

$$10 - 2 + 3 - 8 + 4 - 6$$

But we can rewrite it in a more convenient order, taking care to maintain each number's sign, and this will make the arithmetic easier.

$$\begin{aligned} &10 - 2 + 3 - 8 + 4 - 6 \\ &\quad = 10 + 3 + 4 - 2 - 8 - 6 \\ &\quad = 17 - 16 \\ &\quad = 1 \end{aligned}$$

Simplify: $120 - 2 \times 14 \times 5 + 9 \times 3 - 8 + 10 - 9 \div 3$.

$$\begin{aligned} &120 - 2 \times 5 \times 14 + 9 \times 3 - 8 + 10 - 9 \div 3 \\ &= 120 - 10 \times 14 + 9 \times 3 - 8 + 10 - 9 \div 3 \\ &= 120 - 140 + 27 - 8 + 10 - 3 \\ &= 120 + 27 + 10 - 140 - 8 - 3 \\ &= 157 - 140 - 8 - 3 \\ &= 17 - 8 - 3 \\ &= 9 - 3 \\ &= 6 \end{aligned}$$

Notice that in the second step, we multiplied the 2 by the 5 before we multiplied by the 14. That made the math easier because it's easier to multiply by 10 than by just about any other number.

You'll have to be careful at first as you practice manipulating arithmetic like this, but since having a strong command of the material will greatly improve both your speed and accuracy, it is well worth the effort! Success on the GMAT often demands a creative problem-solving approach.

The Associative Property

Like the commutative property, the associative property pertains to either the addends in a sum or the factors in a product. The **associative property of addition** or **multiplication** states that changing the grouping (parentheses or brackets) of addends in a sum or the grouping of factors in a product does not change the resulting sum or product.

$a + (b + c) = (a + b) + c$, where a, b, and c are any real numbers.

$a \times (b \times c) = (a \times b) \times c$, where a, b, and c are any real numbers.

Sometimes, you can use the associative law (together with the commutative law) creatively to make your life easier. For example, you may want to look for numbers that are easier to add first, then add the rest. In the following problem, we can move the numbers into groups that add up to 10 to make our arithmetic easier.

$$\begin{aligned}
&3 + 9 + 1 + 2 + 7 + 8 \\
&= 3 + (9 + 1) + 2 + 7 + 8 \\
&= 3 + (9 + 1) + 7 + (2 + 8) \\
&= (3 + 7) + (9 + 1) + (2 + 8) \\
&= 10 + 10 + 10 \\
&= 30
\end{aligned}$$

We would have gotten 30 if we had added the numbers in the original order as well, but it would have been a bit more challenging. The difference is even more notable when the numbers are larger! Try to find pairs of numbers that add up to 100 in the following: 48 + 36 + 81 + 70 + 64 + 30 + 52 + 19.

Answer:

$$\begin{aligned}
&48 + 36 + 81 + 70 + 64 + 30 + 52 + 19 \\
&\quad = (48 + 52) + (36 + 64) + (81 + 19) + (70 + 30) \\
&\quad = 100 + 100 + 100 + 100 \\
&\quad = 400
\end{aligned}$$

Again, simply adding the numbers in the original order would also have yielded 400. By combining them in this order, however, we make the problem much easier. Of course, not all problems will provide such perfect pairings, but it is worth your time to look for ways to make the problems easier. The payoff in speed and accuracy will be enormous down the line.

To explore the associative property of multiplication, consider the expression $7 \times 20 \times 5 \times 8$. Notice that $20 \times 5 = 100$, so change the grouping to make the multiplication easier: $7 \times (20 \times 5) \times 8$. Now evaluate from left to right: $7 \times 100 = 700$. Finally, $700 \times 8 = 5{,}600$.