An Introduction to Mechanical Engineering

Part 1

companion WEB

Michael Clifford, Richard Brooks, Alan Howe, Andrew Kennedy, Stewart McWilliam, Stephen Pickering, Paul Shayler & Philip Shipway

ROUTLEDGE

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Introduction

Engineering is not merely knowing and being knowledgeable, like a walking encyclopaedia; engineering is not merely analysis; engineering is not merely the possession of the capacity to get elegant solutions to non-existent engineering problems; engineering is practicing the art of the organized forcing of technological change.

Dean Gordon Brown

This book is written for undergraduate engineers and those who teach them. It contains concise chapters on solid mechanics, materials, fluid mechanics, thermodynamics, electronics and dynamics, which provide grounding in the fundamentals of mechanical engineering science. An introduction to mathematics is covered in the companion publication, *An Introduction to Mathematics for Engineers* by Stephen Lee, also published by Hodder Education.

The material in this book is supported by an accompanying website: www.hodderplus.co.uk/mechanicalengineering.

The authors have over 120 years' experience of teaching undergraduate engineers between them, mostly, but not exclusively, at the University of Nottingham. The material contained within this textbook has been derived from lecture notes, research findings and personal experience from within the lecture theatre and tutorial sessions.

We gratefully acknowledge the support, encouragement and occasional gentle prod from Stephen Halder and Gemma Parsons at Hodder Education, without whom this book would still be a figment of our collected imaginations.

Dedicated to past, present and future engineering students at the University of Nottingham.

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Unit 1

Solid Mechanics

Richard Brooks

UNIT OVERVIEW

- Basic design analysis
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1.1 Basic design analysis

Forces, moments and couples

> A **force** arises from the action (or reaction) of one body on another.

Although a force cannot be directly observed, its effect can be. A typical example is a force arising from the surface contact between two bodies, e.g. one pushing against the other. Two forces actually occur in this situation as shown in Figure 1.1. One is the 'action' of the man on the wall and the other is the 'reaction' of the wall on the man.

> Newton's third law tells us that the action and reaction forces in this situation (and generally) are equal and opposite.

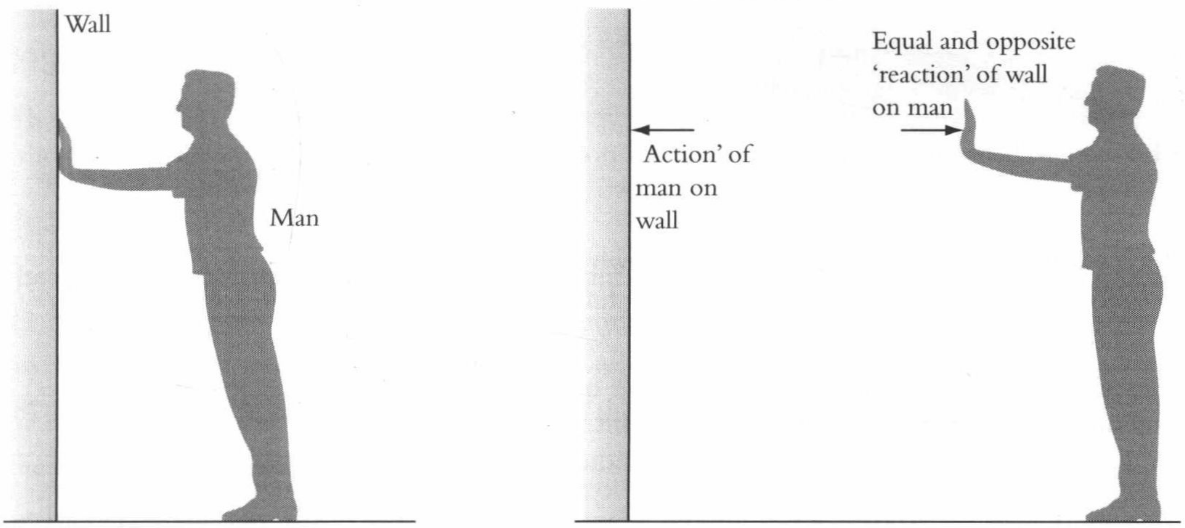


Figure 1.1 Newton's third law

Such contact forces occur where bodies interact with each other; however, they can also occur internally within a single body. In this case, it is the microscopic particles, e.g. molecules, atoms, etc. which contact each other and interact with forces between themselves. For this chapter, we will generally be dealing with macroscopic bodies where the interaction forces occur at external surface contacts.

Another type of force occurring is that which arises from the remote influence of one body on another, such as the force of gravity. The Earth's gravity acting on a person gives rise to a force acting at his or her centre of mass. This type of force is termed the person's **weight** and acts vertically downwards or towards the centre of the Earth. Magnetic attraction is another example of a remote (or non-contact) force arising from the influence of a magnetic field on a body.

The SI unit of force is the newton (N).

A force of 1 N is that force which, when applied to a mass of 1 kg, will result in an acceleration of the mass of 1 m s^{-2}. Thus, in general, a force applied to a body tends to change the state of rest or motion of the body, and the relationship between the resulting motion (acceleration, a) and the applied force, F, is given by Newton's second law, i.e. $F = ma$ where m is the mass of the body. However, in this chapter, we will generally be concerned with bodies in equilibrium, where there is no motion, i.e. static situations. For this to be the case, all forces acting on the body must balance each other out so that there is no resultant force (see the next section on 'equilibrium').

A force has both a **magnitude** and a **direction** and is therefore a vector quantity which can be represented by an arrow as shown in Figure 1.2. The magnitude of the force is represented by a label, e.g. 5 N as shown, or, alternatively, when solving problems graphically, by the length of the arrow. The direction of the force is clearly represented by the orientation of the arrow in space such as the angle θ to the x-direction.

