



Greiner Müller

# GAUGE THEORY OF WEAK INTERACTIONS

Third Edition

弱相互作用规范理论  
第3版

Springer-Verlag

世界图书出版公司

[www.wpcbj.com.cn](http://www.wpcbj.com.cn)

ISBN 7-5062-6587-7



9 787506 265874 >

ISBN 7-5062-6587-7/O · 441

WB6587

定价：82.00元

---

Walter Greiner · Berndt Müller

# GAUGE THEORY OF WEAK INTERACTIONS

With a Foreword by  
D. A. Bromley

Third Revised Edition  
With 123 Figures,  
and 75 Worked Examples and Problems



Springer

世界图书出版公司

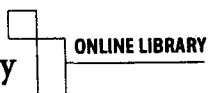
书 名: Gauge Theory of Weak Interactions  
作 者: W. Greiner, B. Müller  
中 译 名: 弱相互作用的规范理论  
出 版 者: 世界图书出版公司北京公司  
印 刷 者: 北京世图印刷厂  
发 行: 世界图书出版公司北京公司 (北京朝内大街 137 号 100010)  
联系电话: 010-64015659, 64038347  
电子信箱: kjsk@vip.sina.com  
开 本: 16 开 印 张: 26.5  
出版年代: 2003 年 12 月  
书 号: 7-5062-6587-7/O · 441  
版权登记: 图字:01-2003-8009  
定 价: 82.00 元

世界图书出版公司北京公司已获得 Springer-Verlag 授权在中国大陆  
独家重印发行。

---

W. Greiner · B. Müller  
GAUGE THEORY OF WEAK INTERACTIONS

Physics and Astronomy



<http://www.springer.de/phys/>

**Springer**

*Berlin  
Heidelberg  
New York  
Barcelona  
Hong Kong  
London  
Milan  
Paris  
Singapore  
Tokyo*

---

Greiner  
**Quantum Mechanics**  
An Introduction 3rd Edition

Greiner  
**Quantum Mechanics**  
Special Chapters

Greiner · Müller  
**Quantum Mechanics**  
Symmetries 2nd Edition

Greiner  
**Relativistic Quantum Mechanics**  
Wave Equations 3rd Edition

Greiner · Reinhardt  
**Field Quantization**

Greiner · Reinhardt  
**Quantum Electrodynamics**  
2nd Edition

Greiner · Schramm · Stein  
**Quantum Chromodynamics**  
2nd Edition

Greiner · Maruhn  
**Nuclear Models**

Greiner · Müller  
**Gauge Theory of Weak Interactions**  
3rd Edition

Greiner  
**Mechanics I**  
(in preparation)

Greiner  
**Mechanics II**  
(in preparation)

Greiner  
**Classical Electrodynamics**

Greiner · Neise · Stöcker  
**Thermodynamics  
and Statistical Mechanics**

---

## Foreword to Earlier Series Editions

More than a generation of German-speaking students around the world have worked their way to an understanding and appreciation of the power and beauty of modern theoretical physics – with mathematics, the most fundamental of sciences – using Walter Greiner's textbooks as their guide.

The idea of developing a coherent, complete presentation of an entire field of science in a series of closely related textbooks is not a new one. Many older physicists remember with real pleasure their sense of adventure and discovery as they worked their ways through the classic series by Sommerfeld, by Planck and by Landau and Lifshitz. From the students' viewpoint, there are a great many obvious advantages to be gained through use of consistent notation, logical ordering of topics and coherence of presentation; beyond this, the complete coverage of the science provides a unique opportunity for the author to convey his personal enthusiasm and love for his subject.

The present five-volume set, *Theoretical Physics*, is in fact only that part of the complete set of textbooks developed by Greiner and his students that presents the quantum theory. I have long urged him to make the remaining volumes on classical mechanics and dynamics, on electromagnetism, on nuclear and particle physics, and on special topics available to an English-speaking audience as well, and we can hope for these companion volumes covering all of theoretical physics some time in the future.

