SCHAUM'S OUTLINE SERIES

THEORY and PROBLEMS

of

STRENGTH OF MATERIALS

by WILLIAM A. NASH

including

430 solved problems

Completely Solved in Detail

SCHAUM PUBLISHING CO.

NEW YORK

SCHAUM'S OUTLINE OF

THEORY AND PROBLEMS

of

STRENGTH OF MATERIALS

•

by

WILLIAM A. NASH, PH.D.

Professor of Engineering Mechanics

University of Florida

•

SCHAUM PUBLISHING CO.

611 BROADWAY, NEW YORK 12

Preface

Although some of the fundamentals of the statics of rigid bodies were known to the scientists of ancient Greece, no serious thought was given to the problem of the deformations of even the simplest structures until the time of the Renaissance. Then Leonardo da Vinci (1452-1519) and later Galileo (1564-1642) became interested in the statics of deformable bodies and in the mechanical properties of common engineering materials. Galileo's book "Two New Sciences" presented the first written discussion of the properties of structural materials and also the first treatment of the strength of beams. Although some of Galileo's conclusions do not agree with modern ideas, his early work stimulated considerable interest in this new field. In 1678 Robert Hooke (1635-1702) formulated his very famous and exceedingly simple relationship between force and deformation which was perhaps more influential than any other single factor in the development of the theory of Strength of Materials. Hooke's law of proportionality between deformation and force so greatly simplified mathematical analysis that thereafter progress in this field was quite rapid. Jacob Bernoulli (1654-1705) determined the differential equation of a laterally loaded bar, and later Leonard Euler (1707-1783) continued the study of bending action of beams and also investigated the buckling of an axially compressed bar. The first comprehensive discussion of the fiber stresses in a laterally loaded beam was presented in 1776 by Coulomb (1736-1806) and later this same author laid the foundations of the theory for the torsion of bars. Navier (1785-1836) further clarified the problem of bending of beams, and it might perhaps be said that Coulomb and Navier are largely responsible for the presentation of the material that today we call Strength of Materials.

Chronologically, the development of the science of Strength of Materials followed largely after the development of the laws of statics. Statics considered the external effects of a force acting on a body, i.e. the tendency of the forces to change the state of motion of the body. Strength of Materials treats the internal effects of the force, i.e. the state of deformation and stress set up within the boundaries of the body. Briefly, the science of Strength of Materials provides a more comprehensive explanation of the behavior of solids under load than the student has considered previously. Even so, there are many important problems that are beyond the scope of an undergraduate course on this topic and they are reserved for more sophisticated treatments offered in graduate courses under the names of Theory of Elasticity, Theory of Elastic Stability, Theory of Plasticity, Photoelasticity, Theory of a Continuous Media, and a host of other titles. The subject matter of many of these graduate courses is prerequisite to carrying out an ever-increasing number of intricate design problems for industry and is even more essential in research considerations.

This book is designed to supplement standard texts, primarily to assist students in acquiring a more thorough knowledge and proficiency in this basic field. The contents are divided into chapters covering duly-recognized areas of theory and study. Each chapter begins with a summary of the pertinent definitions, principles and theorems, followed by graded sets of solved and supplementary problems. Derivations of formulas and proofs of theorems are included among the solved problems. The problems have been chosen and solutions arranged so that the principles are clearly established. They serve to illustrate and amplify the theory, provide the repetition of basic principles so vital to effective teaching, and bring into sharp focus those fine points without which the student continually feels himself on unsafe ground.

The author is deeply indebted to his wife, Verna B. Nash, for her inspiration and continued assistance in proofreading and in the preparation of the manuscript. He is also indebted to Mr. Roy W. Gregory for painstaking work in the preparation of all drawings and to Mr. Henry Hayden for technical assistance and typographical arrangement. Particular thanks are extended to Prof. Odd Albert of the Polytechnic Institute of Brooklyn for many valuable suggestions and critical review of the entire manuscript.

WILLIAM A. NASH

Gainesville, Florida
September, 1957

Contents

此为试读，需要完整PDF请访问： www.ertongbook.com

CHAPTER 1

Tension and Compression

INTERNAL EFFECTS OF FORCES

In this book we shall be concerned primarily with what might be called the *internal* effects of forces acting on a body. The bodies themselves will no longer be considered to be perfectly rigid as was assumed in statics; instead, the calculation of the deformations of various shape bodies under a variety of loads will be one of the primary concerns of this study of strength of materials.

AXIALLY LOADED BAR. Perhaps the simplest case to consider at the start will be that of an initially straight metal bar of constant cross-section loaded at its ends by a pair of oppositely directed collinear forces coinciding with the longitudinal axis of the bar and acting through the centroid of each cross-section. For static equilibrium the magnitudes of the forces must be equal. If the forces are directed away from the bar, the bar is said to be in *tension*, if they are directed toward the bar, a state of *compression* exists. These two conditions are illustrated in Figure 1. Under the action of this pair of applied forces, internal resisting forces are set up within the bar and their characteristics may be studied by imagining a plane to be passed through the bar anywhere along its length and oriented perpendicular to the longitudinal axis of the bar. Such a plane is designated as *a-a* in Figure 2*a*. For reasons to be discussed later, this plane should not be "too close" to either end of the bar. If for purposes of