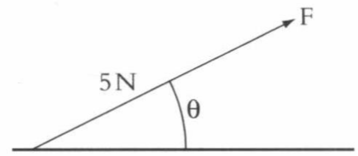


Figure 1.2 **Force as a vector**

Thus, when considering problems in two dimensions, two scalar quantities are required to describe a force, i.e. its magnitude and direction – in the above case 5 N and θ° respectively. To aid the analysis of systems with several forces, the forces are often resolved into their *components* in two perpendicular directions, as shown in Figure 1.3 for the force F. The x- and y-directions are commonly chosen, although resolving in other (perpendicular) directions relevant to the boundaries of a body may be more convenient for a specific problem. From Figure 1.3 the magnitudes of the two components in the x- and y-directions are given by:

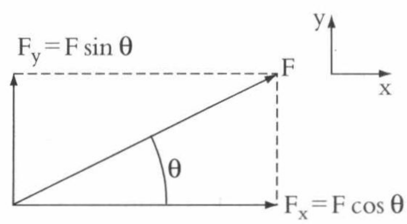


Figure 1.3 **Resolving the force vector into components**

$$F_x = F \cos \theta$$
$$F_y = F \sin \theta \qquad (1.1)$$

With this representation there are still two scalar quantities describing the force, in this case, F_x and F_y.

The **moment** of a force about a point is equal to the product of the magnitude of the force and the perpendicular distance from the point to the line of action of the force.

This is illustrated in Figure 1.4, where the moment, M, of force F, about point O, is given by:

$$M = F.d \qquad (1.2)$$

An example of a device which creates a moment is a spanner, also shown in Figure 1.4. The hand applies the force, F, at one end and imparts a moment, $M = F.d$, on the nut at the other end, O.

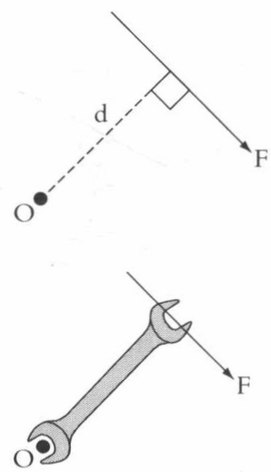


Figure 1.4 **Moment of a force applied by a spanner**

A **couple** is a special case of a moment of a force and arises from a pair of equal and opposite parallel forces acting on a body but not through the same point, as shown in Figure 1.5. If the two forces, F, act at a distance d apart, then the magnitude of couple C, about any point, is given by:

$$C = F.d \qquad (1.3)$$

As the two forces, F, in Figure 1.5, are equal and opposite, their sum is zero and the body on which they act is *not* translated. However, they do create a couple which tends to *rotate* the body. Therefore, a consequence of a couple acting on a body is to impart pure rotation. For this reason, the term 'pure moment' is often used instead of 'couple'.

An example of a device which creates a couple is a wheel nut wrench, also shown in Figure 1.5. Here, the hands apply forces, F, in and out of the page at both ends of one arm of the wrench, imparting a turning couple on a locked nut at O.

When a couple or moment is applied at a point on a body its effect is 'felt' at all other points within the body. This can be illustrated with the cantilever beam shown in Figure 1.6 where a couple of 5 kN m is applied at end A. If we assume that the couple is created by the application of two equal and opposite 5 kN forces, 1 m apart, acting through a rigid bar attached to the beam at A, we can determine the influence that these forces also have at points B and C, at 5 m and 10 m from A respectively.

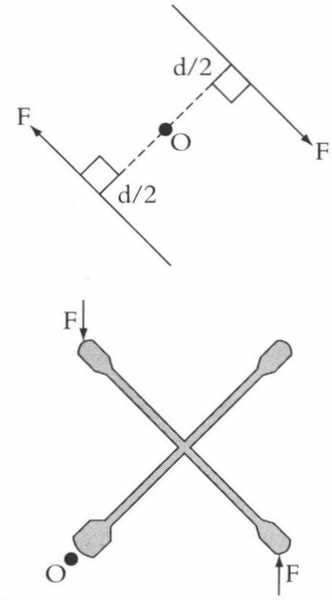


Figure 1.5 Couple and wheel nut wrench (forces act in and out of page)

Taking moments about B:

$M_B = 5\text{ kN}.(5\text{ m} + 0.5\text{ m}) - 5\text{ kN}.(5\text{ m} - 0.5\text{ m})$

$= 27.5 - 22.5 = 5\text{ kN m}$

Taking moments about C:

$M_C = 5\text{ kN}.(10\text{ m} + 0.5\text{ m}) - 5\text{ kN}.(10\text{ m} - 0.5\text{ m})$

$= 52.5 - 47.5 = 5\text{ kN m}$

In both cases the effect, i.e. a 5 kN m turning moment, is felt at B and C. In other words, the turning moment felt on the bar is independent of the distance from A.

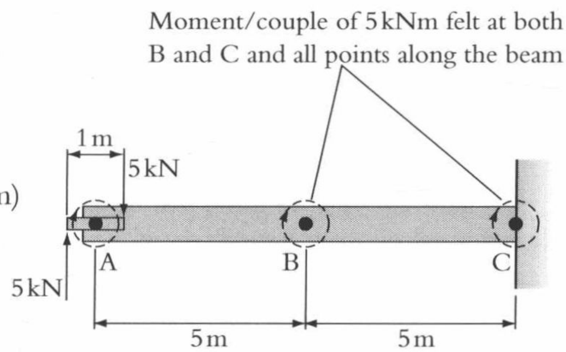


Figure 1.6 Influence of a moment or couple acting at a point

Conditions of equilibrium

For a body to be in equilibrium, it must not translate or rotate. Considering movement in one plane only (i.e. a two-dimensional system), this means the body must not move in the x- or y-directions or rotate about its position. Three conditions are required of the applied forces for this to be the case.