What makes Greiner's volumes of particular value to the student and professor alike is their completeness. Greiner avoids the all too common "it follows that . . ." which conceals several pages of mathematical manipulation and confounds the student. He does not hesitate to include experimental data to illuminate or illustrate a theoretical point and these data, like the theoretical content, have been kept up to date and topical through frequent revision and expansion of the lecture notes upon which these volumes are based.

Moreover, Greiner greatly increases the value of his presentation by including something like one hundred completely worked examples in each volume. Nothing is of greater importance to the student than seeing, in detail, how the theoretical concepts and tools under study are applied to actual problems of interest to a working physicist. And, finally, Greiner adds brief biographical sketches to each chapter covering the people responsible for the development of the theoretical ideas and/or the experimental data presented. It was Auguste Comte (1798–1857) in his *Positive Philosophy* who noted, "To understand a science it is necessary to know its history". This is all too often forgotten in modern physics teaching and the

bridges that Greiner builds to the pioneering figures of our science upon whose work we build are welcome ones.

Greiner's lectures, which underlie these volumes, are internationally noted for their clarity, their completeness and for the effort that he has devoted to making physics an integral whole; his enthusiasm for his science is contagious and shines through almost every page.

These volumes represent only a part of a unique and Herculean effort to make all of theoretical physics accessible to the interested student. Beyond that, they are of enormous value to the professional physicist and to all others working with quantum phenomena. Again and again the reader will find that, after dipping into a particular volume to review a specific topic, he will end up browsing, caught up by often fascinating new insights and developments with which he had not previously been familiar.

Having used a number of Greiner's volumes in their original German in my teaching and research at Yale, I welcome these new and revised English translations and would recommend them enthusiastically to anyone searching for a coherent overview of physics.

Yale University  
New Haven, CT, USA  
1989

*D. Allan Bromley*  
Henry Ford II Professor of Physics



---

## Preface to the Third Edition

Again, we take this opportunity to correct misprints and errors and add new examples and exercises.

We thank several colleagues and students for helpful comments, particularly Dr. Joachim Reinhardt who helped me to improve some exercises and examples and Dipl.-Phys. Constantin Loizides who helped in the preparation of this third edition. Finally, we acknowledge the agreeable collaboration with Dr. H. J. Kölsch and his team at Springer-Verlag, Heidelberg.

Frankfurt am Main  
July 2000

*Walter Greiner*

---

## Preface to the Second Edition

We are pleased to note that our text *Gauge Theory of Weak Interactions* has found many friends among physics students and researchers so that the need for a second edition has arisen. We have taken this opportunity to make several amendments and improvements to the text. A number of misprints and minor errors have been corrected and explanatory remarks have been added at various places. In addition to many other smaller changes the Sects. 6.4 on Cabibbo's theory of flavour mixing, 7.4 on the properties of allowed beta decay, 9.3 on the SU(5) Gauge Theory, and 9.5 on the scale of the SU(5) symmetry breaking have been expanded. Several new examples and exercises in Chaps. 6, 7, and 9 have been added, e.g., on parity violation in inelastic lepton–nucleon scattering or on the running coupling constant in quantum field theory.

We thank several colleagues and students for helpful comments. We also thank Dr. E. Stein and Dr. Steffen A. Bass who have supervised the preparation of the second edition of the book. Finally we acknowledge the agreeable collaboration with Dr. H. J. Kölsch and his team at Springer-Verlag, Heidelberg.

Frankfurt am Main and Durham, NC, USA  
December 1995

Walter Greiner  
Berndt Müller

---

## Preface to the First Edition

Modern theoretical physics has, over the past twenty years, made enormous progress, which may well be compared to the dramatic developments that occurred during the first few decades of this century. Whereas the discoveries of the early twentieth century (quantum mechanics, special and general relativity) concerned the foundations of modern physics, remaking the very concepts on which our view of the laws of nature are based, the recent breakthroughs have provided an almost complete understanding of the basic principles of the fundamental interactions among elementary particles. These principles are laid down in the so-called "Standard Model of Particle Physics" which successfully describes all established experimental data in physics.

