Analysis and Design of Sliding Mode Control Systems

（滑模控制系统的分析与设计）

Liu Leipo

Science Press
Beijing

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Abstract

This book mainly introduces some new design methods and ideas about sliding mode control, which are applied to the differential inclusion systems, nonlinear systems, discrete systems, chaotic systems and the delta operator systems, etc. In particular, the effect of input nonlinearity is fully considered in analyzing and implementing a sliding mode control scheme. Incorporating some control algorithms, such as H_∞ control, passive control, adaptive control and generalized H_2 control, etc, into sliding mode control extends the application range.

This book is suitable for professional researchers in the fields of control science and engineering, industrial automation, electrical automation and mechanical engineering, but also can be used as a reference material for relevant scientific and technical engineers.

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Preface

Sliding mode control is an effective robust control strategy, which has many attractive features such as robustness to parameter variations and insensitivity to matched disturbances. It is successfully applied to a wide variety of practical engineering systems such as robot manipulators, electrical motors, power systems, aircrafts, underwater vehicles, spacecrafts, flexible space structures, and automotive engines, and so on.

The sliding mode control design generally consists of two stages. Firstly, it is to choose an appropriate sliding surface on which the system has desired properties such as stability, disturbance rejection capability, and tracking ability. That is, the sliding mode dynamics has desired performances. Secondly, a discontinuous control law is designed to force the system state trajectories to the sliding surface in a finite time and remains in it thereafter. Because of the discontinuity of sliding mode controller, high frequency oscillations of the state trajectory known as chattering phenomenon are the major disadvantages to the widespread use of sliding mode control in many practical control systems.

How to reduce chattering is an important and challenging problem. Many meaningful results have been presented to overcome this drawback, such as, boundary layer and reaching law approach. In recent years, I have been working on the research in this field and wanting to write an academic monograph about the current new progress of sliding mode control.

The content of this book mainly originates from my latest research results. Some new sliding surface designs and reaching law approaches are proposed to reduce the chattering. The objective models include differential inclusion systems, nonlinear systems, discrete systems, chaotic systems, delta operator systems and so on. In practice, due to physical limitation, there do exist nonlinearities in the control input, input nonlinearities, such as saturation, quantization, backlash, dead-zones, and so on, naturally originate from actuators in system realization and might cause a serious degradation of the system performance. Thus the effect of input nonlinearity is also taken into account. Moreover, when the slope parameters of input nonlinearity are unmeasured, adaptive sliding mode control algorithm is proposed. Besides these, the book also includes generalized H_2 sliding mode control, H_∞ non-fragile observer-based sliding mode control, non-fragile observer-based sliding mode passive control, sliding mode tracking control and disturbance observer-based sliding mode control, etc. These design schemes significantly enrich the sliding mode control theory.

I am indebted to my teachers, colleagues and students for their help in writing this book. I am grateful to the supports of the National Natural Science Foundation of China (under grant No.U1404610 and 61473115) and Young Academic Leaders Plan of Henan University of Science and Technology.

Because of the limitation of my knowledge and research scope, there might be some mistakes in this book. Criticism and suggestions are welcome.

CONTENTS

1 Introduction: An overview of sliding mode control

1.1 Introduction

Before we introduce sliding mode control, variable structure control should be illustrated. As a kind of design strategy in modern control theory, variable structure control has many features, such as flexibility and complexity. It is insensitive to the system variations if parameter variations satisfy certain matching conditions.

In the last decades, variable structure control has been successfully applied to the practical industrial systems, for example, a rigid spacecraft, underwater ships, mobile robots, electric drives, and many other mechanical systems [1-8]. But so far, there is no common definition. The essential property of variable structure control is that the discontinuous feedback control switches on one or more manifolds in the state space. Thus the structure of the feedback system is altered or switched as the state crosses each discontinuity surface. It is generally believed that the variable structure control strategy is that the system structure will change according to certain rules in order to make the system performance satisfy the requirement of desired performance index, when the state trajectory of system passes through different areas in state space. System structure generally refers to the system model described by a mathematical equation (group).Variable structure control system is a feedback control with different structures to make the system achieve an expected dynamic performance on the basis of certain switching logical changes.

Variable structure control is a comprehensive method in modern control theory, according to the different structures of variable structure control systems, there are mainly three kinds of control strategies: Precluding sliding mode variable structure control, switching supervisory control and sliding mode control.

The main idea about precluding sliding mode variable structure control is that the system state trajectory can reach the stable area in advance from any initial state by switching logic of the controller and system performance satisfies the desired index requirements. Though the controller is to be switched, but it does not produce sliding mode.

Switching supervisory control is also called multiple model adaptive contro1. The control strategy is to make the structure or parameter of the system still maintain good performance under the condition of structure or parameters mutation, using multiple switched system model and combining adaptive control and variable structure control. This kind of the controller can speed up the convergence speed of response.

