时代教育 · 国外高校优秀教材精选

(英文版·原书第2版)

实分析引论

Introduction to Real Analysis

(美) Manfred Stoll 著

机械工业出版社
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引进国外优秀原版教材，在有条件的学校推动开展英语授课或双语教学，自然也引进了先进的教学思想和教学方法，这对提高我国自编教材的水平，加强学生的英语实际应用能力，使我国的高等教育尽快与国际接轨，必将起到积极的推动作用。

为了做好教材的引进工作，机械工业出版社特别成立了由著名专家组成的国外高校优秀教材审定委员会。这些专家对实施双语教学做了深入细致的调查研究，对引进原版教材提出了许多建设性意见，并慎重地对每一本将要引进的原版教材一审再审，精选再精选，确认教材本身的质量水平，以及权威性和先进性，以期所引进的原版教材能适应我国学生的外语水平和学习特点。在引进工作中，审定委员会还结合我国高校教学课程体系的设置和要求，对原版教材的教学思想和方法的先进性、科学性严格把关。同时尽量考虑原版教材的系统性和经济性。

这套教材出版后，我们将根据各高校的双语教学计划，举办原版教材的教师培训，及时地将其推荐给各高校选用。希望高校师生在使用教材后及时反馈意见和建议，使我们更好地为教学改革服务。

机械工业出版社

序

理性认识是我们认识客观世界中任何事物的重要的阶段，而“理性思维”正是人们在理性认识过程中所应用的主要的思维方式和方法。一个人的“理性思维”的建立与“直观反映”不同，它不是先天具有的而是后天培养的。虽然许多学科对人的理性思维的培养都有一定的作用，但数学则起着任何其他学科都无法替代的作用。特别是那些基础性和应用性都很强的一些大学数学基础课程，如数学分析、线性代数、微分几何等在这方面的效用更是突出。

早在16世纪初，我国古代科学家徐光启在他与意大利传教士利马窦(Matteo Ricci)合译《几何原本》时就指出：“此书为宜，能令学理者袪其浮气，待其精心，学事者资其定法，发其巧思，故举世无一人不当学。”通过对大学数学教改历史的回顾和教育理论的研究，数学教育界在基础数学课程对培养学生理性思维的重要性的认识上，基本上取得了一致，同时也得到了许多其他学科专家们的认同，大家都感到应该在非数学专业的大学数学课程中适当加强这部分数学内容。目前有一些大学也正在这方面进行改革尝试，而其中遇到的突出矛盾是如何在不多的学时中讲授最切合需要的数学内容。在我阅读的一批国内外有关教材中，本书为我们处理这一问题提供了一个有价值的参考，也为那些希望对数学的理性思维方法有进一步了解的读者提供一本很好的入门教材。

这本书的主要特点首先在选材上，本书名为“实分析”，但与我国以往流行的实变函数教材有较大不同，它既有数学分析和实变函数的基本内容，又有近代数学中最基础的概念和方法。这对于理解数学方法和进一步学习近代数学课程有入门的效用。

本书另一特点是注意可读性。数学中许多概念比较抽象、难懂，如一致收敛性、一般集合的测度等。该书在讲述这些概念时，注意从简单到复杂，从具体到一般，平和好懂。数学中有些证明和方法技巧性较高，初学者很难理解，如 Weierstrass 逼近定理和 Fourier 级数收敛定理的证明。本书采用了近代数学中的基本方法，对这些定理作统一处理。这样，使读者既能容易搞懂定理证明本身，又领会了一些近代数学方法的思想。

本书还有一个特点是能面向不同水平的读者。前六章是最基础的内容，为一般的读者所设计，后四章是进一步的选读题目，是为高要求的读者准备的。在前六章的习题中提出的一些较深入的研究性问题，可为后四章的学习起到铺垫作用。

总的来说，无论是对希望在数学学习上进一步深入的读者，还是对大学数学课程的教师都是值得一读的。

谭泽光
清华大学数学科学系

Preface

The subject of real analysis is one of the fundamental areas of mathematics, and is the foundation for the study of many advanced topics, not only in mathematics, but also in engineering and the physical sciences. A thorough understanding of the concepts of real analysis has also become increasingly important for the study of advanced topics in economics and the social sciences. Topics such as Fourier series, measure theory, and integration are fundamental in mathematics and physics as well as engineering, economics, and many other areas.

