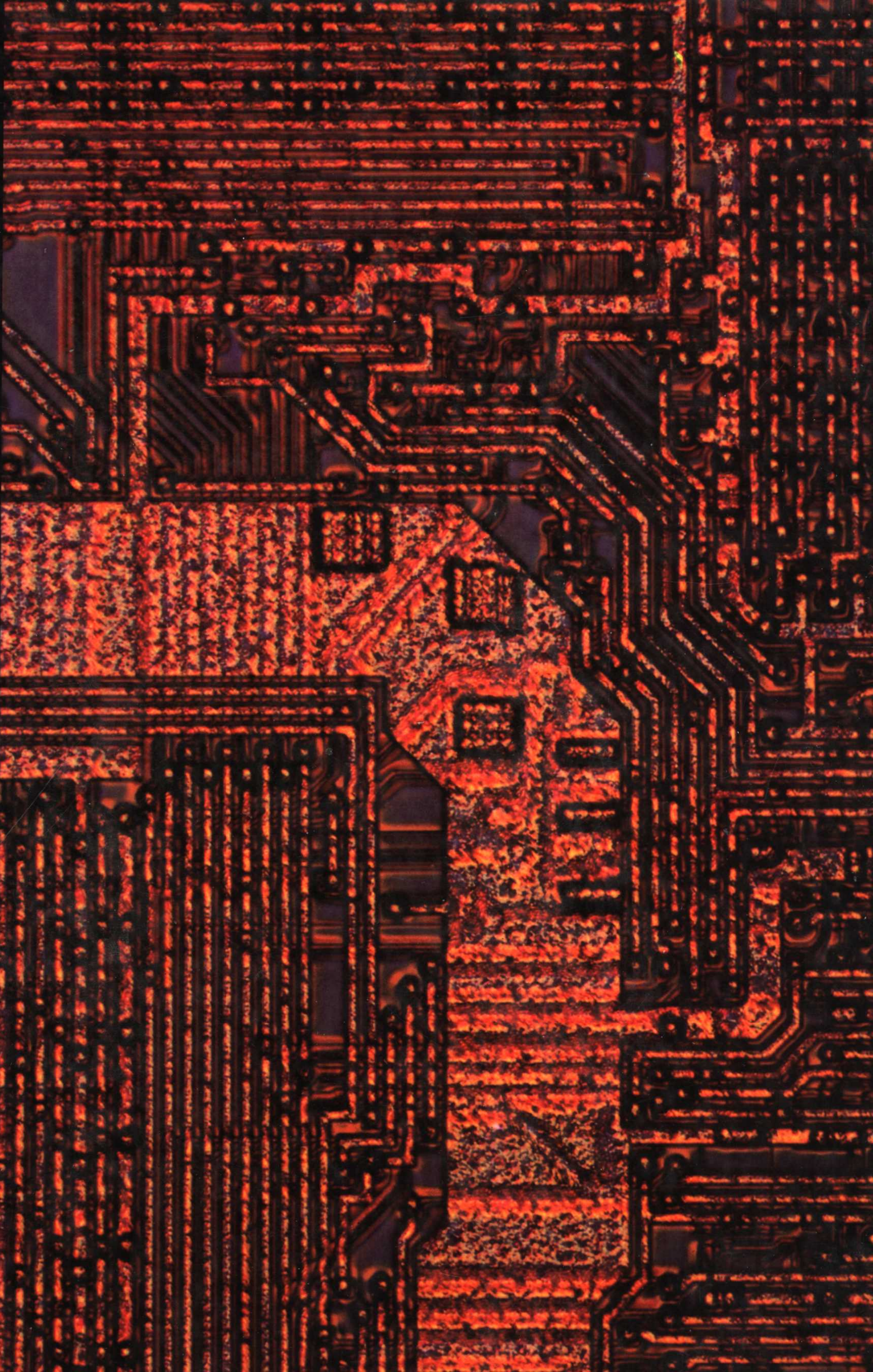
COLLEGE ALGEBRA 3RD EDITION

KARL J. SMITH & PATRICK J. BOYLE



College Algebra
Third Edition

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To Jack and Rosamond with love and affection
In loving memory of Frank and Maureen

Preface

Following the launching of *Sputnik* in the 1950s, mathematics went through dynamic changes that culminated in the "new math" of the 1960s. The goal of the new math was to create in the student a fundamental understanding of mathematics. But in so doing, we lost sight of the practicality of mathematics, and consequently the results of that grand experiment in the new math proved to be less than expected.