This book mainly focuses on sliding mode control, although this subject has already been treated in many papers, surveys, or books [9-15]. It remains the object of many studies (theoretical or various related applications). Sliding mode control was first studied intensively in the 1960's by Russian authors, notably Emel'yanov and Utkin, although early work was also done by Fliigge-Lotz in the 1950's. So far, the research and development of sliding mode control have been greatly accelerated in both theory and applications. The study objects have been extended to discrete systems [16-20], time delay systems [21-25], stochastic systems [26-30], large-scale systems [31-35], differential inclusion systems [36-39] and so on.

1.2 The basic concepts of sliding mode control

We start with a motivating example to illustrate the conceptual framework. Consider the following two-dimensional system:

$$\ddot{x} = a\dot{x} + u, \quad a > 0$$

Let $x_1 = x$ and $\dot{x}_1 = x_2$. Then the above equation can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = ax_2 + u \end{cases} \tag{1.1}$$

where x_1 and x_2 are state variables, u is the control input.

We take the controller as

$$u = -\varphi x_1 \tag{1.2}$$

where the parameter φ is chosen as b or $-b$, $b > 0$.

Case 1 If $\varphi = b$, then system (1.1) can be rewritten as

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -bx_1 + ax_2 \end{cases} \tag{1.3}$$

Obviously, the eigenvalues of the system (1.3) are a pair of conjugate complex roots: the real parts are positive. A sketch of phase locus is shown in Fig. 1.1, so the origin of phase plane is an unstable focus.

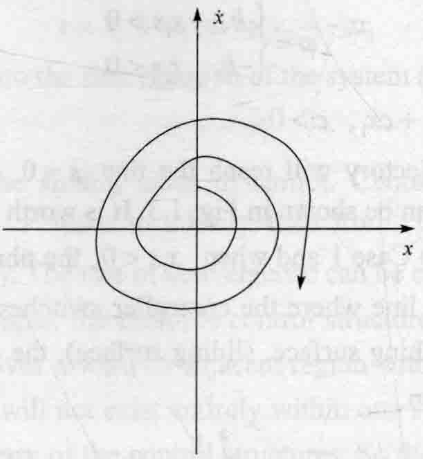

Fig. 1.1 Phase locus ($\varphi = b$)

Case 2 If $\varphi = -b$, then system (1.1) is

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = bx_1 + ax_2 \end{cases} \tag{1.4}$$

The eigenvalues of the system (1.4) are real numbers: one positive and one negative. Similar to Case 1, Fig. 1.2 shows a sketch of phase locus, the origin of phase plane is an unstable saddle point.

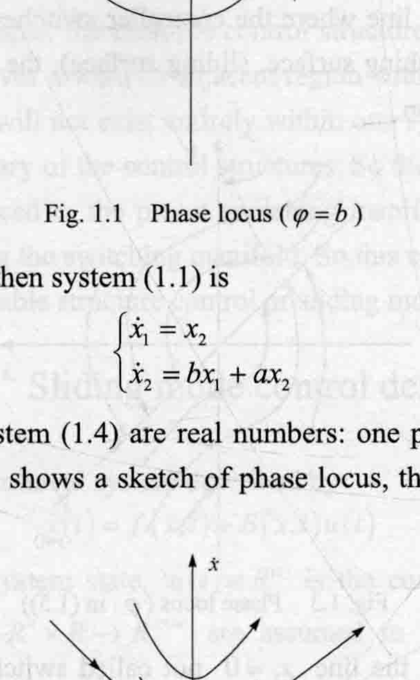

Fig. 1.2 Phase locus ($\varphi = -b$)

From the above discussion, neither of the systems (1.3) and (1.4) is stable, according to these two structures. But if the above two feedback control laws are combined with a certain rule, then we will find a wonderful change of the phase trajectories.

Choose the gain of the controller (1.2) as

$$\varphi = \begin{cases} b, & x_1 s > 0 \\ -b, & x_1 s < 0 \end{cases} \tag{1.5}$$

where the function $s = x_2 + cx_1$, $c > 0$.

Then the system trajectory will reach the line $s = 0$ from the initial point and converge to zero, which can be shown in Fig. 1.3. It is worth noting that when $x_1 s > 0$, the phase locus belongs to Case 1 and when $x_1 s < 0$, the phase locus is in Case 2. The line $s = 0$ is a boundary line where the controller switches, called switching line (or switching manifold, switching surface, sliding surface), the corresponding function s is called switching function.