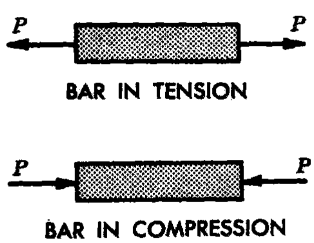


Fig. 1

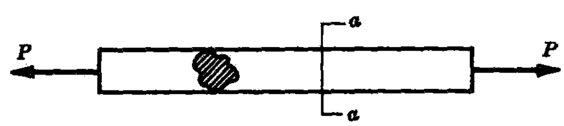


Fig. 2*a*

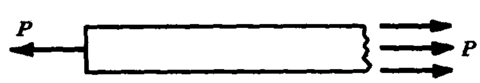


Fig. 2*b*

analysis, the portion of the bar to the right of this plane is considered to be removed, as in Figure 2*b*, then it must be replaced by whatever effect it exerts upon the left portion. By this technique of introducing a cutting plane, the originally internal forces now become external with respect to the remaining portion of the body. For equilibrium of the portion to the left this "effect" must be a horizontal force of magnitude P. However, this force P acting normal to the cross-section *a-a* is actually the resultant of distributed forces acting over this cross-section in a direction normal to it.

DISTRIBUTION OF RESISTING FORCES. At this point it is necessary to make some assumption regarding the manner of variation of these distributed forces, and since the applied force *P* acts through the centroid it is commonly assumed that they are uniform across the cross-section. Such a distribution is probably never realized exactly because of the random orientation of the crystalline grains of which the bar is composed. The exact value of the force acting on some very small element of area of the cross-section is a function of the nature and orientation of the crystalline structure at that point. However, over the entire cross-section the variation is described with reasonable engineering accuracy by the assumption of a uniform distribution.

NORMAL STRESS. Instead of speaking of the internal force acting on some small element of area, it is perhaps of more significance and better for comparative purposes, to treat the normal force acting over a *unit* area of the cross-section. The intensity of normal force per unit area is termed the normal *stress* and is expressed in units of force per unit area, e.g. lb/in^2. The phrase *total stress* is sometimes used to denote the resultant axial force in pounds. If the forces applied to the ends of the bar are such that the bar is in tension, then *tensile stresses* are set up in the bar; if the bar is in compression we have *compressive stresses*. It is essential that the line of action of the applied end forces pass through the centroid of each cross-section of the bar.

TEST SPECIMENS. The axial loading shown in Figure 2*a* occurs frequently in structural and machine design problems. To simulate this loading in the laboratory, a test specimen is held in the grips of either an electrically driven gear type testing machine or a hydraulic machine. Both of these machines are commonly used in materials testing laboratories for applying axial tension.

In an effort to standardize materials testing techniques the American Society for Testing Materials, commonly abbreviated A.S.T.M., has issued specifications that are in common use throughout this country. More than a score of different type specimens are prescribed for various metallic and non-metallic materials for both axial tension and axial compression tests. For the present only two of these will be mentioned here, one for metal plates thicker than 3/16 in. and appearing as in Figure 3, the other for metals over 1.5 in. thick and having the appearance shown in Figure 4. The dimensions shown are those specified by the A.S.T.M. but the ends of the test specimens may be of any shape to fit the grips of the testing machine applying the axial load. As may be seen from these figures, the central portion of the specimen is somewhat smaller than the end regions so that failure will not take place in the gripped portion. The rounded fillets shown are provided so that no so-called stress concentrations will arise at the transition between the two lateral dimensions. Ordinarily, a standard gage length in which elongations are measured is marked by punching two very small holes into the surface of the bar either 2 in. or 8 in. apart as shown.

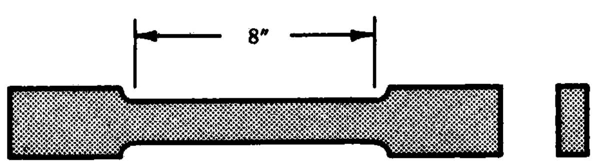


Fig. 3

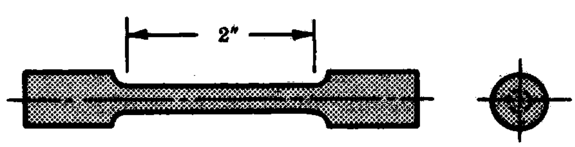


Fig. 4

NORMAL STRAIN. Let us suppose that one of these tension specimens has been placed in a tension-compression testing machine and tensile forces gradually applied to the ends. The total elongation over the gage length may be measured at any predetermined increments of the axial load by a mechanical strain gage and from these values the elongation per unit length, which is termed *normal strain* and denoted by ϵ, may be found merely by dividing the total elongation Δ by the gage length L, i.e. $\epsilon = \Delta/L$. The strain is usually expressed in units of inches per inch and consequently is dimensionless. The phrase *total strain* is sometimes used to denote the elongation in inches.

STRESS-STRAIN CURVE. As the axial load is gradually increased in increments the total elongation over the gage length is measured at each increment of load and this is continued until fracture of the specimen takes place. Knowing the original cross-sectional area of the test specimen the *normal stress*, denoted by s, may be obtained for any value of the axial load merely by the use of the relation

$$s = \frac{P}{A}$$

where P denotes the axial load in pounds, and A the original cross-sectional area. Having numerous pairs of values of normal stress s and normal strain ϵ, the experimental data obtained may be plotted with these quantities considered as ordinate and abscissa respectively. This is a *stress-strain diagram* of the material for this type of loading. Such a diagram may take many widely different forms, but shown in Figure 5 are several typical plots for common engineering materials. For a metal such as low-carbon structural steel the data will plot approximately as shown in Figure 5*a*, for a so-called brittle material such as cast iron the plot appears as in Figure 5*b*, while for rubber the diagram of 5c is typical.