> These three conditions of equilibrium are:
> (i) the sum of all the acting forces in the x-direction must be zero, i.e. $\Sigma F_x = 0$.
> (ii) The sum of all the acting forces in the y-direction must be zero, i.e. $\Sigma F_y = 0$.
> (iii) The sum of all the moments about any point must be zero.

Resultants of forces

When a number of forces act at a point on a body, their resultant force can be determined either algebraically or graphically.

Algebraic method

The algebraic method for determining the resultant of a number of forces has the following steps:

(i) Resolve all forces into their x- and y-components.
(ii) Sum the x-components (ΣF_x) and the y-components (ΣF_y).
(iii) Determine the magnitude and direction of the resultant force from ΣF_x and ΣF_y.

The following example illustrates the method.

Figure 1.7 shows three forces F_A, F_B and F_C acting at a point A. Determine the magnitude and direction of the resultant force at A.

The components of the forces are,

$$F_{Ax} = 0 \qquad F_{Ay} = 4\text{ kN}$$
$$F_{Bx} = -8\text{ kN} \qquad F_{By} = 0$$
$$F_{Cx} = -6.\cos 60° = -3\text{ kN} \qquad F_{Cy} = -6.\sin 60° = -5.196\text{ kN}$$

Summing these components in the x- and y-directions,

$$\Sigma F_x = 0 - 8 - 3 = -11\text{ kN}$$
$$\Sigma F_y = 4 + 0 + -5.196 = -1.196\text{ kN}$$

(note the −ve values indicating that the resultant forces act in the −ve x and −ve y directions)

The magnitude, F_R, of the resultant of ΣF_x and ΣF_y is,

$$F_R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
$$= \sqrt{(-11)^2 + (-1.196)^2}$$
$$= 11.064\text{ kN}$$

The angle, θ (with respect to the x-axis), of the resultant force is,

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$
$$= \tan^{-1}\left(\frac{-1.196}{-11}\right)$$
$$= 6.2° \text{ to the negative } x\text{-direction as shown in Figure 1.7.}$$

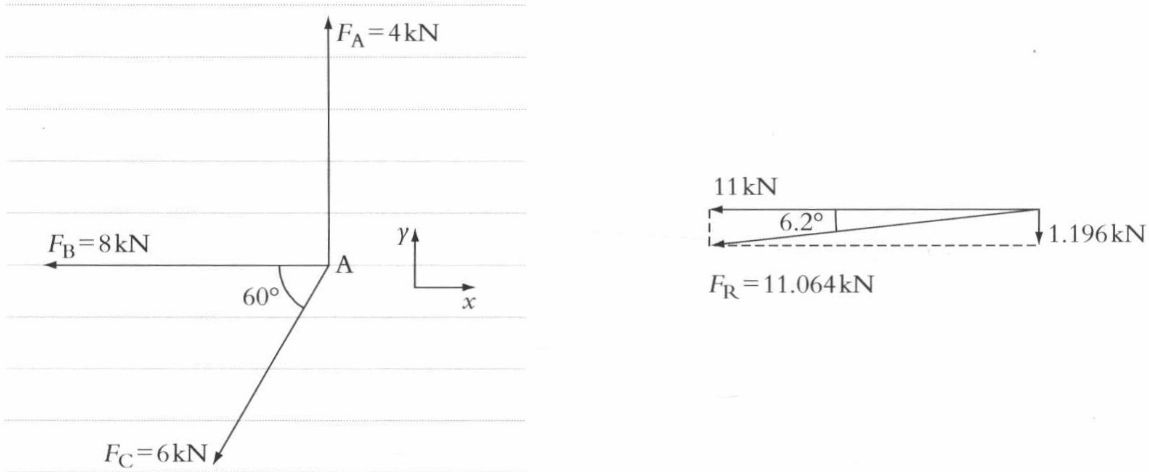


Figure 1.7 Resultant of a number of forces acting at a point

Graphical Method

The procedure for the graphical method of determining the resultant of a number of forces is shown in Figure 1.8 for the problem given above.

Firstly, draw to scale each of the three vector forces, F_A, F_B and F_C, following on from each other, as shown in the figure. The resultant force, F_R, is the single vector force that joins the start point A to the finishing point B, i.e. that closes the polygon of forces. Its magnitude and direction (θ) may be measured off from the scale vector diagram.

(NB: it does not matter in which order the three vectors are drawn in the diagram.)

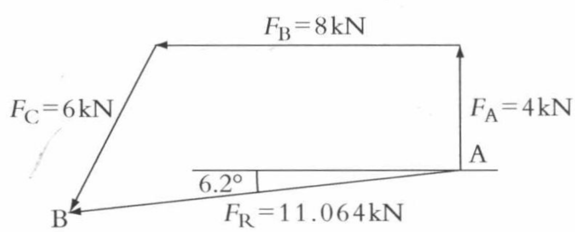


Figure 1.8 Resultant of forces acting at a point – graphical method

The graphical method is useful to give a quick approximate solution, whereas the algebraic method normally takes longer but will yield an exact result.

Frictional forces

Consider a solid body, i.e. a block, weight W, resting on the ground but in equilibrium under the action of an applied force, F_A, as shown in Figure 1.9. In general, where the body contacts the ground there will be a reaction force (from the ground) acting on the body. This reaction force has two components as follows:

(i) a tangential force, F, termed the *friction force*;

(ii) a normal force, N.

As the body is in equilibrium, these two components of the reaction force counterbalance the applied force, F_A, and the weight of the body, W, to prevent any movement. (NB: the body's weight is given by its mass × the acceleration of gravity, i.e. Mg and acts at the centre of mass.)