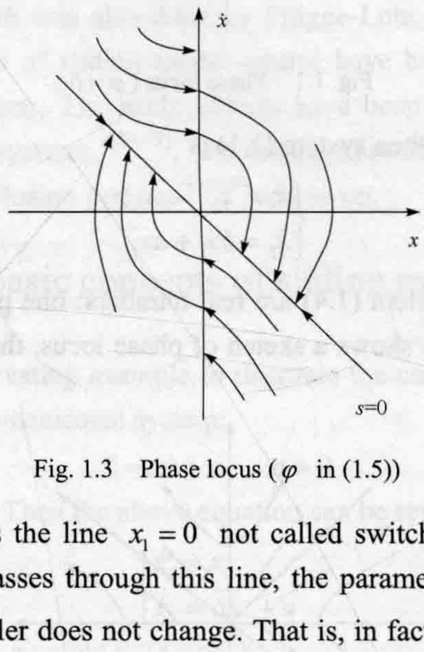

Fig. 1.3 Phase locus (φ in (1.5))

Furthermore, why is the line $x_1 = 0$ not called switching line? This is that because the system state passes through this line, the parameter φ has switching, but the symbol of the controller does not change. That is, in fact, the controller can be described by

$$u = -b|x_1| \operatorname{sgn} s \tag{1.6}$$

where the signum function sgn(·) is defined by

$$\operatorname{sgn}(s) = \begin{cases} 1, & s > 0 \\ 0, & s = 0 \\ -1, & s < 0 \end{cases}$$

From (1.5) and (1.6), the structure of the controller (1.5) is not changed, so the line $x_1 = 0$ is not called switching line.

The motion on the manifold $s = 0$ is called sliding mode. In the sliding mode, we have

$$s = x_2 + cx_1 = 0 \Rightarrow x_2 = -cx_1 \tag{1.7}$$

Substituting $x_2 = -cx_1$ into the first equation of the system (1.1) yields to

$$\dot{x}_1 = -cx_1$$

This equation is called the sliding mode dynamics. Choosing $c > 0$ guarantees the state x_1 tends to zero as t tends to infinity, then from (1.7), the state x_2 tends to zero as t tends to infinity. The rate of convergence can be controlled by choice of c.

From the above example, the multiple control structures are designed so that the state trajectory always moves toward an adjacent region with a different controller, and so the ultimate trajectory will not exist entirely within one system structure. Instead, it will slide along the boundary of the control structures. So the main idea is that the system state trajectory is forced to the preset switching manifold by the controller, and then reaches the zero along the switching manifold. So this control with a sliding mode is called sliding mode variable structure control or sliding mode control for short.

1.3 Sliding mode control design

Consider an affine nonlinear system described by

$$\dot{x}(t) = f(x,t) + B(x,t)u(t) \tag{1.8}$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ is the control input. The functions $f: R^n \times R \mapsto R^n$ and $B: R^n \times R \mapsto R^{n\times m}$ are assumed to be continuous and sufficiently smooth.

The switching function is selected as $s = s(x,t) \in R^m$.

The control law $u = u(x,t)$ will switch in the switching manifold by

$$u_i(x,t) = \begin{cases} u_i^+(x,t), & s_i(x,t) > 0, \\ u_i^-(x,t), & s_i(x,t) < 0, \end{cases} \quad i = 1,\cdots,m \tag{1.9}$$

where $u(x,t) = [u_1(x,t),\cdots,u_m(x,t)]^{\mathrm{T}}$ and $s(x,t) = [s_1(x,t),\cdots,s_m(x,t)]^{\mathrm{T}}$, $u_i(x,t)$ and $s_i(x,t)$ are smooth for $i = 1,\cdots,m$.

Note that because of the discontinuity of the control law (1.9), the classical theory of ordinary differential equations is unable to explain the existence and uniqueness of the solution of differential equation (1.8) with discontinuous right-hand sides. That is, the solution of the system (1.8) is known to exist and be unique if the control law u is a Lipschitz function and so continuous according to ordinary differential equation theory. Consequently, many researchers begin to look for appropriate mathematical tools to solve this problem, alternative approaches and construction of solutions can be found in Filippov's work and differential inclusion theory. These conclusions are omitted here and the readers interested in these contents may see [40]-[45].

From Section 1.2, the sliding mode control scheme involves two steps:

(1) A switching manifold (i.e., the sliding surface) $s(x,t)=0$ is designed such that the system state trajectory presents desirable behavior when the system state is confined to this manifold.

(2) A sliding mode control law $u=u(x,t)$ is constructed so that the system state trajectory reaches and stays on the manifold.

The sliding mode control scheme can be shown in Fig. 1.4.

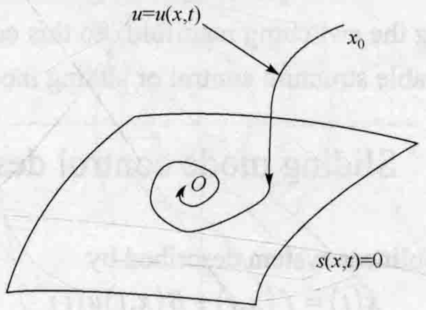

Fig. 1.4 A schematic diagram of sliding mode controller design

Because sliding mode control law is not continuous, it has the ability to drive state trajectory to the switching manifold in finite time. Once the state trajectory reaches the sliding surface, the system takes on the character of the sliding mode, that is, the system may be asymptotically stable on this manifold.