Due to the increased importance of real analysis in many diverse subject areas, the typical first semester course on this subject has a varied student enrollment in terms of both ability and motivation. From my own experience, the audience typically includes mathematics majors, for whom this course represents the only rigorous treatment of analysis in their collegiate career, and students who plan to pursue graduate study in mathematics. In addition, there are mathematics education majors who need a strong background in analysis in preparation for teaching high school calculus. Occasionally, the enrollment includes graduate students in economics, engineering, physics, and other areas, who need a thorough treatment of analysis in preparation for additional graduate study either in mathematics or their own subject area. In an ideal situation, it would be desirable to offer separate courses for each of these categories of students. Unfortunately, staffing and enrollment usually make such choices impossible.

In the preparation of the text there were several goals I had in mind. The first was to write a text suitable for a one-year sequence in real analysis at the junior or senior level, providing a rigorous and comprehensive treatment of the theoretical concepts of analysis. The topics chosen for inclusion are based on my experience in teaching graduate courses in mathematics, and reflect what I feel are minimal requirements for successful graduate study. I get to the least upper bound property as quickly as possible, and emphasize this important property in the text. For this reason, the algebraic properties of the rational and real number systems are treated very informally, and the construction of the real number system from the rational numbers is included only as a miscellaneous exercise. I have attempted to keep the proofs as concise as possible, and to let the subject matter progress in a natural manner. Topics or sections that are not specifically required in subsequent chapters are indicated by a footnote.

My second goal was to make the text understandable to the typical student enrolled in the course, taking into consideration the variations in abilities, background, and motivation. For this reason, Chapters 1 through 6 have been written with the intent to be accessible to the average student, while at the same time challenging the more talented student through the exercises. The basic topological concepts of open, closed, and compact sets, as well as limits of sequences and functions, are introduced for the real numbers only. However, the proofs of many of the theorems, especially those involving topological concepts, are presented in a manner that permits easy extensions to more abstract settings. These chapters also include a large number of examples and more routine and computational exercises. Chapters 7 through 10 assume that the students have achieved some level of expertise in the subject. In these chapters, function spaces are introduced and studied in greater detail. The theorems, examples, and exercises require greater sophistication and mathematical maturity for full understanding. From my own experiences, these are not unrealistic expectations.

The book contains most of the standard topics one would expect to find in an introductory text on real analysis—limits of sequences, limits of functions, continuity, differentiation, integration, series, sequences and series of functions, and power series. These topics are basic to the study of real analysis and are included in most texts at this level. In addition, I have included a number of topics that are not always included in comparable texts. For instance, Chapter 6 contains a section on the Riemann-Stieltjes integral, and a section on numerical methods. Chapter 7 also includes a section on square summable sequences and a brief introduction to normed linear spaces. Both of these concepts appear again in later chapters of the text.

In Chapter 8, to prove the Weierstrass approximation theorem, I use the method of approximate identities. This exposes the student to a very important technique in analysis that is used again in the chapter on Fourier series. The study of Fourier series, and the representation of functions in terms of series of orthogonal functions, has become increasingly important in many diverse areas. The inclusion of Fourier series in the text allows the student to gain some exposure to this important subject, without the necessity of taking a full semester course on partial differential equations. In the final chapter I have also included a detailed treatment of Lebesgue measure and the Lebesgue integral. The approach to measure theory follows the original method of Lebesgue, using inner and outer measure. This provides an intuitive and leisurely approach to this very important topic.

The exercises at the end of each section are intended to reinforce the concepts of the section and to help the students gain experience in developing their own proofs. Although the text contains some routine and computational problems, many of the exercises are designed to make the students think about the basic concepts of analysis, and to challenge their creativity and logical thinking. Solutions and hints to selected exercises are included at the end of the text. These problems are marked by an asterisk (*).

At the end of each chapter I have also included a section of notes on the chapter, miscellaneous exercises, and a supplemental reading list. The notes in many cases provide historical comments on the development of the subject, or discuss topics not included in the chapter. The miscellaneous exercises are intended to extend the subject matter of the text or to cover topics that, although important, are not covered in the chapter itself. The supplemental reading list provides references to topics that relate to the subject under discussion. Some of the references provide historical information; others provide alternative solutions of results or interesting related problems. Most of the articles appear in the *American Mathematical Monthly* or *Mathematics Magazine*, and should be easily accessible for students' reference.