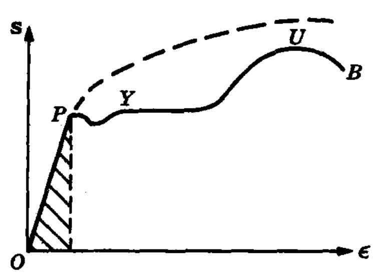


Fig. 5*a*

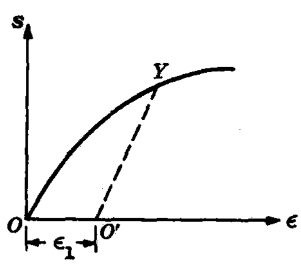


Fig. 5*b*

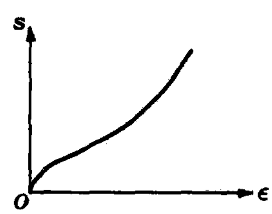


Fig. 5*c*

DUCTILE AND BRITTLE MATERIALS. Metallic engineering materials are commonly classed as either *ductile* or *brittle* materials. A *ductile material* is one having a relatively large tensile strain up to the point of rupture (for example, structural steel or aluminum) whereas a *brittle material* has a relatively small strain up to this same point. An arbitrary strain of 0.05 in/in is frequently taken as the dividing line between these two classes of materials. Cast iron and concrete are examples of brittle materials.

HOOKE'S LAW. For a material whose stress-strain curve is similar to that shown in Figure 5*a*, it is evident that the relation between stress and strain is linear for comparatively small values of strain. This linear relation between elongation and the axial force causing it (since each of these quantities differs only by a constant from the strain or the stress) was first noticed by Sir Robert Hooke in 1678 and bears the name of *Hooke's Law*. To describe this initial linear range of action of the material we may consequently write

$$s = E\epsilon$$

where E denotes the slope of the straight-line portion (OP) of the stress-strain curve in Figure 5*a*.

MODULUS OF ELASTICITY. The quantity E, *i.e.* the ratio of the unit stress to the unit strain, is the *modulus of elasticity* of the material in tension, or, as it is often called, *Young's Modulus*. Values of E for various engineering materials are tabulated in handbooks. Since the unit strain ϵ is a pure number (being a ratio of two lengths) it is evident that E has the same units as does the stress, for example lb/in^2. For many common engineering materials the modulus of elasticity in compression is very nearly equal to that found in tension. It is to be carefully noted that the behavior of materials under load as discussed in this book is restricted (unless otherwise stated) to this linear region of the stress-strain curve.

MECHANICAL PROPERTIES OF MATERIALS

The stress-strain curve shown in Figure 5*a* may be used to characterize several strength characteristics of the material. They are:

PROPORTIONAL LIMIT. The ordinate to the point P is known as the *proportional limit*, i.e. the maximum stress that may be developed during a simple tension test such that the stress is a linear function of strain. For a material having the stress-strain curve shown in Figure 5*b* there is no proportional limit.

ELASTIC LIMIT. The ordinate to a point almost coincident with P is known as the *elastic limit*, i.e., the maximum stress that may be developed during a simple tension test such that there is no permanent or residual deformation when the load is entirely removed. For many materials the numerical values of the elastic limit and the proportional limit are almost identical and the terms are sometimes used synonymously. In those cases where the distinction between the two values is evident the elastic limit is almost always greater than the proportional limit.

ELASTIC RANGE. That region of the stress-strain curve extending from the origin to the proportional limit.

PLASTIC RANGE. That region of the stress-strain curve extending from the proportional limit to the point of rupture.

YIELD POINT. The ordinate to the point Y at which there is an increase in strain with no increase in stress is known as the *yield point* of the material. After loading has progressed to the point Y, yielding is said to take place. Some materials exhibit two points on the stress-strain curve at which there is an increase of strain without an increase of stress. These are called *upper* and *lower yield points*.

ULTIMATE STRENGTH OR TENSILE STRENGTH. The ordinate to the point U, the maximum ordinate to the curve, is known either as the *ultimate strength* or the *tensile strength* of the material.

BREAKING STRENGTH. The ordinate to the point B is called the *breaking strength* of the material.

MODULUS OF RESILIENCE. The work done on a unit volume of material, as a simple tensile force is gradually increased from zero to such a value that the proportional limit of the material is reached, is defined as the *modulus of resilience*. This may be calculated as the area under the stress-strain curve from the origin up to the proportional limit and is represented as the shaded area in Figure 5*a*. The units of this quantity are in-lb/in^3. Thus, resilience of a material is its ability to absorb energy in the elastic range.

MODULUS OF TOUGHNESS. The work done on a unit volume of material as a simple tensile force is gradually increased from zero to the value causing rupture is defined as the *modulus of toughness*. This may be calculated as the entire area under the stress-strain curve from the origin to rupture. Toughness of a material is its ability to absorb energy in the plastic range of the material.