1.3.1 Reaching condition

The vital part of sliding mode controller design is to choose a control law u so that it drives the state trajectory to sliding surface and maintains it on this surface once it has been reached. For the system (1.8) with $m=1$, the reaching condition can be expressed by

$$\lim_{s\to 0^+} \dot{s} < 0 \quad \text{and} \quad \lim_{s\to 0^-} \dot{s} > 0$$

or

$$s\dot{s} < 0 \tag{1.10}$$

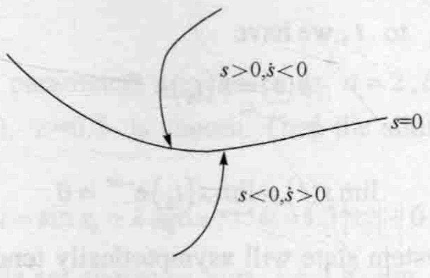

Fig. 1.5 The reaching condition $s\dot{s}<0$

The reaching condition (1.10) can be shown in Fig. 1.5. But this condition is not sufficient to guarantee the system state reaches sliding surface in finite time.

Example 1.1 Consider the pendulum equation as follows:

$$\ddot{\theta}+\sin\theta+b\dot{\theta}=cu$$

If we take the state variables as $x_1=\theta$ and $x_2=\dot{\theta}$, then a state model of the pendulum can be obtained as

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=-\sin x_1-bx_2+cu\end{cases} \tag{1.11}$$

where the parameters b,c are positive.

The sliding surface is designed as

$$s=x_2+ax_1=0,\quad a>0$$

Thus

$$\dot{s}=-\sin x_1+(a-b)x_2+cu$$

The following control law is

$$u=\frac{1}{c}(\sin x_1+k\operatorname{sgn}s),\quad k>0 \tag{1.12}$$

If $k>\left|(a-b)x_2\right|$ is held, then

$$s\dot{s}<\left[\left|(a-b)x_2\right|-k\right]|s|<0 \tag{1.13}$$

The reaching condition (1.10) is satisfied.

However, if we take the control law as

$$u=\frac{1}{c}\left[\sin x_1-(a-b)x_2-ks\right],\quad k>0 \tag{1.14}$$

then

$$\dot{s}=-ks \tag{1.15}$$

The reaching condition (1.10) is also satisfied.

Integrating (1.15) from t_0 to t, we have

$$s(t)=s(t_0)\mathrm{e}^{-\alpha t}$$

Thus

$$\lim_{t\to\infty}s(t)=\lim_{t\to\infty}s(t_0)\mathrm{e}^{-\alpha t}=0$$

That is to say, the system state will asymptotically tend to the sliding surface in infinite time rather than finite time.

To avoid this problem, the reaching condition (1.10) can be modified as

$$\begin{cases}\dot{s}>\varepsilon, & s<0\\ \dot{s}<-\varepsilon, & s>0\end{cases}$$

or

$$s\dot{s}<-\varepsilon|s|,\quad \varepsilon>0 \tag{1.16}$$

which can guarantee the system state reaches the sliding surface in a finite time.

Next, we give the specific reaching time.

In fact,

$$s\dot{s}=\frac{1}{2}\cdot\frac{\mathrm{d}s^2}{\mathrm{d}t}=\frac{1}{2}\cdot\frac{\mathrm{d}|s|^2}{\mathrm{d}t}=|s|\cdot\frac{\mathrm{d}|s|}{\mathrm{d}t} \tag{1.17}$$

From (1.16) and (1.17), we have

$$|s|\cdot\frac{\mathrm{d}|s|}{\mathrm{d}t}<-\varepsilon|s| \tag{1.18}$$

If the system state is not on the sliding surface, then $s\neq 0$, that is $|s|\neq 0$.

From (1.18), we have

$$\frac{\mathrm{d}|s|}{\mathrm{d}t}<-\varepsilon \tag{1.19}$$

Integrating (1.18) from 0 to t, we have

$$|s(t)|-|s(0)|<-\varepsilon t \tag{1.20}$$

Let $|s(T)|=0$. The reaching time T is obtained as

$$T<\frac{|s(0)|}{\varepsilon} \tag{1.21}$$

That is to say, if $t>T$, then $s(t)=0$.

Obviously, if the control law is selected as (1.12) with $k=|(a-b)x_2|+\varepsilon$, then (1.13) satisfies the modified condition (1.16). Thus finite time reachability can be