



MODERN ROBOTICS

MECHANICS, PLANNING,
AND CONTROL

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Mechanics, Planning, and Control

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Modern Robotics

Mechanics, Planning, and Control

This introduction to robotics offers a distinct and unified perspective of the mechanics, planning, and control of robots. It is ideal for self-learning, or for courses, as it assumes only freshman-level physics, ordinary differential equations, linear algebra, and a little bit of computing background. Modern Robotics:

- Presents the state-of-the-art screw-theoretic techniques capturing the most salient physical features of a robot in an intuitive geometrical way;
- Includes numerous exercises at the end of each chapter;
- Has accompanying, freely-downloadable software written to reinforce book concepts;
- Provides freely-downloadable video lectures aimed at changing the classroom experience, which students can watch in their own time, whilst class time is focused on collaborative problem-solving;
- Offers instructors the opportunity to design both one- and two-semester courses tailored to emphasize a range of topics, such as kinematics of robots and wheeled vehicles, kinematics and motion planning, mechanics of manipulation, and robot control;
- Can be used either with courses or for self-learning.

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Foreword by Roger Brockett

In the 1870s, Felix Klein was developing his far-reaching Erlangen Program, which cemented the relationship between geometry and group theoretic ideas. With Sophus Lie's nearly simultaneous development of a theory of continuous (Lie) groups, important new tools involving infinitesimal analysis based on Lie algebraic ideas became available for the study of a very wide range of geometric problems. Even today, the thinking behind these ideas continues to guide developments in important areas of mathematics. Kinematic mechanisms are, of course, more than just geometry; they need to accelerate, avoid collisions, etc., but first of all they are geometrical objects and the ideas of Klein and Lie apply. The groups of rigid motions in two or three dimensions, as they appear in robotics, are important examples in the work of Klein and Lie.

In the mathematics literature the representation of elements of a Lie group in terms of exponentials usually takes one of two different forms. These are known as exponential coordinates of the first kind and exponential coordinates of the second kind. For the first kind one has $X = e^{(A_1x_1 + A_2x_2 \dots)}$. For the second kind this is replaced by $X = e^{A_1x_1}e^{A_2x_2} \dots$. Up until now, the first choice has found little utility in the study of kinematics whereas the second choice, a special case having already shown up in Euler parametrizations of the orthogonal group, turns out to be remarkably well-suited for the description of open kinematic chains consisting of the concatenation of single degree of freedom links. This is all nicely explained in Chapter 4 of this book. Together with the fact that $Pe^AP^{-1} = e^{PAP^{-1}}$, the second form allows one to express a wide variety of kinematic problems very succinctly. From a historical perspective, the use of the product of exponentials to represent robotic movement, as the authors have done here, can be seen as illustrating the practical utility of the 150-year-old ideas of the geometers Klein and Lie.

In 1983 I was invited to speak at the triennial Mathematical Theory of Networks and Systems Conference in Beer Sheva, Israel, and after a little thought I decided to try to explain something about what my recent experiences had taught me. By then I had some experience in teaching a robotics course that discussed kinematics, including the use of the product of exponentials representation of kinematic chains. From the 1960s onward e^{At} had played a central role in system theory and signal processing, so at this conference a familiarity, even an affection, for the matrix exponential could be counted on. Given this, it was

natural for me to pick something e^{Ax} -related for the talk. Although I had no reason to think that there would be many in the audience with an interest in kinematics, I still hoped that I could say something interesting and maybe even inspire further developments. The result was the paper referred to in the preface that follows.

In this book, Frank and Kevin have provided a wonderfully clear and patient explanation of their subject. They translate the foundation laid out by Klein and Lie 150 years ago to the modern practice of robotics, at a level appropriate for undergraduate engineers. After an elegant discussion of the fundamental properties of configuration spaces, they introduce the Lie group representations of rigid-body configurations, and the corresponding representations of velocities and forces, used throughout the book. This consistent perspective is carried through foundational robotics topics including the forward, inverse, and differential kinematics of open chains, robot dynamics, trajectory generation, robot control, and more specialized topics such as the kinematics of closed chains, motion planning, robot manipulation, planning and control for wheeled mobile robots, and the control of mobile manipulators.