During the 1970s there was a slow but steady trend toward renewed emphasis on practical, solid mathematics. In the 1980s competency requirements are now being imposed on many students receiving high school diplomas, associate degrees, and even bachelor's degrees. These competency requirements are forcing schools to review their curricula and develop courses that not only provide basic skills, but also develop critical thinking and a fundamental understanding of mathematics. This "back to basics" movement of the 1980s does not seek to reestablish the methods used before *Sputnik*. Instead, there is a deep concern to present practical, solid mathematics while keeping the goal of giving our students a fundamental understanding of mathematics. It is not enough simply to present the superficial "monkey see, monkey do" type of exercises. The problem sets must be designed to develop the student's thought processes.

This *renewed mathematics* is the goal of our series of algebra textbooks. In this text we carefully and deliberately present exposition, examples, and problems that have a threefold purpose:

1. No-nonsense drill problems teach the students essential mathematical skills; this edition provides a *significant* number of new drill problems. The A and B drill problems are graded in difficulty and are presented as matched pairs or triplets of problems. The C problems give practice at a more difficult level. Each chapter has a summary and review problem set that includes over 100 problems for additional practice and study.
2. Applied problems teach the student some of the usefulness and practicality of mathematics; nearly every section of this edition has a sampling of applied problems so that the student can gain continued experience in dealing with applied problems. The applied problems are self-contained and do not require any outside knowledge.
3. Problems that actually *teach* the thought process to the student provide a fundamental understanding of mathematical processes in a way that will enhance the student's reasoning ability, not only in mathematics, but also in nonmathematical situations requiring a reasoned conclusion.

The exposition and examples in the text also reflect this goal. The first chapter provides a review of the real number system with exponents as the unifying theme. This chapter will review and refresh the student's manipulative skills. Chapters 2 and 3 review solving and graphing first- and second-degree equations and inequalities in one and two variables. The concept of a function is presented in Chapter 4 and is the unifying theme for the remaining chapters, which are relatively independent. Chapter 5 develops the theory of equations, including a discussion and graphing of both polynomial and rational functions. Chapter 6 focuses on the nature and applications of the exponential and logarithmic functions, rather than on their computational properties. In Chapter 7, we discuss matrices and their properties, the determinant function, and systems of equations. Sequences, series, and the Binomial Theorem are presented in Chapter 8, with combinatorics and probability in Chapter 9. Chapter 10 offers a unique introduction to higher mathematics and the nature of proof. The student is taught how to prove mathematical theorems in the framework of a college algebra course. This provides a smooth transition to more advanced mathematics.

The organization gives maximum flexibility in content, order, and emphasis in order to meet the wide-ranging mathematical needs of the students who enroll in a college algebra course. A logical, deductive approach is emphasized, beginning with a reasonable level of rigor; only those results that further the understanding of the topic are proved. Initially, many arguments are intuitive and many developments are inductive, but the notion of proof grows throughout the book. The final chapter is devoted to the nature of mathematical proof, but elements of this chapter may easily be included earlier as deemed appropriate. For example, the instructor may wish to cover the topic of mathematical induction in conjunction with the chapter on series and sequences.

Full advantage is taken of the availability of hand-held scientific calculators. We feel that the calculator is indispensable for topics such as exponential and logarithmic functions, but we have resisted adding calculator problems with "ugly numbers" simply for the sake of designating them "calculator problems." Calculators should be used as a natural extension of other available tools, such as a pencil and paper.

Answers to odd-numbered problems are included in the back of the book. An instructor's manual is available, which gives the answers to all the problems.

Our thanks go to the reviewers of this edition: William Anderson of East Texas State University, Elton Beougher of Fort Hays State University, Martin J. Brown of Jefferson Community College, J. S. Collins of Embry-Riddle University, Libby W. Holt of Florida Junior College, Harlan Koca of Washburn University, Fr. Micah E. Kozoil of Saint Vincent College, Helen Kriegsman of Pittsburg State University, and Wayne Mackey of Johnson County Community College. And our continued gratitude goes to the reviewers of the previous edition: James E. Arnold of the University of Wisconsin, David Barwick of Macon Junior College, Robert A. Chaffer of Central Michigan University, Jean Coover of West Virginia University, Allen Hesse of Rochester Community College, Joseph B. Hoffert of Drake University, William Perry of Texas A & M University, David Wend of Montana State University, and Charles R. Williams of Midwestern University.

