ARITHMETIC & CALCULATORS

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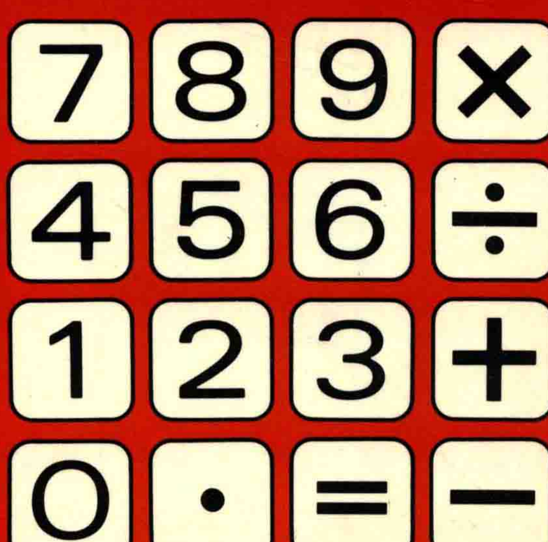

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Arithmetic and Calculators

HOW TO DEAL WITH ARITHMETIC IN THE CALCULATOR AGE

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Arithmetic and Calculators

Contents

Prelude

To our dear readers:

This is a book about arithmetic—about the arithmetic of the calculator age.

We are not going to overwhelm you with rules about carrying and borrowing or with a list of terms like "*trial divisors*," although we shall surely try some divisors. We shall not make a fuss about addends, subtrahends, and minuends. Instead, we shall concentrate on what makes arithmetic tick and what arithmetic can do for you—the kinds of problems arithmetic can solve and how to solve them.

The arithmetic itself is to be done in large part by a calculator. You will need an inexpensive electronic calculator to enjoy this book. A calculator cannot tell you how to do a problem. It *can help* you learn. It *can help* you to apply what is important in arithmetic. We think your calculator may change your attitude toward arithmetic. It should make arithmetic more useful to you. It will free you from the drudgery of calculating. You will be free to concentrate on the applications of arithmetic. It will give you a new tool to solve everyday problems. And we hope that it will give you some fun in playing and experimenting with numbers.

Let's begin with a psychological experiment. How do you respond to these questions?

What do you think of when you think of numbers?

How do you picture the counting numbers,

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, . . . ?

Express your ideas in a few words, or if you like, draw a picture! Let your feeling toward numbers go wild! If you can, get some other persons to answer these questions.

Compare your answers. You will discover that people give different and often personal answers to these questions. We are sure you will find these answers, interesting and informative. Afterward, but please not before, you may want to look at Appendix A, where we have recorded some of the responses given by an adult class in arithmetic.

There is no unique image of counting numbers that all people carry around. Indeed, there are many different impressions of numbers and arithmetic. This means that, to perform arithmetic operations, you may want to call upon your own personal view of how operations work.

Many of our likes and dislikes are shaped by our earliest contacts. In the case of arithmetic, this contact often comes from unhappy experiences in early school learning situations. Too often our natural desires to quantify our experiences are snubbed by demands for a rote recall of number facts for which no need has been established. As we grow older, our feelings and frustrations toward arithmetic grow into a helpless confusion in a sea of numbers. Now with the aid of these relatively inexpensive calculators, we have the capability of dealing swiftly and correctly with the processes of arithmetic and can concentrate on their use. Incidentally, the development of these electronic calculators is the first technological breakthrough in computation to become available to most people since the abacus.

Another reward for having a calculator at hand is that you will be able to play and to experiment with numbers. You can use a calculator to discover things about numbers not easily revealed through hand computation. In this way, you will see things about numbers that you would not be able to appreciate by hand. In this way, a calculator is to arithmetic what a microscope is to nature. With a microscope, you see things that you cannot with the unaided eye.

In arithmetic, observations are often expressed through the recognition of patterns. Here are some elementary examples.

Example 0.1: Counting by 2's.

2,	4,	6,	8,	10,
12,	14,	16,	18,	20,
22,	24,	26,	28,	30,

. . .