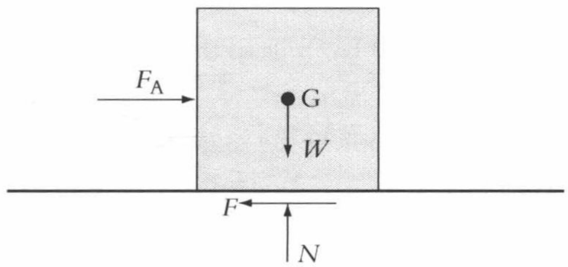


Figure 1.9 **Frictional force (F) and normal force (N) at point of contact between a block and the ground**

The frictional force, F, exists because of the rough nature of the contact surface between the body and the ground. In some cases, where the contact is smooth or lubricated, the frictional force will be negligible and there will be a normal reaction force only. This is a special case only found under certain circumstances, e.g. contact surfaces in a lubricated bearing.

If the applied force, F_A, is slowly increased, the frictional component, F, will also increase to maintain equilibrium. At some point the applied force will become sufficiently large to overcome the frictional force and cause movement of the body. Up to this point of 'slip' between the surfaces, a relationship exists between the frictional force and the normal reaction force as follows:

$$F \leqslant \mu N \tag{1.4}$$

Note the 'less than or equal to' sign indicates that a limiting condition can occur. This limiting condition is the point of slip, at which point $F = \mu N$. Thus, the maximum value of F, i.e. the limiting frictional force, is proportional to N. The constant of proportionality, μ, is termed the **coefficient of static friction** and its value depends on the roughness of the two contacting surfaces and hence the contacting materials. Typical values are in the range 0.1 – 1.0, where a lower value indicates a smoother surface and reduced friction. Values outside this range can occur for some material contact surfaces e.g. lubricated surfaces can have values lower than 0.1 while stick-slip surfaces, such as rubber on a hard surface, can have values in the range 1–10.

A number of important observations can now be stated about the frictional force, F:

(i) F cannot exceed μN;

(ii) the direction of F always *opposes* the direction in which subsequent motion would take place if slip occurred;

(iii) the magnitude of F is independent of the size of the contact area between the contacting surfaces;

(iv) if slip does occur, the magnitude of F is independent of the velocity of sliding between the two contact surfaces.

Although in this chapter we will be concerned primarily with static friction up to the point of slip, if slip does occur, the coefficient of dynamic friction (also called the kinetic frictional coefficient, μ_k) is usually marginally lower than the coefficient of static friction. In the sliding (i.e. slipping) condition the limiting form of equation (1.4) still applies, i.e. $F = \mu_k N$.

Free body diagrams

To analyse the forces in more complex systems, such as assemblies of components or structures containing many different elements, it is normal to break down the problem into separate free bodies.

Figure 1.10 shows two bodies, body A positioned on top of body B which itself is located on the ground. To analyse this problem for forces, we separate the two bodies and draw on each all the external forces acting as shown in the figure. The aim is to solve for the unknown reaction forces between the two bodies and between body B and the ground.

Thus, for body A, the external forces are its weight, W_A, acting at its centre of mass ($W_A = M_A.g$) and the vertical reaction force, R_B, from body B. There is no horizontal friction force at the contact between the bodies because there are no horizontal forces acting.

For body B, there is also its weight, $W_B = M_B.g$, again acting at its centre of mass, the action force, R_A, acting downwards from A and the reaction force, R_G, acting upwards from the ground.

Newton's third law tells us that $R_A = R_B$, i.e. 'for every action there is an equal and opposite reaction'.

We can now look at the *equilibrium* of each body in turn:

For body A, $\Sigma F_y = 0 \quad \therefore \; R_B = W_A$

and for body B $\Sigma F_y = 0 \quad \therefore \; R_G = R_A + W_B = R_B + W_B = W_A + W_B$

It is no surprise that the reaction force at the ground is equal to the sum of the weights of the two bodies. This is necessary to maintain the system in equilibrium.

Although this is a simple problem, it clearly illustrates the value of separating the two bodies, allowing us to solve for the unknown reaction force between the bodies. The diagrams of each separate body are referred to as **freebody diagrams** (FBDs).

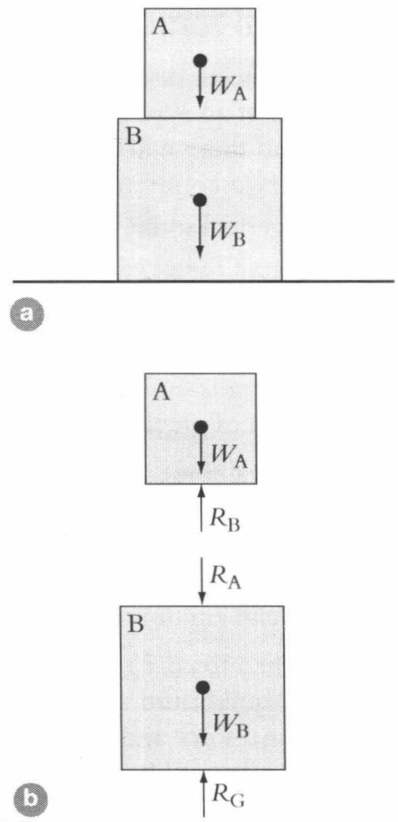


Figure 1.10 **Free body diagrams**

Key points about free body diagrams:

(i) A free body diagram, as the name implies, is a diagram of a free body which shows *all* the external forces acting on the body.

(ii) Where several bodies (or subcomponents) interact as part of a more complex system, each body should be drawn separately, and interacting bodies should be replaced at their contact points with suitable reaction forces and/or moments.

General design principles

A number of general principles related to force analysis can be applied in design to simplify problems. In this section we will consider several of these principles.

Principle of transmissibility

A force can be moved along its line of action without affecting the static equilibrium of the body on which it acts. This principle of transmissibility is illustrated in Figure 1.11 where the equilibrium of the body is the same whether it is subjected to a pushing force or a pulling force acting along the same line of action. It should be pointed out that, although static equilibrium is the same in each case, the internal forces within the body will be different.

Figure 1.11 **Principle of transmissibility**

Statically equivalent systems

A load system can be replaced by another one, provided the static behaviour of the body on which they act is the same. Such load systems are termed **statically equivalent**.