I am confident that this book will be a valuable resource for a generation of students and practitioners of robotics.

Roger Brockett
Cambridge, Massachusetts, USA
November 2016

Foreword by Matthew Mason

Robotics is about turning ideas into action. Somehow, robots turn abstract goals into physical action: sending power to motors, monitoring motions, and guiding things towards the goal. Every human can perform this trick, but it is nonetheless so intriguing that it has captivated philosophers and scientists, including Descartes and many others.

What is the secret? Did some roboticist have a eureka moment? Did some pair of teenage entrepreneurs hit on the key idea in their garage? To the contrary, it is not a single idea. It is a substantial body of scientific and engineering results, accumulated over centuries. It draws primarily from mathematics, physics, mechanical engineering, electrical engineering and computer science, but also from philosophy, psychology, biology and other fields.

Robotics is the gathering place of these ideas. Robotics provides motivation. Robotics tests ideas and steers continuing research. Finally, robotics is the proof. Observing a robot's behavior is the nearly compelling proof that machines can be aware of their surroundings, can develop meaningful goals, and can act effectively to accomplish those goals. The same principles apply to a thermostat or a fly-ball governor, but few are persuaded by watching a thermostat. Nearly all are persuaded by watching a robot soccer team.

The heart of robotics is motion – controlled programmable motion – which brings us to the present text. *Modern Robotics* imparts the most important insights of robotics: the nature of motion, the motions available to rigid bodies, the use of kinematic constraint to organize motions, the mechanisms that enable general programmable motion, the static and dynamic character of mechanisms, and the challenges and approaches to control, programming, and planning motions. *Modern Robotics* presents this material with a clarity that makes it accessible to undergraduate students. It is distinguished from other undergraduate texts in two important ways.

First, in addressing rigid-body motion, *Modern Robotics* presents not only the classical geometrical underpinnings and representations, but also their expression using modern matrix exponentials, and the connection to Lie algebras. The rewards to the students are two-fold: a deeper understanding of motion, and better practical tools.

Second, *Modern Robotics* goes beyond a focus on robot mechanisms to address the interaction with objects in the surrounding world. When robots make contact

with the real world, the result is an *ad hoc* kinematic mechanism, with associated statics and dynamics. The mechanism includes kinematic loops, unactuated joints, and nonholonomic constraints, all of which will be familiar concepts to students of *Modern Robotics*.

Even if this is the only robotics course students take, it will enable them to analyze, control, and program a wide range of physical systems. With its introduction to the mechanics of physical interaction, *Modern Robotics* is also an excellent beginning for the student who intends to continue with advanced courses or with original research in robotics.

Matthew T. Mason

Pittsburgh, Pennsylvania, USA

November 2016

Preface

It was at the IEEE International Conference on Robotics and Automation in Pasadena in 2008 when, over a beer, we decided to write an undergraduate textbook on robotics. Since 1996, Frank had been teaching robot kinematics to Seoul National University undergraduates using his own lecture notes; by 2008 these notes had evolved to the kernel around which this book was written. Kevin had been teaching his introductory robotics class at Northwestern University from his own set of notes, with content drawn from an eclectic collection of papers and books.

We believe that there is a distinct and unifying perspective to the mechanics, planning, and control for robots that is lost if these subjects are studied independently or as part of other more traditional subjects. At the 2008 meeting, we noted the lack of a textbook that (a) treated these topics in a unified way, with plenty of exercises and figures, and (b), most importantly, was written at a level appropriate for a first robotics course for undergraduates with only freshman-level physics, ordinary differential equations, linear algebra, and a little bit of computing background. We decided that the only sensible recourse was to write such a book ourselves. (We didn't know then that it would take us more than eight years to finish the project!)