These are the so-called "*even*" numbers. An even number is one that is a multiple of 2. Numbers that end in a 2, 4, 6, 8, or 0 are multiples of 2. This fact is suggested by the table above for we can see how the rows repeat. Thus, for example, 48 is an even number because it ends in an 8. Indeed,

$$48 = 24 \times 2.$$

Example 0.2: Counting by 5's.

$$\begin{array}{rr} 5, & 10, \\ 15, & 20, \\ 25, & 30, \\ & \ldots \end{array}$$

These numbers are the multiples of 5. The pattern of the table suggests that any number ending in a 5 or a 0 is a multiple of 5. Thus, for example, 75 is a multiple of 5. Indeed,

$$75 = 15 \times 5$$

Example 0.3: Some special products. Explain what is happening here:

8×9	$= 72$	
8×99	$= 792$	
8×999	$= 7992$	
8×9999	$= 79992$	
8×99999	$= 7$ ______ 2	(Fill in the blank)
8×999999	$=$ ______	(Write in the answer)
$8 \times$ ______	$= 79999992$	(Fill in the blank)

If you see the pattern, then you can make a good guess even without checking the answer.

Other interesting patterns arise in division.

Example 0.4: Division by 3; the decimal part. (It will help if you use the calculator. If its operation is unfamiliar, go on to Chapter 1 and then return to this example when you are ready.) Begin systematically. We recorded the answers as they will appear on an 8-digit calculator.

$1 \div 3 = 0.3333333$

$2 \div 3 = 0.6666666$

$3 \div 3 = 1.$ (or 1.0000000 on some calculators)

$4 \div 3 = 1.3333333$

$5 \div 3 = 1.6666666$

$6 \div 3 = 2.$ (or 2.0000000 on some calculators)

$7 \div 3 = 2.3333333$

$8 \div 3 = 2.6666666$

$9 \div 3 = 3.$ (or 3.0000000 on some calculators)

and so on.

Do you see a pattern? Can you predict the decimal part of $11 \div 3$? of $13 \div 3$?

Example 0.5: Division by 7; the decimal part. What is the decimal form of the remainder when a whole number is divided by 7?

$1 \div 7 = 0.1428571$

$2 \div 7 = 0.2857142$

$3 \div 7 = 0.4285714$

$4 \div 7 = 0.5714285$

$5 \div 7 = 0.7142857$

$6 \div 7 = 0.8571428$

$7 \div 7 = 1.$ (or 1.0000000 on some calculators)

$8 \div 7 = 1.1428571$

$9 \div 7 = 1.$ ______ (Complete the answer!)

Do you see a pattern—or perhaps several patterns? Can you predict the decimal part of $15 \div 7$?

Here are some things you might notice about Example 0.5. The decimal part always begins and ends with the same digit. The sequence of the digits is always the same, but the starting place differs.

Thus in each part,

1 is followed by 4

4 is followed by 2

2 is followed by 8

8 is followed by 5

5 is followed by 7

7 is followed by 1

and now the cycle repeats.

If you like these patterns, try divisions by 13 and look for similar patterns.

Example 0.6: Magic squares. The following arrangement of the whole numbers from 1 to 9 has been known for a long time. We can imagine that arrangements of this type were first found by doodling—very much as we are suggesting playing around with a calculator. To find what is magic about the arrangement below, add the numbers in each row and column.

8	1	6	row sum = ______
4	9	2	row sum = ______
3	5	7	row sum = ______

Perhaps that's not so impressive. Now add the sums of each column. Each column sums to 15 also! Does this seem surprising?

Now consider this arrangement of the same numbers.

2	7	6
9	5	1
4	3	8

Again add the numbers in each row and column. The result is 15. But there is more magic! Add the numbers on each of the diagonals as indicated by the dotted lines in the square above. Again 15! (More magic and more magic squares will be discussed in Chapter 11.)

There may seem to be much that is magic about numbers and arithmetic. There is! Sometimes solutions to difficult problems call for insight and a stroke of cleverness born of intuition and experience. But more often, systematic methods for the analysis of problems are available. Indeed, it is the power of such systems that make mathematics a useful discipline. These systems are based on universal mathematical concepts and constructions. Together they compose the world of ordered beauty that is mathematics. So, before we can fly on wings of magic we must walk along the roads leading to these basic mathematical principles. Your calculator will help you gain the experience and technique so necessary to the development of insight and intuition. Join us on this journey.

1

Getting Started on Your Calculator

1.1 A Look at Your Calculator

Electronic calculators come in many sizes and shapes. They differ in the kinds of mathematical functions they perform and in the complexity of problems they solve with these functions. In this book we shall suppose that you have a calculator that—

1. accepts eight-digit numbers for computation (an "*8-place*" calculator);
2. performs the four basic arithmetic operations, addition (+), subtraction (−), multiplication (×), and division (÷);
3. uses "*floating-point*" arithmetic; and
4. uses "*algebraic logic.*"

Many calculators have features to supplement these basic features. Some have special keys for square roots, percents, reciprocals, and powers. Others will compute logarithms and trigonometric functions. Some calculators have one or more memory registers, activated by pressing memory keys (M), which permit partial computations to be stored for later recall while still other calculations are performed. Some machines have "*fixed-point*" arithmetic. For these, the results of computations are displayed to a fixed number of decimals—say, to hundredths. For others, the position of the decimal point may vary from problem to problem according to the computations. These have a "*floating-point*" mode. Then again, some machines can use either mode. The floating-point mode is the more flexible and will have the greatest use for us in studying arithmetic. We shall illustrate some of these extra features in special sections as we need them.