At present, we know four fundamental interactions among elementary particles: the *strong nuclear* interaction (mediated by the exchange of mesons or – at a deeper level – of gluons), the *electromagnetic* interaction (mediated by photon exchange), the *weak nuclear* interaction (mediated by the exchange of the recently discovered W and Z bosons and, like the strong interaction, of short range), and *gravity*. Experimental searches have so far failed to uncover forces other than those four, although we cannot exclude the existence of other, very weak or short-ranged interactions.

The search for a common origin of all interactions is an ultimate (maybe *the* ultimate) goal of physics. Ever since Einstein's failed search for a unified field theory, it has been the dream of theoretical physicists to condense all laws of physics into a single fundamental equation, which contains all known interactions as special cases. This development had had its first dramatic success with Maxwell's theory of electromagnetism, which had combined the laws of electricity and magnetic interactions into a single set of equations which, in modern notation, take the beautifully simple form:  $\partial_\nu F^{\mu\nu} = j^\mu$ ,  $\partial_\nu \tilde{F}^{\mu\nu} = 0$ . The disparate phenomena of electricity and magnetism suddenly had become recognized as inseparable parts of a more general interaction. Maxwell's equations had predicted the existence of electromagnetic waves. These were discovered shortly afterwards and today form the basis of the global communication network, without which modern life and modern science could not be imagined.

A comparable breakthrough occurred twenty years ago when Glashow, Salam, Weinberg, and others recognized a deep relation between the electromagnetic and the weak nuclear interaction and showed that they could be derived from a unified theory. These lecture notes deal with the ideas and insight that led to this modern unification, and introduce the student to the phenomena that played a central role in this development. We begin with a detailed exposition of Fermi's theory of

beta decay and discuss the successes and shortcomings of this remarkable theory. The importance of the consideration of fundamental symmetries is illustrated by the violation of parity invariance, leading up to the  $(V-A)$  theory of weak interactions. Numerous solved problems and examples demonstrate various aspects of the weak interaction and provide an opportunity to apply the newly learned material.

The central part of the lectures introduces us to the concept of gauge theories, based on the generalization of the symmetry principle to local symmetries. The present volume may be regarded as continuation of volume 2 of this series: "Quantum Mechanics—Symmetries", extending the concepts of continuous symmetry groups to gauge transformations. The application of the gauge principle to weak isospin and hypercharge results in the unified electroweak gauge theory. The concepts of spontaneous symmetry breaking, charged and neutral currents, and mixing angles, are introduced and discussed in broad detail. Many aspects are illustrated with examples selected from current research fields, such as the problem of neutrino mixing with its application to the solar neutrino flux. Additional chapters are concerned with the applications of the electroweak gauge theory to hadronic decays and to the nuclear beta decay, where the presentation is systematically based on the quark model first introduced in volume 2. A separate chapter deals with the phenomenon of CP violation.

Only a few years after the formulation of the electroweak gauge theory, it was discovered that the strong interactions are also based on a set of equations that closely resembles those of the unified electroweak theory. This immediately fostered speculations that electroweak and strong interactions could be the low-energy manifestations of a "grand unified" gauge theory. The last section of our book contains an extended introduction on the principles underlying the search for such unified theories. We discuss the  $SU(5)$  model of Georgi and Glashow, the simplest unified gauge theory, and show how model building is constrained by experimental data. The presentation is broad and self-contained as usual in this series, introducing the student to the new concepts and formal techniques without unnecessary ballast. A detailed derivation of proton decay is presented, and the question of anomaly freedom is discussed. The book concludes with an outlook on supersymmetric unification in the light of recent precision measurements of the electroweak and strong gauge coupling constants.