PERCENTAGE REDUCTION IN AREA. The ratio of the decrease in cross-sectional area from the original area upon fracture divided by the *original* area and multiplied by 100 is termed *percentage reduction in area*. It is to be noted that when tensile forces act upon a bar, the cross-sectional area decreases but calculations for the normal stress are usually made upon the basis of the original area. This is the case for the curve shown in Figure 5*a*. As the strains become increasingly larger it is more important to consider the instantaneous values of the cross-sectional area (which are decreasing), and if this is done the *true* stress-strain curve is obtained. Such a curve has the appearance shown by the dotted line in Figure 5*a*.

PERCENTAGE ELONGATION. The ratio of the increase in length (of the gage length) after fracture divided by the initial length, multiplied by 100 is the *percentage elongation*. Both the percentage reduction in area and the percentage elongation are considered to be measures of the *ductility* of a material.

WORKING STRESS. The above-mentioned strength characteristics may be used to select a so-called *working stress*. Throughout this book all working stresses will be within the elastic range of the material. Frequently such a stress is determined merely by dividing either the stress at yield or the ultimate stress by a number termed the *safety factor*. Selection of the safety factor is based upon the designer's judgment and experience. Specific safety factors are sometimes specified in building codes. See Problems 4, 12, 13.

The non-linear stress-strain curve of a brittle material, shown in Figure 5*b*, characterizes several other strength measures that cannot be introduced if the stress-strain curve has a linear region. They are:

YIELD STRENGTH. The ordinate to the stress-strain curve such that the material has a predetermined permanent deformation or "set" when the load is removed is called the *yield strength* of the material. The permanent set is often taken to be either 0.002 or 0.0035 in. per in. These values are of course arbitrary. In Figure 5*b* a set ϵ_1 is denoted on the strain axis and the line $O'Y$ is drawn parallel to the initial tangent to the curve. The ordinate to Y represents the yield strength of the material, sometimes called the *proof stress*.

TANGENT MODULUS. The slope of the tangent to the stress-strain curve at the origin of the plot is known as the *tangent modulus* of the material.

There are other characteristics of a material that are useful in design considerations. They are:

COEFFICIENT OF LINEAR EXPANSION. This is defined as the change of length per unit length of a straight bar subject to a temperature change of one degree. The value of this coefficient is independent of the unit of length but does depend upon the temperature scale used. Usually we will consider the Fahrenheit scale, in which case the coefficient denoted by α is given for steel, for instance, as 6.5×10^{-6} per F°. Temperature changes in a structure give rise to internal stresses just as do applied loads. See Problems 5 and 8.

POISSON'S RATIO. When a bar is subject to a simple tensile loading there is an increase in length of the bar in the direction of the load, but a decrease in the lateral dimensions perpendicular to the load. The ratio of the strain in the lateral direction to that in the axial direction is defined as *Poisson's Ratio*. It is denoted in this book by the Greek letter μ. For most metals it lies in the range 0.25 to 0.35. See Problems 16, 17, 18, 19, 20,

GENERAL FORM OF HOOKE'S LAW. The simple form of Hooke's Law has been given for axial tension when the loading is entirely along one straight line, i.e. uni-axial. Only the deformation in the direction of the load was considered and it was given by

$$\epsilon = \frac{s}{E}$$

In the more general case an element of material is subject to three mutually perpendicular normal stresses s_x, s_y, s_z, which are accompanied by the strains ϵ_x, ϵ_y, ϵ_z respectively. By superposing the strain components arising from lateral contraction due to Poisson's effect upon the direct strains we obtain the general statement of Hooke's Law:

$$\epsilon_x = \frac{1}{E}[s_x - \mu(s_y + s_z)]$$

$$\epsilon_y = \frac{1}{E}[s_y - \mu(s_x + s_z)]$$

$$\epsilon_z = \frac{1}{E}[s_z - \mu(s_x + s_y)]$$

See Problems 17 and 20.

CLASSIFICATION OF MATERIALS

This entire discussion has been based upon the assumptions that two characteristics prevail in the material. They are that we have a:

HOMOGENEOUS MATERIAL, one having the same elastic properties (E, μ) at all points in the body, and an

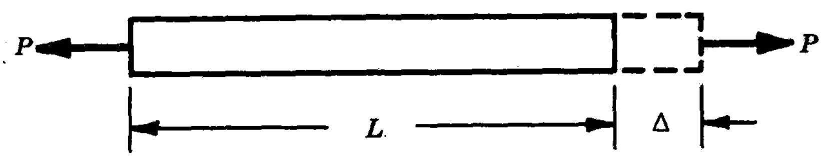
ISOTROPIC MATERIAL, one having the same elastic properties in all directions at any one point of the body. Not all materials are isotropic. If a material does not possess any kind of elastic symmetry it is called *anisotropic*, or sometimes *aeolotropic*. Instead of having two independent elastic constants (E, μ) as an isotropic

此为试读,需要完整PDF请访问: www.ertongbook.com

material does, such a substance has 21 elastic constants. If the material has three mutually perpendicular planes of elastic symmetry it is said to be *orthotropic*. The number of independent constants is 9 in this case. This book considers only the analysis of isotropic materials.

SOLVED PROBLEMS

1. Determine the total elongation of an initially straight bar of length L, cross-sectional area A, and modulus of elasticity E if a tensile load P acts on the ends of the bar.



The unit stress in the direction of the force P is merely the load divided by the cross-sectional area, i.e., $s = P/A$. Also the unit strain ϵ is given by the total elongation Δ divided by the original length, i.e. $\epsilon = \Delta/L$. By definition the modulus of elasticity E is the ratio of s to ϵ, i.e.,

$$E = \frac{s}{\epsilon} = \frac{P/A}{\Delta/L} = \frac{PL}{A\Delta} \qquad \text{or} \qquad \Delta = \frac{PL}{AE}$$

Note that Δ has the units of length, perhaps in. or ft.