Look at your calculator and become familiar with its operation. Because of the wide variation in calculators, it is impossible to write detailed instructions to fit every machine. Read the operation manual accompanying your machine and practice with it.

Still, we must agree on a language and symbolism to convey the arithmetical procedures we want to study in this book. So let us have a look at the parts of a calculator and do some easy exercises to gain familiarity and to fix our notation for its operation.

1.2 The Physical Set-Up

Look at your calculator. Find the "off-on" switch. It won't work unless you turn it on! To save batteries, turn it off when you have completed a calculation and recorded the final answer.

Prominently displayed on your calculator is a "window." The numbers you wish to manipulate will appear here as you enter them into the calculator. The answer to your computations will appear in this window also.

The rest of the face of your calculator is occupied by its keys. Each key is labeled to show its function. Some sophisticated machines give two or more different functions for the same key. We shall assume that you have a fairly simple machine with one function per key.

Most calculators have a central cluster of *numerical* keys, which look like this:

7	8	9
4	5	6
1	2	3
0		

In machines using "*algebraic logic*," these numerical keys are surrounded by operational or command keys, including decimal-point (.), equal (=), addition (+), subtraction (−), multiplication (×), and division (÷) keys. In this book, use of these keys will be indicated by the following symbols.

$\boxed{\cdot}$ $\boxed{=}$ $\boxed{+}$ $\boxed{-}$ $\boxed{\times}$ $\boxed{\div}$

On many machines, the command keys are arranged as follows:

7	8	9	÷
4	5	6	×
1	2	3	−
0	·	=	+

There are other keys as well. The clear (C) key permits you to begin a new problem by erasing all numbers associated with any previous calculation. Turning the machine off and then on also has that same effect, because electrical impulses are necessary to keep a number in a register. Another key that most calculators have is a "*clear entry*" (CE) key, which permits you to correct a mistake in the number you are entering without disturbing other numbers or calculations already stored elsewhere in the machine. On some machines, this function is combined with the (C) key. Read your operating manual. If your machine has a memory, there are special keys to control it.

Some machines have a *constant* key or switch (K). When you activate this switch, you may make several multiplications (or divisions) with the same factor (or divisor). For example, in calculating these products.

$$8 \times 2, \quad 4 \times 2, \quad 36 \times 2,$$

the constant switch causes the machine to remember the constant factor, 2, and so this factor need not be re-entered for each multiplication. Many machines have this capability as an automatic feature of their logic, so that a separate switch is unnecessary.

1.3 Entering Numbers into Your Calculator

Here are some easy exercises to get started with your calculator.

Example 1.1: Enter 325.

Put the number, 325, into the calculator so that, for example, something could be added to it.

Action		*Display*
Turn your calculator **ON**	Read the display:	0.
Press numerical key, **3**	Read the display:	3
Press numerical key, **2**	Read the display:	32
Press numerical key, **5**	Read the display:	325

You are not done yet! The calculator does not know that there are only three digits in your number. For all it knows, you might have been trying to enter

3257895674893.23456283185394247780125663!

You must tell the calculator that you have finished entering a number.

Press operation key **(=)** Read the display: *325.*

Pressing the (=) key tells the machine that a number has been entered. Some machines will, at this point, add the decimal point as we have indicated to acknowledge that the number is complete. Other machines "blink" as you press the (=) key. As a matter of fact, you can end the entry of a number by pressing any operation key. The key you choose should depend on what you want to do next. However, because any other operation key may follow an (=) command, it is a good one for us to use here.

In the future, we shall describe the actions in Example 1.1 simply as follows.

325 [=]

In this book, numerals printed as

0 1 2 3 4 5 6 7 8 9

designate numbers that either have been entered into a calculator or are the results of calculator computations.

Example 1.2: Enter 3.25.

This shows how to use the decimal point. We would use it in working with $3.25.

Enter 3.25	*Action*	*Display*	*Remarks*
	Press **(C)**	0.	Clear all registers.
Key in 3.25	Press **3**	3	
	Press **(.)**	3.	
	Press **2**	3.2	
	Press **5**	3.25	
	Press **(=)**	3.25	Complete entry.

or, more briefly,

3.25 [=]