Figure 1.12 shows a number of loads (five in total) each of 5 kN acting on a beam structure in such a way as to be evenly distributed along the length of the beam. If we are not interested in the internal forces developed within the beam but only the equilibrium of the beam as a whole, then this loading system can be replaced by a simpler point load of 25 kN applied at the centre of the beam. The two load systems are statically equivalent and the equilibrium conditions for the beam will apply, whichever of the loading systems is assumed.

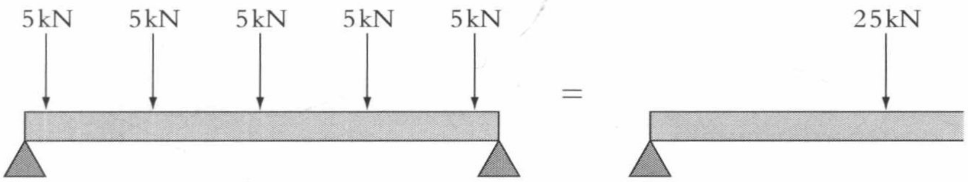


Figure 1.12 **Statically equivalent**

Two-force principle

> The **two-force principle** states that, for a two-force body (i.e. a body with forces applied at two points only) to be in equilibrium, both forces must act along the same line of action.

This is illustrated in Figure 1.13 where Body A is subjected to two forces, F_1 and F_2, *not* acting along the same line of action. Taking moments about point A, the application point for F_1, it is clear that there is a resultant moment and the body cannot be in equilibrium. For it to be in equilibrium, the distance d must be zero. This is the case for Body B, where F_1 and F_2 act along the same line and cannot therefore generate a moment. In addition, F_1 must equal F_2.

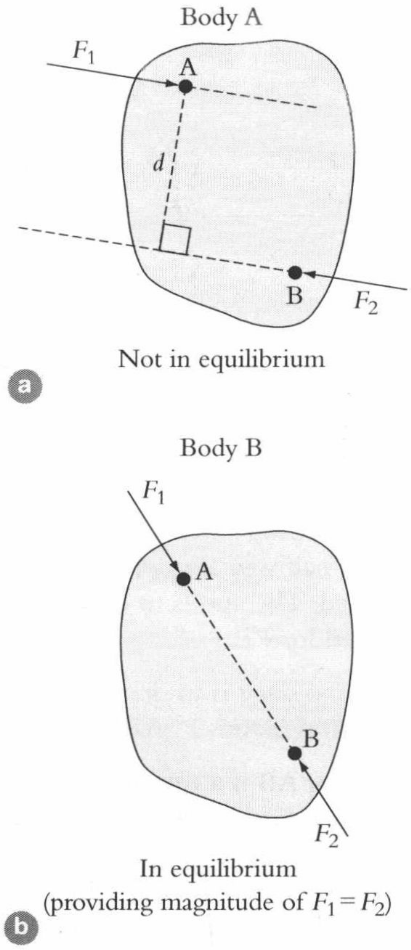


Figure 1.13 **Two-force principle**

Three-force principle

> The **three-force principle** states that for a three-force body (i.e. a body with forces applied at three points only) to be in equilibrium, the lines of action of these forces must pass through a common point.

This is illustrated in Figure 1.14 where Body A is subjected to three forces, F_1, F_2 and F_3 not acting along the same line of action. Taking moments about point O, where the lines of action of F_1 and F_2 meet, it is clear that there is a resultant moment arising from F_3 and the body cannot be in equilibrium. For it to be in equilibrium, the distance d must be zero. This is the case for Body B, where the lines of action of F_1, F_2 and F_3 meet at O and there cannot be a resulting moment. In addition, the vector sum of F_1, F_2 and F_3 must be zero.

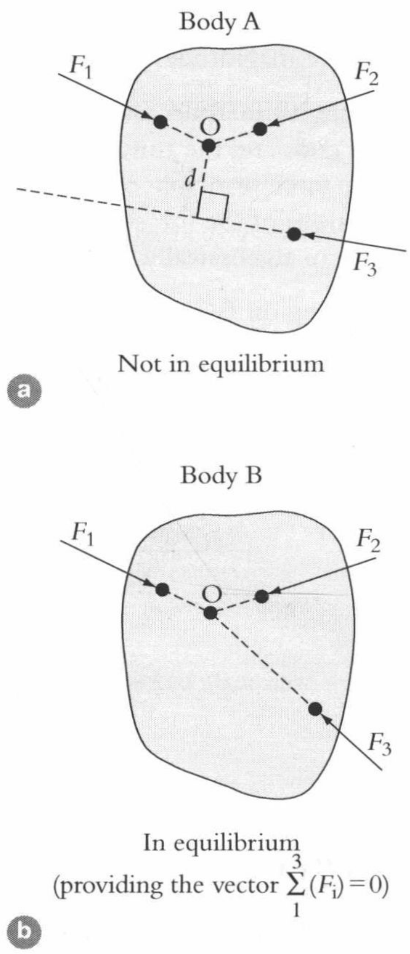


Figure 1.14 **Three-force principle**

Pin-jointed structures

A pin-jointed structure, as shown in Figure 1.15, comprises an assembly of several members, which are joined together by frictionless pin joints. Such a joint cannot transmit moments due to the free rotation of the pin. This simplification is found in practice to be valid for many structures and enables the analysis of forces within the structure to be significantly simplified. The objective is usually to determine the forces occurring at each of the pin joints in the structure, and this is achieved by considering equilibrium of individual members and the structure as a whole.
A solution can be obtained algebraically or graphically.