These lectures make an attempt to familiarize the student with the developments of modern particle physics by providing a conceptually simple, yet rigorous introduction combined with hands-on experience through exercises and examples. They grew out of advanced graduate courses presented at the Johann Wolfgang Goethe-Universität in Frankfurt am Main and the Vanderbilt University in Nashville, Tennessee during the years 1982–85. The volume is designed as a self-contained introduction to gauge theories. Of course, much of the material is based on the framework of relativistic quantum field theory; it is desirable that the student has at least a working familiarity with the theory of quantum electrodynamics (volume 4 of this series). Some important and often used equations and relations are collected in appendices.

Our special gratitude goes to Dr. Matthias Grabiak and Professor Dr. Andreas Schäfer for their help with the examples and exercises. Several students have helped to convert the material from the stage of informal lecture notes to a textbook. For

this first English edition we have enjoyed the help of Dipl.-Phys. Jürgen Augustin, Dipl.-Phys. Maria Berenguer, Dr. Oliver Graf, Dipl.-Phys. Christian Hofmann, cand. Phys. Markus Hofmann, Dipl.-Phys. André Jahns, Dipl.-Phys. Kyong-Ho Kang, Dipl.-Phys. Ullrich Katcher, Dipl.-Phys. Jürgen Klenner, cand. Phys. Yaris Pürsün, cand. Phys. Matthias Rosenstock, Dipl.-Phys. Jürgen Schaffner, Dipl.-Phys. Alexander Scherdin, cand. Phys. Christian Spieles and Dipl.-Phys. Mario Vidović. Miss Astrid Steidl drew the graphs and pictures. To all of them we express our sincere thanks.

We would especially like to thank Dipl. Phys. Raffaele Mattiello and Dr. Béla Waldhauser for their overall assistance in the preparation of the manuscript. Their organizational talent and advice in technical matters have contributed decisively to the successful completion of this work.

Frankfurt am Main, July 1993  
Durham, USA, July 1993

*Walter Greiner*  
*Berndt Müller*

---

## Contents

<b>1. The Discovery of the Weak Interaction</b> . . . . .	<b>1</b>
1.1 The Universal Fermi Interaction . . . . .	2
1.2 The Non-conservation of Parity . . . . .	10
1.3 Bibliographical Notes . . . . .	22
<b>2. Leptonic Interactions</b> . . . . .	<b>25</b>
2.1 The Current-Current Interaction (Charged Currents) . . . . .	25
2.2 The Decay of the Muon . . . . .	27
2.3 The Lifetime of the Muon . . . . .	38
2.4 Parity Violation in the Muon Decay . . . . .	44
2.5 The Michel Parameters . . . . .	53
2.6 The Tau Lepton . . . . .	71
2.7 Bibliographical Notes . . . . .	76
<b>3. Limitations of Fermi Theory</b> . . . . .	<b>79</b>
3.1 Neutral Currents . . . . .	79
3.2 Scattering of a Muon Neutrino by an Electron . . . . .	80
3.3 $\bar{\nu}_\mu - e^-$ Scattering . . . . .	83
3.4 High-Energy Behaviour of Neutrino-Electron Scattering . . . . .	86
3.5 Supplement: Scattering Formalism for Spin- $\frac{1}{2}$ Particles . . . . .	95
3.6 Divergences in Higher-Order Processes . . . . .	102
3.7 Bibliographical Notes . . . . .	104
<b>4. The Salam-Weinberg Theory</b> . . . . .	<b>107</b>
4.1 The Higgs Mechanism . . . . .	107
4.2 The Yang-Mills Field . . . . .	119
4.3 The Feynman Rules of Yang-Mills Theory . . . . .	129
4.4 The Glashow-Salam-Weinberg Model of Leptons . . . . .	146
4.5 Spontaneous Symmetry Breaking: The Higgs Sector . . . . .	154
4.6 Hidden $SU(2) \times U(1)$ Gauge Invariance . . . . .	168
4.7 Bibliographical Notes . . . . .	175
<b>5. Some Properties of the Salam-Weinberg Theory of Leptons</b> . . . . .	<b>177</b>
5.1 Decay of the Charged Boson $W^-$ . . . . .	177
5.2 The Process $e^+e^- \rightarrow Z^0 \rightarrow \mu^+\mu^-$ . . . . .	182