Thanks also go to Phyllis Niklas, Joan Marsh, Carl Brown, Craig Barth, and Karen Sharp for all their contributions to the book. We especially appreciate the loving support of our wives, Linda and Theresa.

Karl J. Smith
Patrick J. Boyle

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*Section 10.5 does not require Sections 10.1–10.4.

CHAPTER 1

Numbers and Expressions

Gottlob Frege
(1848–1925)

The laws of number . . . are not the laws of nature . . . , they are the laws of the laws of nature.

Gottlob Frege,
Foundations of Science,
edited by N. Campbell, Dover, New York, 1957, p. 292.

Historical Note

Gottlob Frege was a German logician and mathematician who, in 1884, wrote a book on *The Fundamental Laws of Arithmetic* because he was not satisfied with its basic concepts. Have you ever tried to explain or define what we mean by the number five? Try it if you want an exercise in frustration. In his book, Frege attempted to put arithmetic on a sound logical basis and tried to answer questions such as "What is five?" His logical development depended heavily on the notion of "the set of all sets." After many years of work on this project, he received a letter from Bertrand Russell which contained Russell's famous "set of all sets paradox." This letter forced Frege to close his book with the statement: "A scientist can hardly encounter anything more undesirable than to have the foundation collapse just as the work is finished. I was put in this position by a letter from Mr. Bertrand Russell when the work was almost through the press."

The "set of all sets paradox" can be summarized by the following question: "The barber in a certain small town shaves those persons and only those persons who do not shave themselves. Who shaves the barber?"

Chapter Overview

This chapter will serve as a quick review for much of the material contained in a previous algebra course. The main thread tying the material together is the idea of exponents. Using positive integer exponents, we review polynomials, simplifying expressions, and factoring. Next, we review rational expressions by considering integral exponents. Finally, we allow exponents to be rational, which leads to a review of irrational expressions and radicals.

1.1 Sets of Numbers

You are probably familiar with the various sets of numbers used in mathematics, so they are simply summarized for you in Figure 1.1 and Table 1.1 to refresh your memory.

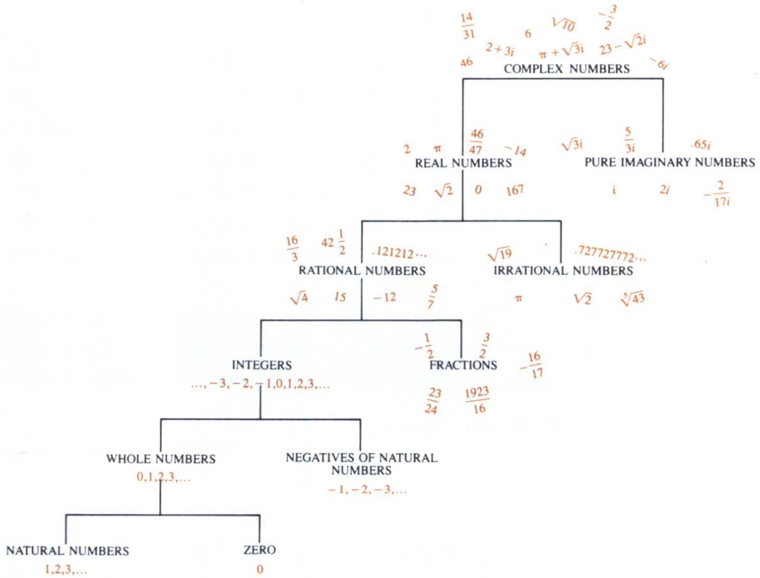


Figure 1.1

This course will focus attention on the set of real numbers, which is easily visualized by using a **coordinate system** called a **number line** (Figure 1.2). A **one-to-one correspondence** is established between all real numbers and all points on a number line.