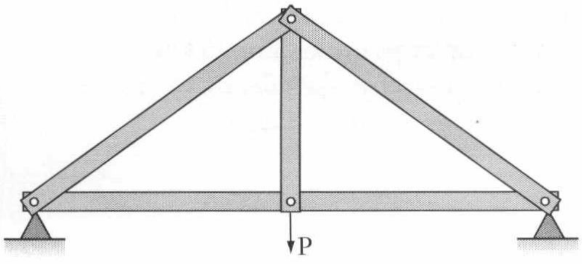


Figure 1.15 **A pin-jointed structure**

Algebraic solution to a pin-jointed structure problem

Consider a wall bracket comprising a simple two-bar pin-jointed structure, ABC, as shown in Figure 1.16. Both members are of equal length and inclined to the horizontal at 45°. The joints at A, B and C are all pin joints, and the lower member, BC, is subjected to a vertical downward load, P, acting half way along its length. The weights of the members may be ignored. The aim is to determine the forces at A, B and C in terms of the applied load P.

The first stage is to draw the freebody diagrams for the two members, also shown in Figure 1.16.

Member AB is a two-force member as forces act only at the two ends of the member. For equilibrium of a two-force member, the directions of the forces, R_A and R_B, must be along the same line, i.e. along the axis of the member. As AB is clearly in tension, the directions are as indicated in the figure. Note that for some problems it may not be possible to establish on simple inspection whether a member is in tension or compression. This is not a problem, because if the forces are drawn incorrectly, say in compression rather than tension, the analysis will, in that case, result in a negative magnitudes for the forces.

Moving to member BC, this is a three-force member with forces acting at both ends and the third force, P, acting at the centre of the member. The three-force principle could be used for this member to establish the directions of the forces; however, we will not do so, as we are solving the problem algebraically.

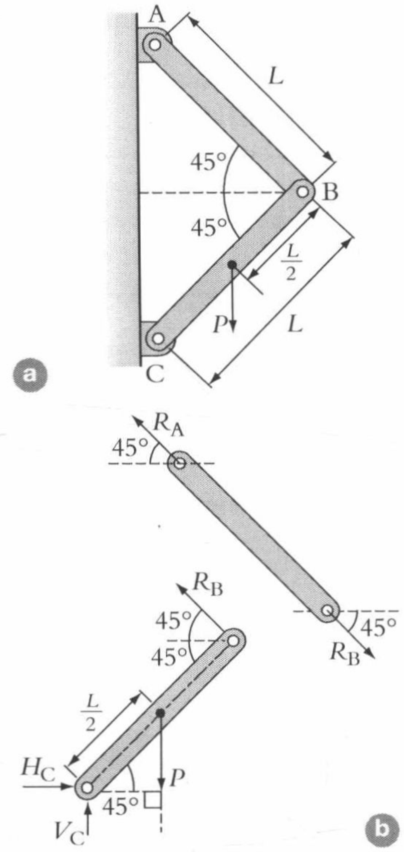


Figure 1.16 **Pin-jointed structure – algebraic solution**

R_B acting on BC at joint B must be equal and opposite to R_B acting on AB at joint B (Newton's third law). As we do not know the direction of the force at C, we will give it two unknown components, H_C and V_C, as shown in Figure 1.16. Now looking at the equilibrium of BC:

horizontal forces $\quad H_C - R_B \cos 45° = 0$

$$\therefore\ H_C = 0.707\ R_B \qquad (1.5)$$

vertical forces $\quad V_C - P + R_B \sin 45° = 0$

$$\therefore\ V_C = P - 0.707\ R_B \qquad (1.6)$$

moments about C $\quad P.\left(\frac{L}{2}\right).\cos 45° - R_B.L = 0$

$$\therefore\ R_B = 0.354P$$

$$\text{and } R_A = R_B = 0.354P$$

Both R_A and R_B act at 45° to the horizontal, i.e. along the line of AB.

Substituting for R_B into (1.5) and (1.6) gives,

$$H_C = 0.25P \text{ and } V_C = 0.75P$$

The magnitude of the resultant force R_C is:

$$R_C = \sqrt{(H_C)^2 + (V_C)^2} = \sqrt{(0.25P)^2 + (0.75P)^2} = 0.791P$$

and the angle of R_C with respect to the horizontal is:

$$\tan\theta_C = \frac{V_C}{H_C} = \frac{0.75P}{0.25P} = 3$$

$$\theta_C = 71.6°$$

Thus, the magnitudes and directions of all three forces at the joints A, B and C have been determined.

Geometrical solution to a pin-jointed structure problem

Figure 1.17 shows a schematic model of a crane boom supported by a pin-joint at one end, A, and a cable attached to the other end, C, inclined at an angle of 30° to the horizontal. The member is assumed to be weightless but carries a vertical downwards load, P, part way along its length at position B. The aim is to determine the tension, T, in the cable and the magnitude and direction of the reaction force, R_A.

The first stage is to draw the freebody diagram (to a suitable scale) of the member ABC, also shown in Figure 1.17. The freebody diagram should include all forces acting on the member. The directions of P and the cable tension, T, are known and can be drawn in immediately. However, the direction of R_A is not known. To find this, the three-force principle can be used because the member has three forces acting on it and, for equilibrium, all three forces must meet at a point. Thus, the line of action of P should be extended to intersect the line of action of T at point O. Then the line of action of R_A must also pass through the point O to satisfy the three-force principle. AO therefore defines the direction of R_A, shown in the figure as the angle α to the horizontal. From a scale drawing, α can be measured as 44°, or alternatively this value can be calculated by trigonometry in triangles ABO and BCO.

The above stage only yields the direction of the unknown force, R_A. Its magnitude and the magnitude of the tension T are still required. To find these, a 'force polygon' is drawn. In this case it is actually a force triangle as there are only three forces, but the general term is 'force polygon'. The triangle comprises the vectors of the three forces, P, T and R_A, drawn following on from each other to form a closed triangle as shown in Figure 1.17. The triangle must close as the member, ABC, is in equilibrium under the action of the three forces and their sum must be zero. To draw the triangle, firstly draw the vector representing P vertically downwards. Its end points define two points of the triangle. Next, at one end of the vector P, draw a line representing the direction of T. At the other end of P, draw a line representing the direction of R_A (NB: it does not matter which end each of the force directions are drawn from, as long as it is a different end for each force). These two lines intersect at the third point of the triangle and the lengths of the other two sides (not P) give the magnitudes of T and R_A respectively. Note also that the direction of R_A is upwards as it must close the triangle.