A second motivation for this book, and one that we believe sets it apart from other introductory treatments on robotics, is its emphasis on modern geometric techniques. Often the most salient physical features of a robot are best captured by a geometric description. The advantages of the geometric approach have been recognized for quite some time by practitioners of classical screw theory. What has made these tools largely inaccessible to undergraduates – the primary target audience for this book – is that they require an entirely new language of concepts and constructs (screws, twists, wrenches, reciprocity, transversality, conjugacy, etc.), and their often obscure rules for manipulation and transformation. However, the mostly algebraic alternatives to screw theory often mean that students end up buried in the details of calculation, losing the simple and elegant geometric interpretation that lies at the heart of what they are calculating.

The breakthrough that made the techniques of classical screw theory accessible to a more general audience arrived in the early 1980s, when Roger Brockett showed how to describe kinematic chains mathematically in terms of the Lie group structure of rigid-body motions (Brockett, 1983b). This discovery allowed

one, among other things, to re-invent screw theory simply by appealing to basic linear algebra and linear differential equations. With this “modern screw theory” the powerful tools of modern differential geometry can be brought to bear on a wide-ranging collection of robotics problems, some of which we explore here, and others of which are covered in the excellent but more advanced graduate textbook by Murray et al. (1994).

As the title indicates, this book covers what we feel to be the fundamentals of robot mechanics, together with the basics of planning and control. A thorough treatment of all the chapters would likely take two semesters, particularly when coupled with programming assignments or experiments with robots. The contents of Chapters 2–6 constitute the minimum essentials, and these topics should probably be covered in sequence.

The instructor can then selectively choose content from the remaining chapters. At Seoul National University, the undergraduate course M2794.0027 Introduction to Robotics covers, in one semester, Chapters 2–7 and parts of Chapters 10–12. At Northwestern, ME 449 Robotic Manipulation covers, in an 11-week quarter, Chapters 2–6 and 8, and then touches on different topics in Chapters 9–13 depending on the interests of the students and instructor. A course focusing on the kinematics of robot arms and wheeled vehicles could cover Chapters 2–7 and 13, while one on kinematics and motion planning could additionally include Chapters 9 and 10. A course on the mechanics of manipulation would cover Chapters 2–6, 8, and 12, while another on robot control would cover Chapters 2–6, 8, 9, and 11. If the instructor prefers to avoid dynamics (Chapter 8), the basics of robot control (Chapters 11 and 13) can be covered by assuming that the velocity at each actuator is controlled, not the forces and torques. A course focusing only on motion planning could cover Chapters 2 and 3, Chapter 10 in depth (possibly supplemented by research papers or other references cited in that chapter), and Chapter 13.

To help the instructor choose which topics to teach and to help the student keep track of what she has learned, we have included summaries at the ends of chapters and a summary of important notation and formulas used throughout the book in Appendix A. For those whose primary interest in this text is as an introductory reference, we have attempted to provide a reasonably comprehensive, though by no means exhaustive, set of references and bibliographic notes at the end of each chapter. Some of the exercises provided at the end of each chapter extend the basic results covered in the book and, for those who wish to probe further, these should be of interest in their own right. Some of the more advanced material in the book can be used to support independent study projects.

Another important component of the book is the software, which is written to reinforce the concepts in the book and to make the formulas operational. The software was developed primarily by Kevin’s ME 449 students at Northwestern and is freely downloadable from <http://modernrobotics.org>. Video lectures that accompany the textbook will also be available at the website. The intention

of the video content is to help the instructor to “flip” the classroom: students watch brief lectures on their own time, rewinding as needed, and class time is focused more on the collaborative problem-solving that has traditionally occurred between classes. This way, the professor is present when the students are applying the material and discovering the gaps in their understanding; this creates the opportunity for interactive mini-lectures addressing the concepts that need most reinforcing. We believe that the added value of the professor is greatest in this interactive role, not in delivering a lecture in the same way that it was delivered the previous year. This approach has worked well for Kevin’s introduction to mechatronics course, <http://nu32.org>.

The video content is generated using the Lightboard, <http://lightboard.info>, created by Michael Peshkin at Northwestern University. We thank him for sharing this convenient and effective tool for creating instructional videos.

We have also found the V-REP robot simulation software to be a valuable supplement to the book and its software. This simulation software allows students to explore interactively the kinematics of robot arms and mobile manipulators and to animate trajectories that are the result of exercises on kinematics, dynamics, and control.