Table 1.1
Sets of Numbers

Name	Symbol	Set
Counting numbers Natural numbers	**N**	{1, 2, 3, 4, . . .}
Whole numbers	**W**	{0, 1, 2, 3, 4, . . .}
Integers	**Z**	{. . . , −3, −2, −1, 0, 1, 2, 3, . . .}
Rational numbers	**Q**	Numbers that can be written in the form p/q, where p and q are integers with $q \neq 0$;* they are also characterized by numbers whose decimal representations either terminate or repeat
Irrational numbers	**Q′**	Numbers whose decimal representations do not terminate or repeat
Real numbers	**R**	Numbers that are either rational or irrational
Pure imaginary numbers	**I**	Numbers of the form bi, where b is a nonzero real number and $i^2 = -1$
Complex numbers	**C**	Numbers of the form $a + bi$, where a and b are real numbers and $i^2 = -1$

*Set-builder notation can be used for many of these definitions. For example, we can write

$\{p/q \mid p \text{ and } q \in \mathbf{Z}, q \neq 0\}$

for the definition of the set of rational numbers. It is read "set of all numbers p/q such that p and q are integers and q does not equal 0." A review of sets and set notation is given in Appendix A.

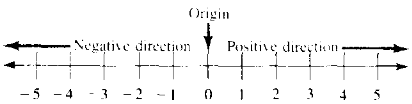


Figure 1.2

1. Every point on the line corresponds to precisely one real number.
2. For each real number, there corresponds one and only one point.

A point associated with a particular number is called the **graph** of that number. Numbers associated with points to the right of the origin are called **positive real numbers,** and those associated with points to the left are called **negative real numbers.**

Axioms concerning order of real numbers and formal definitions of inequality are considered in Chapter 10, but it is assumed that you are familiar with the following informal definitions:

$a = b$ Is read "a is equal to b" and means the graphs of both a and b are the same point.

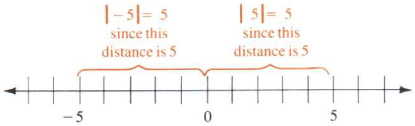
$a < b$ Is read "a is less than b" and means the graph of a is to the left of the graph of b on a number line.

$a > b$ Is read "a is greater than b" and means $b < a$.

$a \leq b$ Is read "a is less than or equal to b" and means $a = b$ or $a < b$.

$a \geq b$ Is read "a is greater than or equal to b" and means $a = b$ or $a > b$.

$|a|$ Is read "the absolute value of a" and means the distance between the graph of a and the origin. For example:



Example 1 **a.** $|2| = 2$ **b.** $|-2| = 2$ **c.** $|-\frac{2}{3}| = \frac{2}{3}$

d. $|\sqrt{5}| = \sqrt{5}$ **e.** $-|2| = -2$ **f.** $-|\pi| = -\pi$ □

Since we will want to use the notion of absolute value in a variety of contexts, we will now give an algebraic definition of absolute value that is equivalent to the geometric one using distances given above.

ABSOLUTE VALUE

The **absolute value** of a real number a is defined by

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Example 2 Find the absolute values using the algebraic definition.

a. $|2| = 2$ Since $2 \geq 0$

b. $|-2| = -(-2) = 2$ Since $-2 < 0$

c. $-|5| = -5$ Since $5 \geq 0$

d. $-|-46| = -[-(-46)] = -46$ Since $-46 < 0$

e. $|\pi - 3| = \pi - 3$ Since $\pi - 3 \geq 0$

f. $|3 - \pi| = -(3 - \pi) = -3 + \pi = \pi - 3$ Since $3 - \pi < 0$

g. $|\sqrt{20} - 4| = \sqrt{20} - 4$ By definition, $\sqrt{20}$ is the positive square root of 20. We know that $4^2 = 16$ and $5^2 = 25$, so the positive square root of 20 is between 4 and 5; this means $\sqrt{20} - 4$ is positive.

h. $|\sqrt{20} - 5| = -(\sqrt{20} - 5) = 5 - \sqrt{20}$ Since $\sqrt{20} - 5$ is negative (see the reasoning in part g) □

Some properties of absolute value will be considered later (in Section 2.3).