2. A surveyors' steel tape 100 ft long has a cross-section of 0.250 in. by 0.03 in. Determine the elongation when the entire tape is stretched and held taut by a force of 12 lb. The modulus of elasticity is $30 \cdot 10^6$ lb/in^2.

$$\text{Elongation } \Delta = \frac{PL}{AE} = \frac{(12)(100 \cdot 12)}{(0.250)(0.03)(30 \cdot 10^6)} = 0.0640 \text{ in.}$$

3. A steel bar of cross-section 1 in^2 is acted upon by the forces shown in Figure (*a*). Determine the total elongation of the bar. For steel, $E = 30 \cdot 10^6$ lb/in^2.

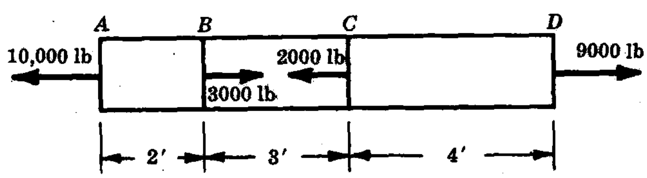


Fig. (*a*)

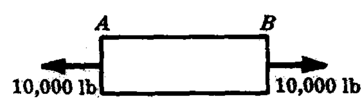


Fig. (*b*)

The entire bar is in equilibrium, hence all portions of it are also. The portion of the bar between A and B has a resultant force of 10,000 lb acting over every cross-section, hence a free-body diagram of this 2 ft length appears as in Figure (*b*) above. The force at the right end of this segment must be 10,000 lb to maintain equilibrium with the applied force at the left end. The elongation of this portion is given by

$$\Delta_1 = \frac{PL}{AE} = \frac{10,000(24)}{(1)(30 \cdot 10^6)} = 0.0080 \text{ in.}$$

The force acting in the segment between B and C is found by considering the algebraic sum of the forces to the left of a section between B and C. This indicates that a resultant force of 7000 lb acts to the left, i.e. the section has a tensile force acting upon it. This same result could of course have been obtained by considering the algebraic sum of the forces to the right of this section. Consequently the free-body diagram of the segment BC appears as in Figure (*c*) below.

The elongation of this portion is given by $\Delta_2 = \dfrac{7000(36)}{(1)(30\cdot10^6)} = 0.0084$ in.

Similarly, the force acting over any cross-section between C and D must be 9000 lb to maintain equilibrium with the applied load at D. The free-body diagram of the segment CD appears as in Fig. (d).

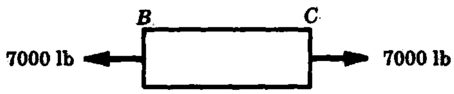


Fig. (c)

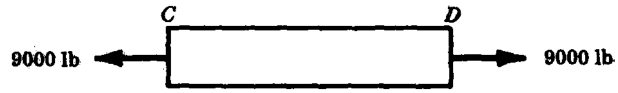


Fig. (d)

The elongation of this portion is given by $\Delta_3 = \dfrac{(9000)(48)}{(1)(30\cdot10^6)} = 0.0144$ in.

The total elongation is consequently $\Delta = 0.0080 + 0.0084 + 0.0144 = 0.0308$ in.

4. The Howe truss shown in Fig. (a) supports the single load of 120,000 lb. If the working stress of the material in tension is taken to be 20,000 lb/in^2, determine the required cross-sectional area of bars DE and AC. Find the elongation of bar DE over its 20 ft length. Assume that the limiting value of the working stress in tension is the only factor to be considered in determining the required area. Take the modulus of elasticity of the bar to be $30\cdot10^6$ lb/in^2.

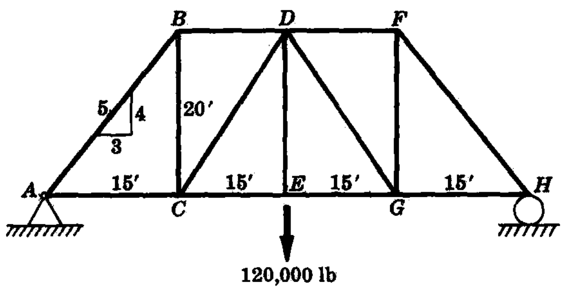


Fig. (a)

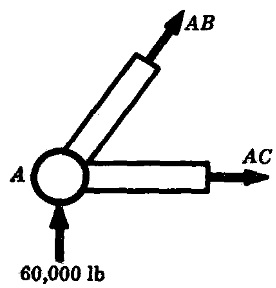


Fig. (b)

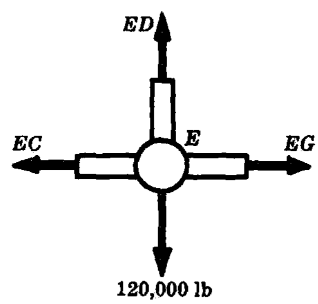


Fig. (c)

This truss is statically determinate both externally and internally, i.e., the reactions at the supports may be determined by the equations of static equilibrium and also the axial force in each bar may be found by a simple statics analysis.