5.3	High-Energy Behaviour of the GSW Theory	197
5.4	Bibliographical Notes	203
<b>6.</b>	<b>Semi-Leptonic Interactions of Hadrons</b>	<b>205</b>
6.1	The World of Hadrons	205
6.2	Phenomenology of Decays of Hadrons	208
6.3	Weak Interactions of Quarks	216
6.4	Cabibbo's Theory of Flavour Mixing	227
6.5	Bibliographical Notes	246
<b>7.</b>	<b>Nuclear Beta Decay</b>	<b>247</b>
7.1	The MIT Bag Model	247
7.2	Beta Decay of the Neutron	253
7.3	Nuclear Beta Decay	260
7.4	Properties of Allowed Beta Decays	263
7.5	Bibliographical Notes	285
<b>8.</b>	<b>The Neutral Kaon System</b>	<b>287</b>
8.1	The Particles $K_S$ and $K_L$	287
8.2	CP Violation	296
<b>9.</b>	<b>Unified Gauge Theories</b>	<b>303</b>
9.1	Introduction: The Symmetry Group $SU(5)$	303
9.2	Embedding $SU(3)_C \times SU(2)_L \times U(1)$ into $SU(5)$	309
9.3	The $SU(5)$ Gauge Theory	326
9.4	Spontaneous Breaking of the $SU(5)$ Symmetry	341
9.5	Determination of the Scale of $SU(5)$ Symmetry Breaking	351
9.6	Proton Decay	365
9.7	Outlook: Extensions of the Standard Model	381
9.8	Bibliographical Notes	385
<b>Appendix</b>		<b>387</b>
A.	Conventions and "Natural" Units	387
B.	The Dirac Equation	388
C.	Feynman Rules	391
D.	Symmetry Transformations	393
<b>Subject Index</b>		<b>397</b>

---

## Contents of Examples and Exercises

1.1	Kinematics of Two-Body Decays . . . . .	3
1.2	Lorentz Transformation of Dirac Operators . . . . .	6
1.3	The Non-relativistic Limit of the Transition Operators . . . . .	8
1.4	Properties of the Helicity Operator . . . . .	12
1.5	Rotation of Helicity Eigenfunctions . . . . .	14
1.6	Left-Handed Dirac Operators . . . . .	19
1.7	The Weyl Equation . . . . .	20
2.1	Neutrino-Electron Exchange Current . . . . .	26
2.2	Proof of (2.31) . . . . .	31
2.3	Calculation of the Averaged Decay Matrix Element . . . . .	32
2.4	A Useful Relation for the Levi-Civita Tensor . . . . .	33
2.5	The Endpoint of the Electron Energy Spectrum in Muon Decay . . . . .	38
2.6	Myon Decay for Finite Neutrino Masses . . . . .	41
2.7	Average Helicity and Parity Violation . . . . .	47
2.8	Angular Distribution and Parity Violation . . . . .	48
2.9	Electron Helicity in Muon Decay . . . . .	49
2.10	CP Invariance in Muon Decay . . . . .	50
2.11	Muon Decay and the Michel Parameters . . . . .	56
2.12	The Fierz Transformation . . . . .	65
2.13	The Discovery of the Tau Lepton . . . . .	74
3.1	Muon Neutrino-Electron Scattering Cross Section . . . . .	84
3.2	The Spin-Averaged Cross Section for Antineutrino-Electron Scattering . . . . .	88
3.3	The Spin-Averaged Cross Section of Muon Neutrino-Electron Scattering . . . . .	92
3.4	High-Energy Scattering . . . . .	94
4.1	The Debye Effect . . . . .	109
4.2	Creation of Mass in Interacting Fields . . . . .	111
4.3	Gauge Invariance of the Lagrangian Corresponding to the Kinetic Energy of the Meson Fields . . . . .	118
4.4	Isospin Rotations . . . . .	124
4.5	Gauge Covariance of Minimal Coupling and of the Field-Strength Tensor . . . . .	126
4.6	Propagators and Gauge Invariance . . . . .	134
4.7	Self-Interaction of Gauge Fields . . . . .	144
4.8	The Gauge-Covariant Formulation of the GSW Theory: Weak Isospin and Weak Hypercharge . . . . .	151