To begin our study of algebra, it is necessary to assume certain properties of equality and inequality. The first is called the **trichotomy property,** or **property of comparison.**

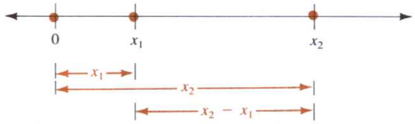
PROPERTY OF COMPARISON

> Given any two real numbers a and b, exactly one of the following holds:
>
> 1. $a = b$ 2. $a < b$ 3. $a > b$

This property relates to any two real numbers and tells you that, if you are given *any* two real numbers, either they are equal or one of them is greater than the other. It establishes order on the number line. We need this property to derive a formula for the distance between two points on a number line. Let P_1 and P_2 be any points on a number line with coordinates x_1 and x_2, respectively. This is usually denoted by $P_1(x_1)$ and $P_2(x_2)$. Then, by the property of comparison, we know that $x_1 = x_2$, $x_1 < x_2$, or $x_1 > x_2$. Consider these possibilities one at a time:

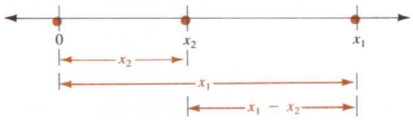
$x_1 = x_2$ The distance between P_1 and P_2 is 0.

$x_1 < x_2$



The distance is $x_2 - x_1$.

$x_1 > x_2$



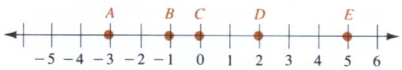
The distance is $x_1 - x_2$.

We can combine these different possibilities into one distance formula by using the idea of absolute value.

DISTANCE ON A NUMBER LINE

> The distance d between points $P_1(x_1)$ and $P_2(x_2)$ is
>
> $$d = |x_2 - x_1|$$

Example 3 Let A, B, C, D, and E be points with coordinates as shown on the number line:



a. Distance from D to E: $|5 - 2| = |3| = 3$

b. Distance from E to D: $|2 - 5| = |-3| = 3$

c. Distance from B to D: $|2 - (-1)| = |3| = 3$

d. Distance from B to A: $|-3 - (-1)| = |-2| = 2$

e. Distance from A to C: $|0 - (-3)| = |3| = 3$

f. Distance from A to D: $|2 - (-3)| = |5| = 5$

g. Distance from E to B: $|-1 - 5| = |-6| = 6$ □

When real numbers are added, the result is called the **sum** and the numbers are called **terms.** When real numbers are multiplied, the result is called the **product** and the numbers multiplied are called **factors.** The result from subtraction is called the **difference,** while that from division is called the **quotient.**

The real numbers, together with the relation of equality and the operations of addition and multiplication, satisfy the **field properties,** which are listed in the box.

FIELD PROPERTIES FOR THE SET **R** OF REAL NUMBERS

Let a, b, and c be real numbers.

	Addition Properties	**Multiplication Properties**
CLOSURE	$a + b$ is a unique real number	ab is a unique real number
COMMUTATIVE	$a + b = b + a$	$ab = ba$
ASSOCIATIVE	$(a + b) + c = a + (b + c)$	$(ab)c = a(bc)$
IDENTITY	There exists a unique real number 0 such that $a + 0 = 0 + a = a$	There exists a unique real number 1 such that $a \cdot 1 = 1 \cdot a = a$
INVERSE	For each real number a, there is a unique real number $-a$ such that $a + (-a) = (-a) + a = 0$	For each *nonzero* real number a, there is a unique real number $1/a$ such that $a\left(\frac{1}{a}\right) = \left(\frac{1}{a}\right)a = 1$
DISTRIBUTIVE	$a(b + c) = ab + ac$	

All these properties of real numbers will be important in our study of algebra, but for now we need to focus upon the commutative, associative, and distributive properties:

COMMUTATIVE	Properties of order
ASSOCIATIVE	Properties of grouping
DISTRIBUTIVE	Property that changes a product to a sum or changes a sum to a product

Example 4 Distinguish among the commutative, associative, and distributive properties.

a. $2 + (3 + 4) = (2 + 3) + 4$ Associative for addition

b. $2 + (3 + 4) = (3 + 4) + 2$ Commutative for addition

c. $2(3 + 4) = 2 \cdot 3 + 2 \cdot 4$ Distributive

d.	$ba + a(b + c) = ba + ab + ac$	Distributive
e.	$ba + a(b + c) = ba + a(c + b)$	Commutative for addition
f.	$ba + a(b + c) = ba + (b + c)a$	Commutative for multiplication
g.	$2\sqrt{3} + 5\sqrt{3} = (2 + 5)\sqrt{3}$	Distributive □

The remaining examples involve the other field properties.