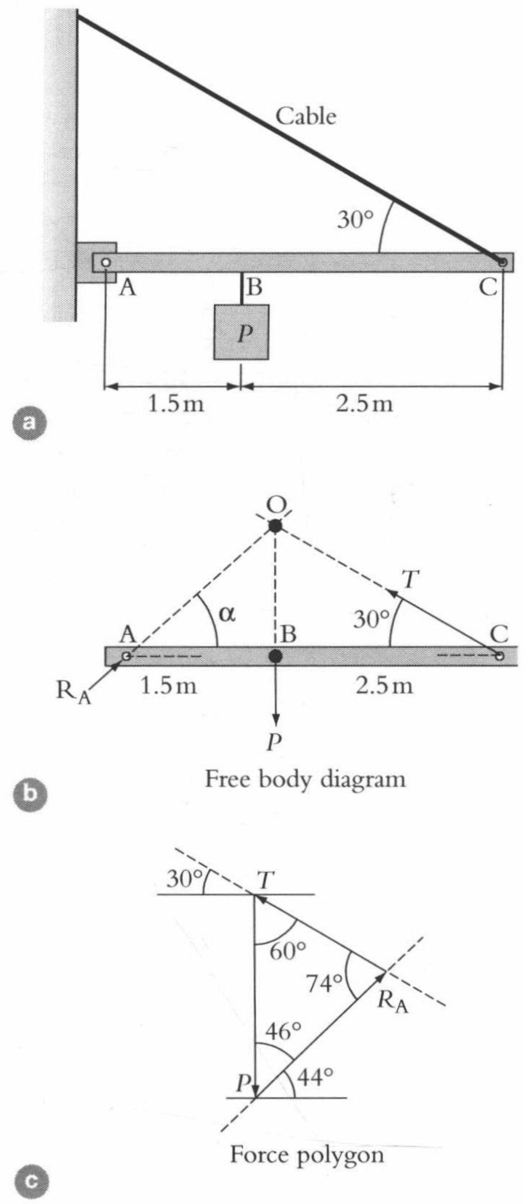


Figure 1.17 Pin-jointed structure – graphical solution

Measuring the force triangle gives the magnitudes of T and R_A, in terms of P, as,

$$T = 0.75P \text{ and } R_A = 0.9P$$

These magnitudes can alternatively be found by using trigonometry as one side and three angles of the triangle are known, allowing the two unknown sides to be determined using the sine rule (see 'Trigonometry' overleaf).

Trigonometry

Trigonometry is often needed to solve pin-jointed structure problems. Both the 'sine rule' and the 'ccosine rule' may be needed to solve for unknown sides and angles in vector polygons (or triangles). These rules are given in Figure 1.18.

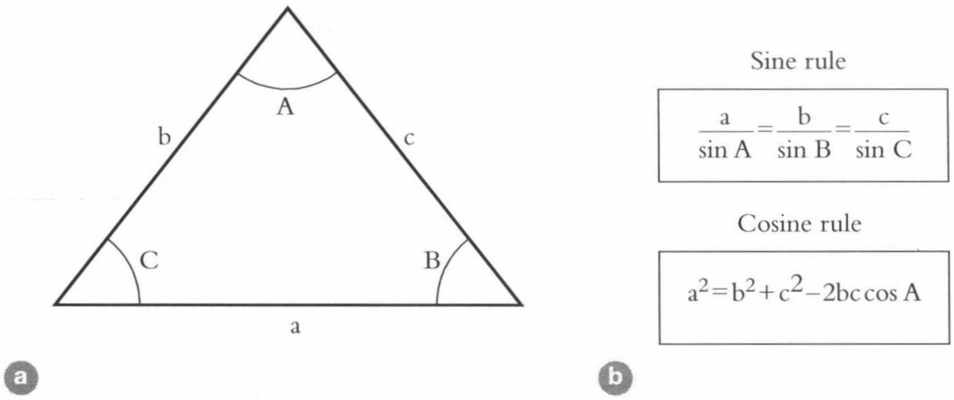


Figure 1.18 Useful trigonometric relationships

Free body diagrams

The inclined edge of block B rests on top of the inclined edge of block A, as shown in Figure 1.19. A horizontal force, *P*, is applied to block A. Block A can slide horizontally along the ground and block B can slide vertically against the wall. All surfaces are frictional contacts with a coefficient of friction μ. The masses of the blocks are M_A and M_B respectively.

Draw the free body diagrams for both blocks when they are just on the point of slipping under the action of the horizontal force *P*.

The solution is also given in Figure 1.19. The two blocks are separated and drawn as free bodies. All contact surfaces contain a normal reaction force and a parallel friction force. As the system is on the point of slip, the frictional force = μ × normal reaction force at each contact surface. Note that where the two blocks contact each other, there are equal and opposite reaction and friction forces (Newton's third law). In this problem, there are two pairs of equal and opposite forces *N* and μ*N* where the two bodies contact. Using the principle of statically equivalent systems, all forces are drawn as point forces for simplification. In reality, they will be distributed over the contact surfaces. The weight of each body is a force acting at its centre of mass.

