Susheng Wang

Microeconomic Theory

微观经济学理论

王苏生　编著

中国人民大学出版社

This book covers microeconomic theory at the Master's and Ph.D levels for students in business schools and economics departments. It concisely covers major mainstream microeconomic theories today, including neoclassical microeconomics, game theory, and information economics.

Introduction

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The Features

1. It is a concise coverage of modern microeconomic theory.

2. It focuses on the mainstream microeconomic topics.

3. It covers some advanced topics that are not well covered in Mas-Colell et al. (1995).

4. Its English is easy to understand for Chinese readers.

About The Author

The author graduated from Nankai University in 1985, got his first job at Mathematics Department of Nankai University, obtained his Ph.D degree in economics at University of Toronto in 1991, and then became an assistant professor at Concordia University in Canada. He moved to Hong Kong University of Science and Technology in 1993 and has been there since.

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Chapter 1

Producer Theory

1.1 Technology

Suppose that a firm has n goods to serve as inputs and/or outputs. If the firm uses y_i^- units of good i as an input and produces y_i^+ units of the good, then $y_i \equiv y_i^+ - y_i^-$ is the **net output** of good i. The firm's **production plan** $y = (y_1, y_2, \cdots, y_n) \in \mathbb{R}^n$ is a list of net outputs of all the goods that it produces as outputs and/or uses as inputs. Practically, we can treat positive numbers in a net output vector as outputs and negative numbers as inputs.

For any two vectors $x, y \in \mathbb{R}^n$, denote

$$x \geqslant y \quad \text{if} \quad x_i \geqslant y_i, \ \forall i$$

$$x > y \quad \text{if} \quad x \geqslant y, x \neq y$$

$$x >> y \quad \text{if} \quad x_i > y_i, \ \forall i$$

A set $Y \subset \mathbb{R}^n$ of production plans that are technologically feasible is called the firm's **production possibility set**. A production plan $y \in Y$ is **technologically efficient** if there is no. $\hat{y} \in Y$ such that. $\hat{y} > y$. [①] A production plan is **economically efficient** if it maximizes profits $\pi = p \cdot y$ for a given price vector p over the production possibility set Y.

We can easily see that economic efficiency implies technological efficiency. Technological efficiency has nothing to do with the market. No matter what the prices of inputs and outputs are, the firm needs to achieve technological efficiency in order to achieve profit maximization; see Figure 1.1. We can think of the firm's choice problem in two steps: the firm has to achieve technological efficiency first and then achieve economic efficiency on the efficient production set (the production frontier). The **production frontier** (**PPF**) is defined to be the set of all technologically efficient production plans.

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① A weak version of technological efficiency is as follows: a production plan $y \in Y$ is **technologically efficient** if there is no. $\hat{y} \in Y$ such that. $\hat{y} >> y$.

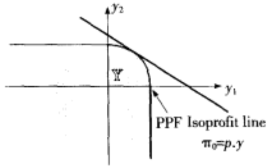

Figure 1.1 Technological Efficiency v.s. Economic Efficiency

Proposition 1.1 *Economic efficiency implies technological efficiency.*

We will concentrate on the case in which the firm produces only one output; we may think of this output as an index of the firm's outputs. The case of multiple outputs will be dealt with later. When there is only one output, we will use $y \in \mathbb{R}_+$ to denote the firms's output and $x \in \mathbb{R}^n_+$ to denote the firm's inputs. Then, a typical production plan is $(y, -x) \in \mathbb{Y}$, where $\mathbb{Y}$ is a set in $\mathbb{R}^{n+1}$. Given any vector $x \in \mathbb{R}^n_+$ of inputs, denote the maximum technologically feasible output as $f(x)$:

$$f(x) \equiv \max_{(y,-x)\in \mathbb{Y}} y$$

We call this function $f: \mathbb{R}^n_+ \to \mathbb{R}_+$ the **production function.** We can easily verify that $y = f(x)$ if and only if $(y, -x)$ is technologically efficient. In this sense, the production function fully characterizes technologically efficient plans. Call curve

$$Q(y) \equiv \{x \in \mathbb{R}^n_+ \mid y = f(x)\}$$

an **isoquant.** The isoquant for output y is the set of all inputs that produces y as the maximum technologically feasible output.