4.9	Lepton Masses . . . . .	161
4.10	The Glashow–Salam–Weinberg Lagrangian . . . . .	161
4.11	Masses of the Vector Bosons . . . . .	163
4.12	The Norm of $T_{\text{vac}}$ . . . . .	166
5.1	Decay of the $Z^0$ Boson . . . . .	180
5.2	The Discovery of the Intermediate Vector Bosons . . . . .	190
5.3	Precision Measurement of the $Z^0$ Boson . . . . .	193
5.4	The s-Wave Contribution to Lepton–Neutrino Scattering . . . . .	202
6.1	The $\pi^\pm$ Decay Rate . . . . .	211
6.2	Concerning V–A Coupling in Pion Decay . . . . .	212
6.3	Suppression of the Electronic Decay Channel in Pion Decay . . . . .	214
6.4	Mixing in Leptonic Families . . . . .	220
6.5	Neutrino Oscillations . . . . .	222
6.6	Proof of (6.55) . . . . .	232
6.7	Absence of Flavour-Changing Neutral Currents . . . . .	233
6.8	Vanishing of Mixed Currents Between d and s Quarks . . . . .	234
6.9	Parity Violation in Lepton–Nucleon Scattering . . . . .	237
6.10	Parity Violation in Atoms . . . . .	241
7.1	The Ground State in the MIT Bag . . . . .	250
7.2	The Mean Square Radius of a Nucleon . . . . .	250
7.3	Parameter Fit to the Hadronic Mass Spectrum . . . . .	252
7.4	Determination of the Antineutrino Mass in Tritium Decay . . . . .	269
7.5	Excitation of the Atomic Electron in the $\beta$ Decay of Tritium . . . . .	271
7.6	Determination of $C_A/C_V$ from the Lifetime of a Neutron . . . . .	272
7.7	An Astrophysical Limit to the Neutrino Mass . . . . .	273
7.8	Double $\beta$ Decay . . . . .	274
7.9	The Majorana Neutrino . . . . .	277
7.10	The Solar Neutrino Problem . . . . .	279
8.1	CP Parity in Kaon Decay . . . . .	290
8.2	Transformation of Kaons Under Space Inversion and Charge Conjugation . . . . .	297
9.1	The Generators of $SU(3) \times SU(2) \times U(1)$ . . . . .	311
9.2	Charge Conjugation . . . . .	316
9.3	The Quintuplet of $SU(5)$ . . . . .	319
9.4	$SU(5)$ Classification of the Remaining Lepton and Quark Families . . . . .	323
9.5	The $SU(5)$ Gauge Bosons . . . . .	327
9.6	Construction of the Lagrangian . . . . .	335
9.7	Colour of the $SU(5)$ Gauge Bosons . . . . .	340
9.8	Minimum of the Higgs Potential . . . . .	343
9.9	Kinetic Energy of the Higgs Field . . . . .	345
9.10	The Running Coupling Constant in Quantum Field Theory . . . . .	357
9.11	Anomaly Freedom . . . . .	359
9.12	Is the Glashow–Salam–Weinberg Theory Anomaly Free? . . . . .	363
9.13	The Interaction Hamiltonian Operator for Proton Decay . . . . .	369