To cover all the chapters in a one-year sequence is perhaps overly ambitious. However, from my own experience in teaching the course, with a judicious choice of topics it is possible to cover most of the text in two semesters. A one-semester course should at a minimum include all or most of the first five chapters, and part or all of Chapter 6 or Chapter 7. The latter chapter can be taught independently of Chapter 6; the only dependence on Chapter 6 is the integral test, and this can be covered without a theoretical treatment of Riemann integration. The remaining topics should be more than sufficient for a full second semester. The only formal prerequisite for reading the text is a standard three- or four-semester sequence in calculus. Even though an occasional talented student has completed one semester of this course during their sophomore year, some mathematical maturity is expected, and the average student might be advised to take the course during their junior or senior year.

Features New to the Second Edition

In content, the second edition remains primarily unchanged from the first. The subject of real analysis has not changed significantly since publication of the first edition. In this edition I have incorporated many of the valuable suggestions from reviewers, instructors, and students. Some new topics have been included, and the presentation of others has been revised.

New examples and revised explanations appear throughout this edition of the text. The second edition also contains additional illustrations and expanded problem sets. The problem sets in all sections of the first six chapters have been expanded to include more routine and computational problems. The challenging problems are still there. With the addition of more routine problems, instructors using this text will have greater flexibility in the assignment of exercises. The supplemental reading lists have all been updated to include relevant articles that have appeared since 1996.

Two of the more substantive changes are the inclusion of a proof of Lebesgue's theorem in Chapter 6, and the addition of an appendix on logic and proofs. In the first edition, Lebesgue's theorem was stated in Section 6.1 and then proved in Chapter 10. At the recommendation of my colleague Anton Schep, I have included a self-contained proof of Lebesgue's theorem as a separate section in Chapter 6. The proof is based on notes that he has used to supplement the text. In the proof, as in the statement of the theorem, the only reference to measure theory is the definition of a set of measure zero. With this change it is now possible not only to state but also to prove this important theorem without first having to develop the theory of Lebesgue measure and integration.

The greatest difficulty facing many students taking a course in real analysis is the ability to write and to understand proofs. Most have never had a course in mathematical logic. For this reason I have included a brief appendix on logic and proofs. The appendix is not intended to replace a formal course in logic; it is only intended to introduce the rules of logic that students need to know in order to better understand proofs. These rules are also crucial in helping students develop the ability to write their own proofs. The various methods of proof are discussed in detail, and examples of each method are included and analyzed. The appendix also includes a section on the use of quantifiers, with special emphasis on the proper negation of quantified sentences. The appendix itself is independent of the text; however, references to it are included throughout the first several chapters of the text. The appendix can be included as part of the course, or assigned as independent reading.

Acknowledgments

I would like to thank the students at South Carolina who have learned this material from me, or my colleagues, from preliminary versions of this text. Your criticisms, comments, and suggestions were appreciated. I am also indebted to those colleagues, especially the late Jeong Yang, who agreed to use the manuscript in their courses.

Special thanks are also due to the reviewers who examined the manuscript for the first edition and provided constructive criticisms and suggestions for its improvement: Joel Anderson, Pennsylvania State University; Bogdam Baishanski, Ohio State University; Robert Brown, University of Kansas; Donald Edmondson, University of Texas at Austin; Kevin Grasse, University of Oklahoma; Harvey Greenwald, California Polytechnic State University; Adam Helfer, University of Missouri, Columbia; Jan Kucera, Washington State University; Thomas Reidel, University of Louisville; Joel Robin, University of Wisconsin, Madison; Stuart Robinson, Cleveland State University; Dan Shea, University of Wisconsin, Madison; Richard B. Sher, University of North Carolina; Thomas Smith, Manhattan College. Your careful reading of the manuscript helped to turn the preliminary drafts into a polished text.

I would also like to thank Carolyn Lee-Davis and the staff at Addison Wesley Longman for their assistance in the preparation of the second edition, and the reviewers for this edition for their comments and recommendations: William Barnier, Sonoma State University; Rene Barrientos, Miami Dade Community College; Denine Burkett, Lock Haven University; Steve Deckelman, University of Wisconsin; Lyn Geisler, Randolph-Macon College; Constant Goutziers, State University of New York; Christopher Heil, Georgia Institute of Technology; William Stout, Salve Regina University. Special thanks go to my colleague, George McNulty, for his careful reading of the appendix. His constructive criticisms and suggestions were appreciated. I am also grateful to the readers who informed me of errors in the first edition, and to the instructors who conveyed to me some of the difficulties encountered while using the book as a text. Hopefully all of the errors and shortcomings of the first edition have been corrected.

Finally, I would especially like to thank my wife, Mary Lee, without whose encouragement this project might never have been completed.