Proposition 1.2 *For the production function $f(\cdot)$, $(y, -x)$ is technologically efficient* $\Leftrightarrow y = f(x)$

Example 1.1 Cobb-Douglas Technology. For $0 \leqslant \alpha \leqslant 1$, let

$$\mathbb{Y} \equiv \{ (y, -x_1, -x_2) \in \mathbb{R}_+ \times \mathbb{R}^2_- \mid y \leqslant x_1^\alpha x_2^{1-\alpha}\}$$

By definition, as shown in Figure 1.2, we have

$$f(x_1, x_2) = x_1^\alpha x_2^{1-\alpha} \quad Q(y) = \{(x_1, x_2) \in \mathbb{R}^2_+ \mid y = x_1^\alpha x_2^{1-\alpha}\}$$

We often have special interest in several important characteristics of a production function, from which some economic insight can be obtained. First, for production function $y = f(x_1, x_2, \cdots, x_n)$, given output y_0, with an increase in x_1, by how much do we need to decrease x_2 to maintain the same level of output y_0? That is, $\left.\frac{\Delta x_2}{\Delta x_1}\right|_{f=\text{const}}$. measures the

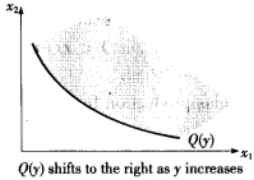

Figure 1.2 Cobb-Douglas Technology

certain substitutability of the two inputs. By differentiating both sides the following equation

$$f(x_1, x_2, \cdots, x_n) = y_0$$

and using the fact that $dy_0 = 0$ and $dx_i = 0$ for $i \geqslant 3$, we have

$$\lim_{\Delta x_1 \to 0} \frac{\Delta x_2}{\Delta x_1}\bigg|_{f=y_0} = \frac{\partial x_2}{\partial x_1} = -\frac{f_{x_1}(x)}{f_{x_2}(x)}$$

Hence, the **marginal rate of transformation** (**MRT**) between x_1 and x_2 at x is defined as

$$\text{MRT}(x) \equiv \frac{fx_1(x)}{fx_2(x)}$$

In Figure 1.3, we can see that the MRT is the slope of the isoquant.

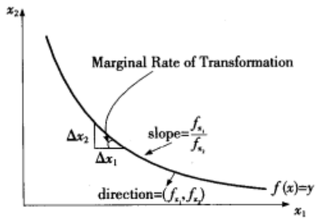

Figure 1.3 The Marginal Rate of Transformation

Example 1.2 Suppose that $f(x_1, x_2) = x_1^a x_2^{1-a}$. Then,

$$\frac{\partial f(x)}{\partial x_1} = a x_1^{a-1} x_2^{1-a} \qquad \frac{\partial f(x)}{\partial x_2} = (1-a) x_1^a x_2^{-a}$$

Therefore, the MRT is $\frac{a}{1-a}\frac{x_2}{x_1}$.

Second, we say that a production function $f: \mathbb{R}_+^n \to \mathbb{R}_+$ has global **constant returns to scale** (**CRS**) if $f(tx) = tf(x)$ for all $x \in \mathbb{R}_+^n$ and $t > 1$. With this production technology, the firm can double its output by doubling its inputs. Similarly, we say that the production function has global **increasing returns to scale (IRS)** or **decreasing returns to scale (DRS)** if, respectively, $f(tx) > tf(x)$ or $f(tx) < tf(x)$ for all $x \in \mathbb{R}^n_+$ and $t > 1$.

Example 1.3 For the Cobb-Douglas production function $f(x_1, x_2) = Ax_1^a x_2^b$, we have

$$f(tx_1, tx_2) = A(tx_1)^a (tx_2)^b = t^{a+b} f(x_1, x_2)$$

We immediately see that the production function has global IRS if $a + b > 1$, global CRS if $a + b = 1$, and global DRS if $a + b < 1$.

