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Equilibrium and Non-Equilibrium Statistical Thermodynamics

平衡态 和非平衡态统计热力学

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EQUILIBRIUM AND NON-EQUILIBRIUM STATISTICAL THERMODYNAMICS

MICHEL LE BELLAC, FABRICE MORTESSAGNE
AND G. GEORGE BATROUNI

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EQUILIBRIUM AND NON-EQUILIBRIUM STATISTICAL THERMODYNAMICS

This book gives a self-contained exposition at graduate level of topics that are generally considered fundamental in modern equilibrium and non-equilibrium statistical thermodynamics.

The text follows a balanced approach between the macroscopic (thermodynamic) and microscopic (statistical) points of view. The first half of the book deals with equilibrium thermodynamics and statistical mechanics. In addition to standard subjects, such as the canonical and grand canonical ensembles and quantum statistics, the reader will find a detailed account of broken symmetries, critical phenomena and the renormalization group, as well as an introduction to numerical methods, with a discussion of the main Monte Carlo algorithms illustrated by numerous problems. The second half of the book is devoted to non-equilibrium phenomena, first following a macroscopic approach, with hydrodynamics as an important example. Kinetic theory receives a thorough treatment through the analysis of the Boltzmann–Lorentz model and of the Boltzmann equation. The book concludes with general non-equilibrium methods such as linear response, projection method and the Langevin and Fokker–Planck equations, including numerical simulations. One notable feature of the book is the large number of problems. Simple applications are given in 71 exercises, while the student will find more elaborate challenges in 47 problems, some of which may be used as mini-projects.

This advanced textbook will be of interest to graduate students and researchers in physics.

MICHEL LE BELLAC graduated from the Ecole Normale Supérieure and obtained a Ph.D. in Physics at the Université Paris-Orsay in 1965. He was appointed Professor of Physics in Nice in 1967. He also spent three years at the Theory Division at CERN. He has contributed to various aspects of the theory of elementary particles and recently has been working on the theory of the quark–gluon plasma. He has written several textbooks in English and in French.

FABRICE MORTESSAGNE obtained a Ph.D. in high-energy physics at the Université Denis Diderot of Paris in 1995, and then was appointed Maître de Conférences at the Université de Nice–Sophia Antipolis. He has developed semi-classical approximations of wave propagation in chaotic systems and was one of the initiators of the ‘Wave Propagation in Complex Media’ research group. In

1998 he extended his theoretical research activities with wave chaos experiments in chaotic optical fibres and microwave billiards.

G. GEORGE BATROUNI obtained a Ph.D. in theoretical particle physics at the University of California at Berkeley in 1983 and then took a postdoctoral fellowship at Cornell University. In 1986 he joined Boston University and later the Lawrence Livermore National Laboratory. He became professor at the Université de Nice-Sophia Antipolis in 1996. He was awarded the Onsager Medal in 2004 by the Norwegian University of Science and Technology. He has made important contributions in the development of numerical simulation methods for quantum field theories and many body problems, and in the study of quantum phase transitions and mesoscopic models of fracture.

This advanced textbook will be of interest to graduate students and researchers in physics. applications are given in 71 exercises, while the student will find more elaborate relations. One notable feature of the book is the large number of problems. Simple method and the Langevin and Fokker-Planck equations, including numerical simulations with general non-equilibrium methods such as linear response, projection of the Boltzmann-Lorentz model and of the Boltzmann equation. The book contains a kinetic theory receives a thorough treatment through the analysis of phenomena, first following a macroscopic approach, with hydrodynamics as an important example. The second half of the book is devoted to non-equilibrium phenomena and the renormalization group, as well as an introduction to numerical methods, with a discussion of the main Monte Carlo algorithms illustrated by numerous problems. The second half of the book is devoted to non-equilibrium phenomena, first following a macroscopic approach, with hydrodynamics as an important example. Kinetic theory receives a thorough treatment through the analysis of phenomena, first following a macroscopic approach, with hydrodynamics as an important example. The second half of the book is devoted to non-equilibrium phenomena and the renormalization group, as well as an introduction to numerical methods, with a discussion of the main Monte Carlo algorithms illustrated by numerous problems. The second half of the book is devoted to non-equilibrium phenomena and the renormalization group, as well as an introduction to numerical methods, with a discussion of the main Monte Carlo algorithms illustrated by numerous problems.

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Preface

This book attempts to give at a graduate level a self-contained, thorough and pedagogic exposition of the topics that, we believe, are most fundamental in modern statistical thermodynamics. It follows a balanced approach between the macroscopic (thermodynamic) and microscopic (statistical) points of view.

The first half of the book covers equilibrium phenomena. We start with a thermodynamic approach in the first chapter, in the spirit of Callen, and we introduce the concepts of equilibrium statistical mechanics in the second chapter, deriving the Boltzmann–Gibbs distribution in the canonical and grand canonical ensembles. Numerous applications are given in the third chapter, in cases where the effects of quantum statistics can be neglected: ideal and non-ideal classical gases, magnetism, equipartition theorem, diatomic molecules and first order phase transitions. The fourth chapter deals with continuous phase transitions. We give detailed accounts of symmetry breaking, discrete and continuous, of mean field theory and of the renormalization group and we illustrate the theoretical concepts with many concrete examples. Chapter 5 is devoted to quantum statistics and to the discussion of many physical examples: Fermi gas, black body radiation, phonons and Bose–Einstein condensation including gaseous atomic condensates.

Chapter 6 offers an introduction to macroscopic non-equilibrium phenomena. We carefully define the notion of local equilibrium and the transport coefficients together with their symmetry properties (Onsager). Hydrodynamics of simple fluids is used as an illustration. Chapter 7 is an introduction to numerical methods, in which we describe in some detail the main Monte Carlo algorithms. The student will find interesting challenges in a large number of problems in which numerical simulations are applied to important classical and quantum models such as the Ising, XY and clock (vector Potts) models, as well as lattice models of superfluidity.

Kinetic theory receives a thorough treatment in Chapter 8 through the analysis of the Boltzmann–Lorentz model and of the Boltzmann equation. The book

ends with general non-equilibrium methods such as linear response, the projection method, the fluctuation-dissipation theorem and the Langevin and Fokker-Planck equations, including numerical simulations.

We believe that one of this book's assets is its large number of exercises and problems. Exercises pose more or less straightforward applications and are meant to test the student's understanding of the main text. Problems are more challenging and some of them, especially those of Chapter 7, may be used by the instructor as mini-research projects. Solutions of a selection of problems are available on the web site.

Statistical mechanics is nowadays such a broad field that it is impossible to review in its entirety in a single volume, and we had to omit some subjects to maintain the book within reasonable limits or because of lack of competence in specialized topics. The most serious omissions are probably those of the new methods using chaos in non-equilibrium phenomena and the statistical mechanics of spin glasses and related subjects. Fortunately, we can refer the reader to excellent books: those by Dorfman [33] and Gaspard [47] in the first case and that of Fisher and Hertz [42] in the second.

The book grew from a translation of a French version by two of us (MLB and FM), *Thermodynamique Statistique*, but it differs markedly from the original. The text has been thoroughly revised and we have added three long chapters: 4 (Critical phenomena), 7 (Numerical simulations) and 9 (Topics in non-equilibrium statistical mechanics), as well as a section on the calculation of transport coefficients in the Boltzmann equation.

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