Example 5 The set of natural numbers is closed for addition since

$m + n$ is a natural number

whenever m and n are natural numbers. On the other hand,

$2 - 5 = -3$

which is *not* a natural number, so the set of natural numbers is not closed for subtraction. □

Example 6 The set $\{0, 1\}$ is not closed for addition because

$1 + 1 = 2$

which is not a member of the set. On the other hand, it *is* closed for multiplication, because

$$0 \cdot 0 = 0 \qquad 1 \cdot 0 = 0$$
$$0 \cdot 1 = 0 \qquad 1 \cdot 1 = 1$$

which shows that all possible products are members of the set. □

Example 7 Identify the field property being used.

a.	$5 \cdot \left(6 \cdot \frac{1}{6}\right) = 5 \cdot 1$	Multiplicative inverse
b.	$5 \cdot 1 = 5$	Multiplicative identity
c.	$(a + 3)\left(\frac{1}{a + 3}\right) = 1, \quad a + 3 \neq 0$	Multiplicative inverse
d.	$(5 \cdot 6) \cdot \frac{1}{6} = 5 \cdot \left(6 \cdot \frac{1}{6}\right)$	Associative for multiplication
e.	$(a + 3) + [-(a + 3)] = 0$	Additive inverse
f.	$5 + [x + (-x)] = 5$	Additive inverse and additive identity □

Problem Set 1.1

Classify each statement in Problems 1–18 as true or false.

A

1. 0 is a natural number.

2. $\sqrt{5}$ is a real number.

3. $-.333\ldots$ is a complex number.

4. $-\frac{1}{4}$ is a complex number.

5. 3.1416 is a rational number.
6. π is a rational number.
7. π is a real number.
8. π is a complex number.
9. .777. . . is a rational number.
10. .121221222. . . is an irrational number.
11. .606606660. . . is a complex number.
12. $\sqrt{-2}$ is a real number.
13. The set of integers is closed with respect to addition.
14. The set of rational numbers is closed with respect to multiplication.
15. The set of irrational numbers is closed with respect to multiplication.
16. The set of real numbers is closed with respect to multiplication.
17. The set $\{-1, 0, 1\}$ is closed with respect to addition.
18. The set $\{-1, 0, 1\}$ is closed with respect to multiplication.
19. Plot the numbers given in Problems 1, 3, 5, 7, and 9 on a number line.
20. Plot the numbers given in Problems 2, 4, 6, 8, and 10 on a number line.

Write each statement in Problems 21–40 without the absolute value symbol.

21. $|17|$
22. $-|23|$
23. $|-23|$
24. $-|-10|$
25. $|\pi - 2|$
26. $|2 - \pi|$
27. $-|\pi - 6|$
28. $|\pi - 12|$
29. $|\sqrt{20} - 4|$
30. $|\sqrt{20} - 5|$
31. $|\sqrt{30} - 5|$
32. $|\sqrt{30} - 6|$
33. $|2\pi - 5|$
34. $|2\pi - 7|$
35. $|x + 3|$ if $x \geqslant 3$
36. $|x + 3|$ if $x < -3$
37. $|y - 5|$ if $y < 5$
38. $|y - 5|$ if $y \geqslant 5$
39. $|5 - 2s|$ if $s > 10$
40. $|4 + 3t|$ if $t < -10$

Find the distance between the pairs of points whose coordinates are given in Problems 41–48.

41. $A(3)$ and $B(21)$
42. $C(-4)$ and $D(-12)$
43. $E(12)$ and $F(-8)$
44. $G(-5)$ and $H(9)$
45. $I(\pi)$ and $J(3)$
46. $K(\pi)$ and $L(4)$
47. $M(\sqrt{5})$ and $N(2)$
48. $P(\sqrt{5})$ and $Q(3)$

B *Identify the property illustrated by each statement in Problems 49–64.*

49. $5(6 + 2) = 5 \cdot 6 + 5 \cdot 2$
50. $4 + 8 = 8 + 4$
51. $15 + (85 + 23) = (85 + 23) + 15$
52. $35 \cdot 1 = 35$
53. $15 + (85 + 23) = (15 + 85) + 23$
54. $35 \cdot \frac{1}{35} = 1$