Manfred Stoll

To the Student

The difference between a course on calculus and a course on real analysis is analogous to the difference in the approach to the subject prior to the nineteenth century and since that time. Most of the topics in calculus were developed in the late seventeenth and eighteenth centuries by such prominent mathematicians as Newton, Leibniz, Bernoulli, Euler, and many others. Newton and Leibniz developed the differential and integral calculus; their successors extended and applied the theory to many problems in mathematics and the physical sciences. They had phenomenal insight into the problems, and were extremely proficient and ingenious in deriving complex formulas. What they lacked, however, were the tools to place the subject on a rigorous mathematical foundation. This did not occur until the nineteenth century with the contributions of Cauchy, Bolzano, Weierstrass, Cantor, and many others.

In calculus, the emphasis is primarily on developing expertise in computational techniques and applications. In real analysis, you will be expected to understand the concepts and to develop the ability to prove results using the definitions and previous theorems. Understanding the concept of a limit, and proving results about limits, will be significantly more important than computing limits. To accomplish this, it is essential that all definitions and statements of theorems be learned precisely. Most of the proofs of the theorems and solutions of the problems are logical consequences of the definitions and previous results; some, however, do require ingenuity and creativity.

The text contains numerous examples and counterexamples to illustrate the particular topics under discussion. These are included to show why certain hypotheses are required, and to help develop a more thorough understanding of the subject. It is crucial that you not only learn what is true, but that you also have sufficient counterexamples at your disposal. I have included hints and answers to selected exercises at the end of the text; these are indicated by an asterisk (*). For some of the problems I have provided complete details; for others I have provided only brief hints, leaving the details to you. As always, you are encouraged to first attempt the exercises, and to look at the hints or solutions only after repeated attempts have been unsuccessful.

At the end of each chapter I have included a supplemental reading list. The journal articles or books are all related to the topics in the chapter. Some provide historical information or extensions of the topics to more general settings; others provide alternative solutions of results in the text, or solutions of interesting related problems. All of the articles should be accessible in your library. They are included to encourage you to develop the habit of looking into the mathematical literature.

An excellent source for additional historical information and biographies of famous mathematicians is the MacTutor History of Mathematics archive at the University of St. Andrews, Scotland. The URL of their webpage is http://www-history.mcs.st-andrews.as.uk/

On reading the text you will inevitably encounter topics, formulas, or examples that may appear too technical and difficult to comprehend. Skip them for the moment; there will be plenty for you to understand in what follows. Upon later reading the section, you may be surprised that it is not nearly as difficult as previously imagined. Concepts that initially appear difficult become clearer once you develop a greater understanding of the subject. It is important to keep in mind that many of the examples and topics that appear difficult to you were most likely just as difficult to the mathematicians of the era in which they first appeared.

The material in the text is self-contained and independent of calculus. I do not use any results from calculus in the definitions and development of the subject matter. Occasionally, however, in the examples and exercises I do assume knowledge of the elementary functions and of notation and concepts that should have been encountered elsewhere. These concepts will be defined carefully at the appropriate place in the text.

Manfred Stoll

Contents

1 The Real Number System

The key to understanding many of the fundamental concepts of calculus, such as limits, continuity, and the integral, is the least upper bound property of the real number system $\mathbb{R}$. As we all know, the rational number system contains gaps. For example, there does not exist a rational number r such that $r^2 = 2$, i.e., $\sqrt{2}$ is irrational. The fact that the rational numbers do contain gaps makes them inadequate for any meaningful discussion of the above concepts.

The standard argument used in proving that the equation $r^2 = 2$ does not have a solution in the rational numbers goes as follows: Suppose that there exists a rational number r such that $r^2 = 2$. Write $r = m/n$ where m, n are integers that are not both even. Thus $m^2 = 2n^2$. Therefore m^2 is even, and hence m itself must be even. But m^2, and hence also $2n^2$, are both divisible by 4. Therefore n^2 is even, and as a consequence n is also even. This, however, contradicts our assumption that not both m and n are even. The method of proof used in this example is proof by contradiction; namely, we assume the negation of the conclusion and arrive at a logical contradiction. A discussion of the various methods of proof is included in Section A.3 of the Appendix.

The above argument shows that there does not exist a rational number r such that $r^2 = 2$. This argument was known to Pythagoras (around 500 B.C.), and even the Greek mathematicians of this era noted that the straight line contains many more points than the rational numbers. It was not until the nineteenth century, however, when mathematicians became concerned with putting calculus on a firm mathematical footing, that the development of the real number system was accomplished. The construction of the real number system is attributed to Richard Dedekind (1831–1916) and Georg Cantor