It is first necessary to determine the vertical reactions at A and H. By symmetry these are each 60,000 lb. A free-body diagram of the joint at A appears as in Fig. (b). In Fig. (b) the unknown forces in the bars have been denoted as AB and AC, the same designations as the bars themselves, and they have been assumed to be tensile forces. In this manner if they are found to be positive they actually indicate tension. If they are found to be negative they indicate compression, and the signs thus obtained are in agreement with the usual sign convention designating tensile forces as positive and compressive forces as negative. Applying the equations of static equilibrium to the above free-body diagram we have

$$\Sigma F_v = 60{,}000 + \frac{4}{5}(AB) = 0 \qquad \text{or} \qquad AB = -75{,}000 \text{ lb}$$

$$\Sigma F_h = \frac{3}{5}(-75{,}000) + AC = 0 \qquad \text{or} \qquad AC = 45{,}000 \text{ lb}$$

Likewise, a free-body diagram of the point at E appears as in Fig. (c) above. From statics,

$$\Sigma F_v = ED - 120{,}000 = 0 \qquad \text{or} \qquad ED = 120{,}000 \text{ lb}$$

The simple consideration of trusses used here assumes all bars are so-called two-force members, i.e., subject to either axial tension or compression and no other loadings.

For axial loading the stress is given by $s = P/A$, where P is the axial force and A the cross-sectional area of the bar. Here, the stress is given as 20,000 lb/in^2 in each bar and the areas are thus given by

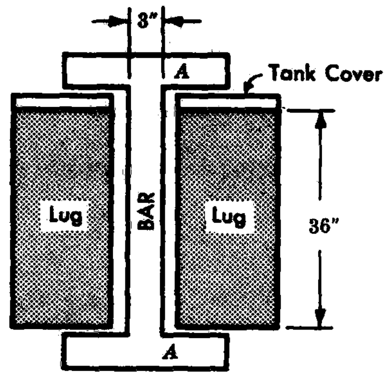
$$A_{DE} = \frac{120,000}{20,000} = 6\ in^2 \quad \text{and} \quad A_{AC} = \frac{45,000}{20,000} = 2.25\ in^2$$

The elongation of a bar under axial tension is given by $\Delta = \dfrac{PL}{AE}$. For bar DE we have

$$\Delta = \frac{(120,000)(240)}{(6)(30\cdot10^6)} = 0.160 \text{ in.}$$

5. A series of prismatic bars of rectangular cross-section 2 × 3 in. is used as a clamping device to secure the top on a cylindrical tank containing fluid under pressure. The outside wall of the pressure tank has projecting lugs welded to it and the prismatic bars fit loosely (in the lateral direction) between adjacent lugs. To secure the clamping effect the bar is purposely machined so that it is too short for its flanges (A) to fit over the tank cover resting on top of the lugs. At room temperature, it fails to clear by 0.10 in. The bar (but not the lugs) is then heated so that it can be slipped over the tank top. After it cools it then exerts a force normal to the tank top.

If the total bearing area at one end of the bar (area in contact with the tank top) is 6 in^2, find the unit pressure each bar exerts on the tank top. Also, find the temperature to which the bar must be heated in order that it just clears the top of the tank cover. The bar is steel, for which $\alpha = 6.5\cdot10^{-6}/°F$.



$$0.10 = (6.5\cdot10^{-6})(36)(\Delta T) \quad \text{from which} \quad \Delta T = 426°\ F.$$

The axial force necessary to stretch the bar this same amount is P where

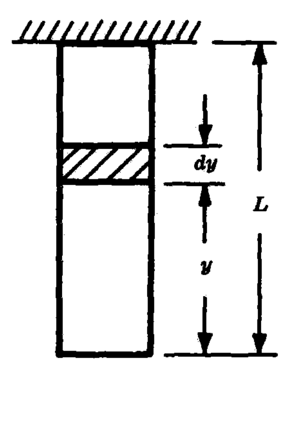
$$0.10 = \frac{P(36)}{(6)(30\cdot10^6)} \quad \text{and} \quad P = 500,000 \text{ lb}$$

The pressure is assumed to be uniformly distributed over the bearing area between the flange and the tank top. Consequently, the pressure is

$$\frac{500,000}{6} = 83,300\ lb/in^2$$

6. Determine the total increase of length of a bar of constant cross-section hanging vertically and subject to it own weight as the only load. The bar is initally straight.

The normal stress (tensile) over any horizontal cross-section is caused by the weight of the material below that section. The elongation of the element of thickness dy shown is



$$d\Delta = \frac{(Ay\gamma)\,dy}{AE}$$

where A denotes the cross-sectional area of the bar and γ its specific weight (weight/unit volume). Integrating, the total elongation of the bar is

$$\Delta = \int_0^L \frac{Ay\gamma\,dy}{AE} = \frac{A\gamma}{AE}\cdot\frac{L^2}{2} = \frac{(A\gamma L)L}{2AE} = \frac{WL}{2AE}$$

where W denotes the total weight of the bar. It is to be noted that the total elongation produced by the weight of the bar is equal to that produced by a load of half its weight applied at the end.

7. A steel wire 1/4 in. in diameter is used for hoisting purposes in building construction. If 500 ft of the wire are hanging vertically, and a load of 300 lb is being lifted at the lower end of the wire, determine the total elongation of the wire. The specific weight of the steel is 0.283 lb/in^3 and $E = 30\cdot10^6$ lb/in^2.