While this book presents our own perspective on how to introduce the fundamental topics in first courses on robot mechanics, planning, and control, we acknowledge the excellent textbooks that already exist and that have served our field well. Among these, we would like to mention as particularly influential the books by Murray et al. (1994), Craig (2004), Spong et al. (2005), Siciliano et al. (2009), Mason (2001), and Corke (2017), and the motion planning books by Latombe (1991), LaValle (2006), and Choset et al. (2005). In addition, the *Handbook of Robotics* (Siciliano and Khatib, 2016), with a multimedia extension edited by Kröger (<http://handbookofrobotics.org>), is a landmark in our field, collecting the perspectives of hundreds of leading researchers on a huge variety of topics relevant to modern robotics.

It is our pleasure to acknowledge the many people who have been the sources of help and inspiration in writing this book. In particular, we would like to thank our Ph.D. advisors, Roger Brockett and Matt Mason. Brockett laid down much of the foundation for the geometric approach to robotics employed in this book. Mason’s pioneering contributions to analysis and planning for manipulation form a cornerstone of modern robotics. We also thank the many students who provided feedback on various versions of this material, in M2794.0027 at Seoul National University and in ME 449 at Northwestern University. In particular, Frank would like to thank Seunghyeon Kim, Keunjun Choi, Jisoo Hong, Jinkyu Kim, Youngsuk Hong, Wooyoung Kim, Cheongjae Jang, Taeyoon Lee, Soocheol Noh, Kyumin Park, Seongjae Jeong, Sukho Yoon, Jaewoon Kwen, Jinhyuk Park, and Jihoon Song, as well as Jim Bobrow and Scott Ploen from his time at UC Irvine. Kevin would like to thank Matt Elwin, Sherif Mostafa, Nelson Rosa, Jarvis Schultz, Jian Shi, Mikhail Todes, Huan Weng, and Zack Woodruff.

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Kevin M. Lynch
Evanston, Illinois, USA

Frank C. Park
Seoul, Korea

November 2016

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The authors consider themselves to be equal contributors to this book. The author order is alphabetical.

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1 Preview

As an academic discipline, robotics is a relatively young field with highly ambitious goals, the ultimate one being the creation of machines that can behave and think like humans. This attempt to create intelligent machines naturally leads us first to examine ourselves – to ask, for example, why our bodies are designed the way they are, how our limbs are coordinated, and how we learn and perform complex tasks. The sense that the fundamental questions in robotics are ultimately questions about ourselves is part of what makes robotics such a fascinating and engaging endeavor.

Our focus in this book is on mechanics, planning, and control for **robot mechanisms**. Robot arms are one familiar example. So are wheeled vehicles, as are robot arms mounted on wheeled vehicles. Basically, a mechanism is constructed by connecting rigid bodies, called **links**, together by means of **joints**, so that relative motion between adjacent links becomes possible. **Actuation** of the joints, typically by electric motors, then causes the robot to move and exert forces in desired ways.

The links of a robot mechanism can be arranged in serial fashion, like the familiar open-chain arm shown in Figure 1.1(a). Robot mechanisms can also have links that form closed loops, such as the Stewart–Gough platform shown in Figure 1.1(b). In the case of an open chain, all the joints are actuated, while in the case of mechanisms with closed loops, only a subset of the joints may be actuated.

Let us examine more closely the current technology behind robot mechanisms. The links are moved by actuators, which typically are electrically driven (e.g., by DC or AC motors, stepper motors, or shape memory alloys) but can also be driven by pneumatic or hydraulic cylinders. In the case of rotating electric motors, these would ideally be lightweight, operate at relatively low rotational speeds (e.g., in the range of hundreds of RPM), and be able to generate large forces and torques. Since most currently available motors operate at low torques and at up to thousands of RPM, speed reduction and torque amplification are required. Examples of such transmissions or transformers include gears, cable drives, belts and pulleys, and chains and sprockets. These speed-reduction devices should have zero or low slippage and **backlash** (defined as the amount of rotation available at the output of the speed-reduction device without motion at