Third, the **elasticity of scale** at x measures the percentage increase in output due to one percentage increase in scale:

$$e(x) \equiv \frac{df(tx)}{dt} \frac{t}{f(x)} \bigg|_{t=1}$$

In other words, $e(x)$ is the percentage increase in output when all inputs have expanded proportionally by 1%. We say that the technology exhibits **local increasing, constant, or decreasing returns to scale if** $e(x)$ is greater, equal, or less than 1, respectively.

Proposition 1.3 *Returns to Scale.*

(1) *For $x \in \mathbb{R}^n_+$, we have*

$$\text{global IRS} \Rightarrow \text{local IRS or CRS}, \ \forall x$$

$$\text{global CRS} \Rightarrow \text{local CRS}, \ \forall x$$

$$\text{global DRS} \Rightarrow \text{local DRS or CRS}, \ \forall x$$

(2) *For $x \in \mathbb{R}_+$, we have*

$$e(x) = \frac{x \cdot f'(x)}{f(x)}$$

implying

$$\text{local IRS} \Leftrightarrow f'(x) > \frac{f(x)}{x}$$

$$\text{local CRS} \Leftrightarrow f'(x) = \frac{f(x)}{x} \qquad (1.1)$$

$$\text{local DRS} \Leftrightarrow f'(x) < \frac{f(x)}{x}$$

(3) *For $x \in \mathbb{R}^n_+$ and $y = f(x)$, we have*

$$e(x) = \frac{\text{AC}(y)}{\text{MC}(y)}$$

implying

$$\begin{aligned} &local\ \text{IRS} \Leftrightarrow \text{AC} > \text{MC} \\ &local\ \text{CRS} \Leftrightarrow \text{AC}=\text{MC} \\ &local\ \text{DRS} \Leftrightarrow \text{AC} < \text{MC} \end{aligned} \tag{1.2}$$

Proof (1) Global IRS implies that, for any $\Delta t > 0$,

$$\frac{f[(1+\Delta t)x]-f(x)}{\Delta t}\frac{1}{f(x)} > \frac{(1+\Delta t)f(x)-f(x)}{\Delta t}\frac{1}{f(x)} = 1$$

that is,

$$e(x) \equiv \frac{df(tx)}{dt}\frac{t}{f(x)}\bigg|_{t=1} = \lim_{\Delta t \to 0+}\frac{f[(1+\Delta t)x]-f(x)}{\Delta t}\frac{1}{f(x)} \geqslant 1$$

Therefore,

global IRS $\Rightarrow$ local IRS or CRS, $\forall\ x$
global CRS $\Rightarrow$ local CRS, $\forall\ x$
global DRS $\Rightarrow$ local DRS or CRS, $\forall\ x$

(2) For $x \in \mathbb{R}_+$, we have $e(x) = \frac{1}{f(x)}x \cdot f'(x)$. By this, we can determine the regions for the three returns to scale on a diagram; see Figure 1.4.

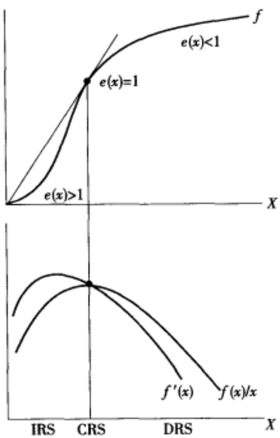

Figure 1.4 Local Returns to Scale

(3) We can also illustrate the returns to scale using cost curves. The cost function is

$$c(y) \equiv \min\{w \cdot x \mid y = f(x)\}$$

The Lagrange function is $\mathcal{L} \equiv w \cdot x + \lambda[y - f(x)]$. The first-order condition (FOC) is

$$w = \lambda Df(x^*)$$

and the Envelope Theorem is

$$c'(y) = \lambda$$

We then have

$$c(y) = w \cdot x^* = \lambda Df(x^*) \cdot x^* = c'(y)[Df(x^*) \cdot x^*]$$

since

$$e(x^*) = \frac{1}{f(x)} Df(x^*) \cdot x^* = \frac{1}{y} Df(x^*) \cdot x^* = \frac{c(y)}{y} \frac{1}{c'(y)} = \frac{\text{AC}}{\text{MC}}$$

Hence, (1.2) holds and it can be illustrated in Figure 1.5.