The total elongation is caused partially by the applied force of 300 lb and partially by the weight of the wire. The elongation due to the 300 lb load is

$$\Delta_1 = \frac{PL}{AE} = \frac{(300)(500\cdot12)}{\frac{\pi}{4}(\frac{1}{4})^2(30\cdot10^6)} = 1.27 \text{ in.}$$

From Problem 6 the elongation due to the weight of the wire is

$$\Delta_2 = \frac{WL}{2AE} = \frac{(\frac{\pi}{4})(\frac{1}{4})^2(500\cdot12)(0.283)(500\cdot12)}{2(\frac{\pi}{4})(\frac{1}{4})^2(30\cdot10^6)} = 0.170 \text{ in.}$$

Consequently, the total elongation is $\Delta = 1.27 + 0.17 = 1.44$ in.

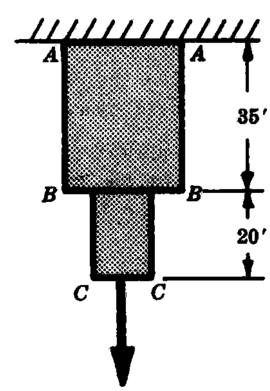
8. A straight aluminum wire 100 ft long is subject to a tensile stress of 10,000 lb/in^2. Determine the total elongation of the wire. What temperature change would produce this same elongation? Take $E = 10\cdot10^6$ lb/in^2 and α (the coefficient of linear expansion) = $12.8\cdot10^{-6}$/F°.

The total elongation is given by $\Delta = \dfrac{PL}{AE} = \dfrac{(10,000)(100\cdot12)}{10\cdot10^6} = 1.20$ in.

A rise in temperature of ΔT would cause this same expansion. Then

$$1.20 = (12.8\cdot10^{-6})(100\cdot12)(\Delta T) \quad \text{and} \quad \Delta T = 78.2^\circ \text{ F.}$$

9. Two prismatic bars are rigidly fastened together and support a vertical load of 10,000 lb as shown. The upper bar is steel having a specific weight of 0.283 lb/in^3, a length of 35 ft, and a cross-sectional area of 10 in^2. The lower bar is brass having a specific weight of 0.300 lb/in^3, a length of 20 ft and a cross-sectional area of 8 in^2. For steel $E = 30\cdot10^6$ lb/in^2, for brass $E = 13\cdot10^6$ lb/in^2. Determine the maximum stress in each material.



The maximum stress in the brass bar occurs just below the junction at section B-B. There, the vertical normal stress is caused by the combined effect of the load of 10,000 lb together with the weight of the entire brass bar below B-B.

The weight of the brass bar is

$$W_b = (20\cdot12)(8)(0.300) = 576 \text{ lb.}$$

The stress at this section is

$$s = \frac{P}{A} = \frac{10,000 + 576}{8} = 1320 \text{ lb/in}^2.$$

The maximum stress in the steel bar occurs at section A-A, the point of suspension, because there the entire weight of the steel and brass bars gives rise to normal stress, whereas at any lower section only a portion of the weight of the steel would be effective in causing stress.

The weight of the steel bar is

$$W_s = (35\cdot12)(10)(0.283) = 1185 \text{ lb.}$$

The stress across section A-A is $s = \dfrac{P}{A} = \dfrac{10,000 + 576 + 1185}{10} = 1180$ lb/in^2.

10. A solid truncated conical bar of circular cross-section tapers uniformly from a diameter d at its small end to D at the large end. The length of the bar is L. Determine the elongation due to an axial force P applied at each end. See Fig. (a)

The coordinate x describes the distance of a disc-like element of thickness dx from the small end. The radius of this small element is readily found by similar triangles:

$$r = \frac{d}{2} + \frac{x}{L}\left(\frac{D-d}{2}\right)$$

The elongation of this disc-like element may be found by applying the formula for extension due to axial loading, $\Delta = PL/AE$. For the element, this expression becomes

$$d\Delta = \frac{P\,dx}{\pi\left[\frac{d}{2} + \frac{x}{L}\left(\frac{D-d}{2}\right)\right]^2 E}$$

The extension of the entire bar is obtained by summing the elongations of all such elements over the bar. This is of course done by integrating. If Δ denotes the elongation of the entire bar,

$$\Delta = \int_0^L d\Delta = \int_0^L \frac{4P\,dx}{\pi\left[d + \frac{x}{L}(D-d)\right]^2 E} = \frac{4PL}{\pi D d E}$$

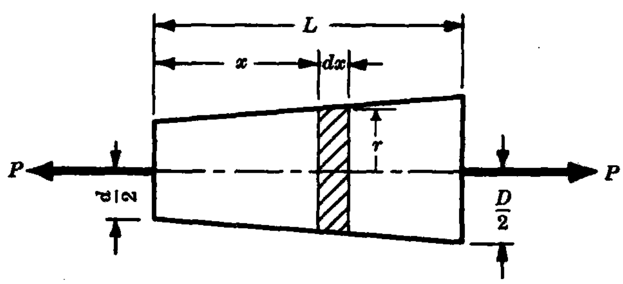


Fig. (a) Prob. 10

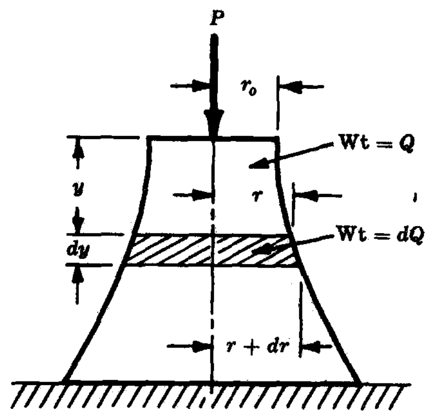


Fig. (b) Prob. 11

11. A body having the form of a solid of revolution supports a load P as shown in Fig. (b). The radius of the upper base of the body is r_0 and the specific weight of the material is γ lb/ft^3. Determine how the radius should vary with the altitude in order that the compressive stress at all cross-sections should be constant. The weight of the solid is not negligible.