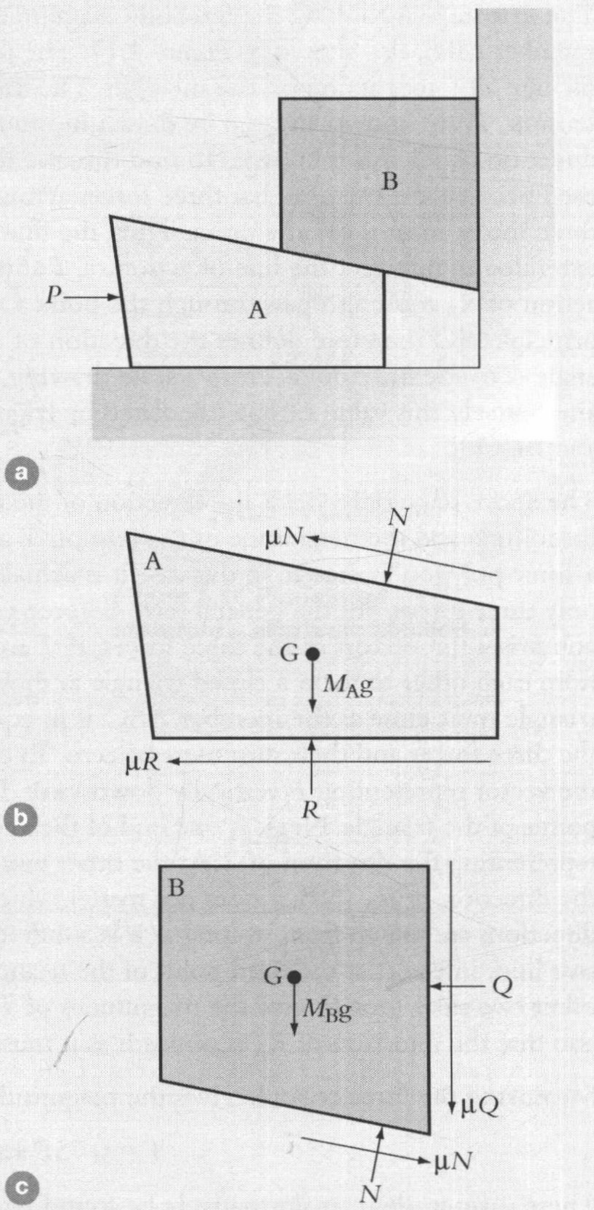


Figure 1.19 Free body diagrams worked example

Equilibrium and frictional forces

Figure 1.20 shows a ladder of weight 500 N resting on two surfaces at points A and B. Dimensions are as indicated. The coefficient of friction at all contacting surfaces is 0.4.

Determine the maximum height to which a man of weight 1200 N can walk up the ladder before slip occurs.

General solution procedure (see Figure 1.20):

(i) Draw the freebody diagram of the ladder.

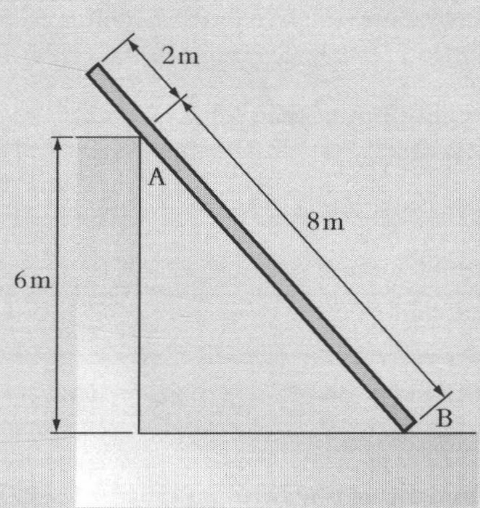
(ii) At the point of slip, assume the man has climbed to a height, h, above the base point, B.

(iii) Draw frictional forces acting opposite to the direction of slip.

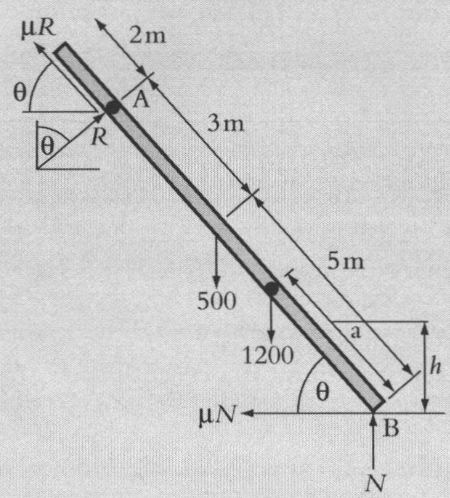
(iv) Assume both points A and B slip at the same time.

(v) At slip, assume friction force = $\mu \times$ normal force.

(vi) Use equilibrium of forces and moments to solve for unknown forces and height h.



a



b

Figure 1.20 **Equilibrium and friction worked example**

Figure 1.20 shows the free body diagram of the ladder with all forces acting, when on the point of slip.

Solving for the angle θ

$$\sin\theta = \frac{6}{8} = 0.75$$

$$\therefore\ \theta = 48.6°$$

$$\text{and}\ \cos\theta = 0.661$$

Equilibrium of vertical forces

$$-1200 - 500 + R\cos\theta + \mu R\sin\theta + N = 0$$

$$-1700 + R.(0.661) + 0.4R.(0.75) + N = 0$$

$$\therefore\ N = 1700 - 0.961R \quad (1.7)$$

Equilibrium of horizontal forces

$$-\mu N + R\sin\theta - \mu R\cos\theta = 0$$

$$-0.4N + R.(0.75) - 0.4R.(0.661) = 0$$

$$\therefore\ N = 1.214R \quad (1.8)$$

From equations (1.7) and (1.8), we obtain

$N = 948.9$ N and $R = 781.6$ N

Taking moments about B

$$-500\cos\theta\,.\,5 - 1200\cos\theta\,.\,(a) + R\,.\,(8) = 0$$

$\therefore\ a = 5.8$ m (i.e. distance up the ladder)

Therefore, height of man, h is

$$h = a\sin\theta = 5.8\ (0.75) = 4.35\ \text{m}$$