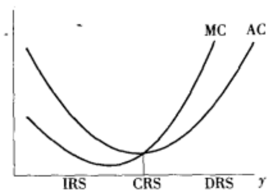

Figure 1.5 Local Returns to Scale

The property in (1.1) is illustrated in Figure 1.4.
The property in (1.2) is illustrated in Figure 1.5.
By the argument for the local returns to scale and Figure 1.4, we immediately have the diagrams for the global returns to scale in Figure 1.6. [1]

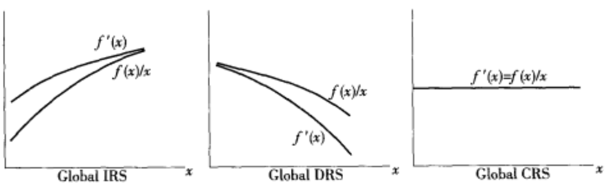

Figure 1.6 Global Returns to Scale

[1] The equation $f'(x) = f(x)/x$ gives the general solution $f(x) = Ax$, where A is an arbitrary constant, implying $f'(x) = A$.

By the argument for the local returns to scale and Figure 1.5, we immediately have the diagrams for the global returns to scale in Figure 1.7.

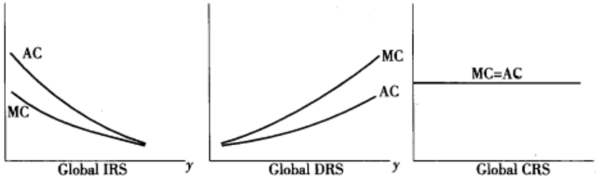

Figure 1.7 Global Returns to Scale

Finally, let $x(w,y)$ be the cost-minimizing input vector for given input price vector w and output y. The **elasticity of substitution** between the two inputs $x_1(w,y)$ and $x_2(w,y)$ is defined to be the ratio of percentage change in $\frac{x_1(w,y)}{x_2(w,y)}$ to the percentage change in $\frac{w_1}{w_2}$:

$$\sigma \equiv -\frac{w_1/w_2}{x_1(w,y)/x_2(w,y)} \frac{\partial[x_1(w,y)/x_2(w,y)]}{\partial(w_1/w_2)}$$

Note that this variable is well defined only if $\frac{x_1(w,y)}{x_2(w,y)}$ is a function of $\frac{w_1}{w_2}$. As shown later, since $x_i(w,y)$ is zero homogenous,① $x_i(w,y)$ is a function of $\frac{w_1}{w_2}$. Therefore, $\frac{x_1(w,y)}{x_2(w,y)}$ is a function of $\frac{w_1}{w_2}$. By the optimality of $x(w,y)$, using (1.5), we can alternatively write this elasticity as

$$\sigma = -\frac{fx_1/fx_2}{x_1/x_2} \frac{\partial(x_1/x_2)}{\partial(f_{x_1}/f_{x_2})}$$

1.2 The Firm's Problem

The firm is assumed to maximize its profits. Profits consist of two distinct parts:

profits = revenue − costs

Revenue is the money received from the sales of the firm's products. Costs are the **economic cost** or more popularly called the **opportunity cost**, which typically includes three components: the cost of labor (and raw materials), the cost of capital (including depreciation), and the cost of land (and natural resources).

In general, suppose that a firm takes n actions accomplished by choosing a vector $a \in \mathbb{R}^n$. The actions may include output levels, labor inputs, capital inputs, and even prices.

① See Appendix A for the definition of homogenous functions.