Let y be measured from the upper base as shown and let Q denote the weight of that portion of the body of altitude y. Then dQ represents the increment to Q in the increment of altitude dy. Let r and $(r+dr)$ denote the radii of the upper and lower surfaces respectively of this horizontal element and A and $(A+dA)$ the corresponding areas. Considering the normal compressive stresses acting over both surfaces of this element, we have

$$\frac{P+Q}{A} = \frac{P+Q+dQ}{A+dA} = s = \text{constant}$$

from which (1) $$\frac{dA}{dQ} = \frac{A}{P+Q} = \frac{1}{s}$$

The increment of area between the upper and lower faces of the element is

$$dA = \pi(r+dr)^2 - \pi r^2 = 2\pi r\,dr$$

The increment of weight is $dQ = \pi r^2 \gamma (dy)$.

Consequently from (1), $\frac{2\pi r(dr)}{\pi r^2 \gamma(dy)} = \frac{1}{s}$. Integrating, $2 \log r = (\frac{\gamma}{s})y + C_1$.

Applying the boundary condition that $r = r_0$ when $y = 0$, we find $C_1 = 2 \log r_0$.

Also from the conditions at the upper base, $s = \frac{P}{\pi r_0^2}$. Finally, $r = r_0 e^{(\frac{\gamma \pi r_0^2 y}{2P})}$

12. Two identical steel bars are pin connected and support a load of 100,000 lb as shown in Fig. (a). Find the required cross-sectional area of the bars so that the normal stress in them is no greater than 30,000 lb/in². Also, find the vertical displacement of the point B. Take $E = 30 \cdot 10^6$ lb/in².

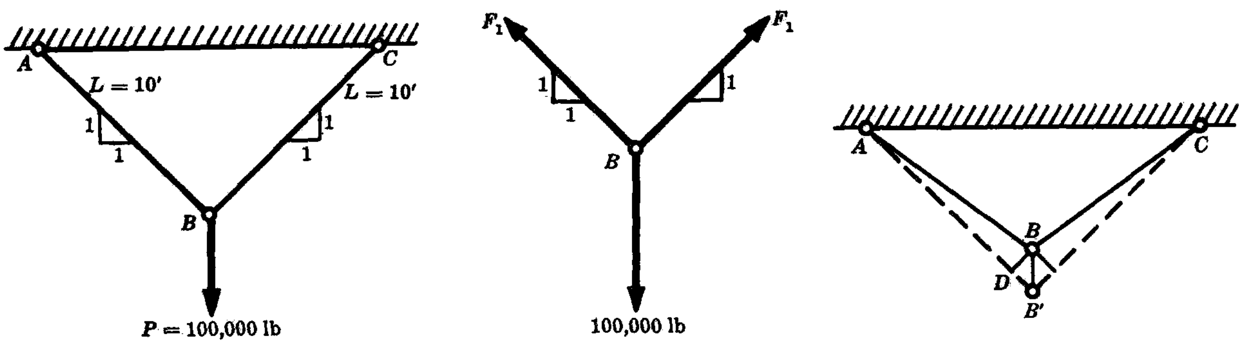


Fig. (a) Fig. (b) Fig. (c)

A free-body diagram of the pin at B is shown in Fig. (b), where F_1 represents the force (lb) in each bar.

From statics, $\Sigma F_v = 2(\frac{1}{\sqrt{2}})F_1 - 100,000 = 0$ or $F_1 = 70,700$ lb.

Hence the required area is $A = \frac{70,700}{30,000} = 2.35$ in².

Because our study of strength of materials is restricted to the case of *small* deformations, the basic geometry of the structure is essentially unchanged. Thus, we can denote the position of the deformed bars by the dotted lines shown in Fig. (c), and the angle $DB'B$ is very nearly 45°. The elongation of the left bar is represented by DB' and is found from the expression for axial extension (Problem 1) to be

$$DB' = \frac{(70,700)(120)}{(2.35)(30 \cdot 10^6)} = 0.120 \text{ in.} \qquad \text{Consequently,} \qquad BB' = \frac{0.120}{\cos 45^\circ} = 0.170 \text{ in.}$$

13. The two steel bars AB and BC are pinned at each end and support the load of 60,000 lb shown in Fig. (a) below. The metal is annealed cast steel, having a yield point of 60,000 lb/in². Safety factors of 2 for tensile members and 3.5 for compressive members are adequate. Determine the required cross-sectional areas of these bars and also the horizontal and vertical components of displacement of point B. Take $E = 30 \cdot 10^6$ lb/in².

A free-body diagram of the joint at B appears as in Fig. (b) below if the unknown forces are assumed to be tensile.

From statics: $\Sigma F_v = -60,000 - BC \sin 30^\circ = 0$ or $BC = -120,000$ lb

$\Sigma F_h = -BA - BC \cos 30^\circ = 0$ or $BA = 104,000$ lb

此为试读,需要完整PDF请访问: www.ertongbook.com