

NATHAN IDA ■ JOÃO P.A. BASTOS

ELECTRO- MAGNETICS AND CALCULATION OF FIELDS

Second Edition

电磁学和场的计算

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Preface

This second edition of *Electromagnetics and Calculation of Fields* was undertaken with the intention of both updating the first edition and, most importantly, introducing the new subject of edge or vector finite elements. We felt that edge elements have matured enough to be included in a text of this type. In addition, the original text was expanded to include an introduction to the finite element method as a general design tool with in-depth discussion of the method's practical aspects, before its application to specific electromagnetic phenomena is introduced. Many of the features of the previous edition as well as the general appearance and flavor were retained while the coverage was extended and the presentation improved.

The text is intended as an introduction to electromagnetics and computation of electromagnetic fields. While many texts on electromagnetics exist, the subject of computation of electromagnetic fields is normally not treated or is treated in a number of idealized examples, with the main emphasis on development of theoretical relations.

"Why another book on electromagnetics?" This is perhaps the first question the reader may ask when opening this book. It is a valid question, because among the many books on electromagnetics some are excellent. We have two answers to this question, answers that have motivated the writing of this book.

The first concerns the method of presentation of electromagnetism. Generally, in classical books the material is presented in the following sequence: electrostatics, magnetostatics, magnetodynamics, and wave propagation, using integral forms of the field equations. As a primary effect of this presentation, the reader is led to think that the knowledge of this science is synonymous with memorizing dozens of formulas. Additionally, an impression that there is no firm connection between these equations lingers in the reader's mind since at each step new postulates are added, seemingly unrelated to previous material. Our opinion is, and we shall try to convey this to the reader, that the electromagnetic formalism is extremely simple and based on very few equations. They are the four "Maxwell equations" which include practically all the existent relationships between the electromagnetic quantities. The only additional relationships that need be considered is the Lorentz force and the material constitutive relations.

Maxwell's four equations are presented in the point, or local form. This treatment provides a powerful tool, because it is valid for any situation and is independent of geometry. When we wish to apply them to a real device, it is more practical to use the integral form of the equations, because applications lead quite

naturally to the notions of line, surface, and volume. In this formalism, the "dozens of formulae" associated with the classical methods of presentation are particular cases of Maxwell's equations as applied to specific geometries. Thus, it seems to us to be more useful to present the Electromagnetic theory based on a few equations, together with the tools needed for their general use. We will show that the electrostatic, magnetostatic, and magnetodynamic applications are particular cases of these four equations. Under certain conditions, Maxwell's equations can be decoupled into two principal groups (electrostatic and magnetic) which can be studied, and used, independently. However, while the electromagnetic formalism is simple, it does not mean that all its phenomena are necessarily easy to solve. There are many complex problems whose solutions are the subject of extensive current inquiry. However, this does not prevent one from trying to present the Electromagnetic theory in a simple, generally applicable way.

Many maintain that this is a science with a definite theoretical character. This is certainly true, but we will try to emphasize its applied aspects associated with a variety of applications. However, it is not an exercise book. There are many good books dedicated exclusively to problem solving. The final sections of most chapters contain a few, carefully chosen examples. These are presented with comments, in order to complement and simplify the understanding of the theoretical material presented in the chapter.

The facts discussed above have motivated the writing of the first part of the book, titled "The Electromagnetic Field and Maxwell's Equations." This part is divided into seven chapters.

In the first chapter we present the mathematical basis necessary to work with the Maxwell's equations in integral and differential forms.

Chapter 2 describes the common basis of electromagnetics, or Maxwell's equations, with special attention to the conditions under which they can be decoupled into two independent groups of equations: one for electrostatic and the other for magnetic phenomena.

In Chapter 3, the electrostatic field equations are studied. The electric field intensity, dielectric strength, and capacitance are presented. In its final section, the finite element method is described, with the goal of proposing a solution of the Laplace equation related to the electric field.

The fourth chapter treats the second group of the decoupled Maxwell equations: the magnetostatic field equations. Here the static aspects of magnetism is presented. The Biot-Savart law, magnetic materials, the notion of inductance, etc. are presented. Permanent magnets are discussed in detail, because of their importance in design of high-performance magnetic devices.

Chapter 5 deals with magnetodynamic phenomena. Field penetration in conductors, eddy currents, eddy current losses, hysteresis losses, and induced currents due to movement are discussed.

In Chapter 6, we discuss the interactions between electromagnetic and mechanical quantities. Among these, we mention here the force on conductors, force on charges (Lorentz force), magnetic field energy, and the Poynting vector.

Special attention is given to Maxwell's force tensor. This aspect of electromagnetic fields is rarely presented in classical books. It is very useful, because it permits, in a simple and elegant way, the calculation of the force exerted on a body by the magnetic field on the surface enclosing the body. We have a particular interest in calculation of forces using Maxwell's force tensor because of its usefulness in numerical calculation of forces.

Chapter 7 deals with what we call high-frequency electromagnetic fields and wave propagation aspects. The distinction between low and high-frequencies is not on frequency but rather on displacement currents. High-frequency applications are those in which displacement currents cannot be neglected, regardless of frequency. Wave propagation, scattering, reflection, transmission, and refraction of plane waves are discussed first, followed by fields in waveguides and cavity resonators. Propagation in dielectrics, lossy dielectrics, and conductors are also presented.

The second answer to the question posed above relates to the second part of the book, entitled "Introduction to the Finite Element Method in Electromagnetics."

The analysis and design of an electrical device requires the knowledge of the electromagnetic phenomena throughout the device. The difficulty of solving Laplace's and Poisson's equations for electrical and magnetic fields was an enormous obstacle in achieving this important goal for a long time. The solution of second order partial differential equations is a special domain in applied mathematics. Methods of solution were limited to a few simple cases until very recently. By 1950, the finite elements method began to emerge as a general method for solution of mechanical engineering problems. Around 1970 this method began to be adapted to electrical engineering applications. The methods employed prior to this, either empirical or approximate, were not satisfactory, especially when the structure had a complex geometry. The use of the finite element method opened a new era in applied electromagnetism and, as a consequence, in electrical engineering, which had important needs in computer-aided design tools. The knowledge of field distribution permits the rational design construction of structures efficiently, economically, and within safety standards.

One of the principal goals of this work is to present the Finite Element method and its application to electromagnetic problems. The didactic approach used here is quite different than in other books on the subject: we present the problems to be solved and then the application of the finite elements to these problems. This means that the principal goal of this book is to solve electromagnetic problems, and the use of finite elements technique is the tool of choice in achieving this end. In this sense, this part of the book can be viewed as a tutorial on problem solving using the finite element method. Although problem solving is the major aspect, the material is presented in detailed fashion to allow the reader to form a firm basis in this important aspects of applied electromagnetics.

The second part of the book is divided into six chapters. The main objective of this part is the didactic presentation of the finite element method which by now has

become an indispensable method for analysis and design of electromagnetic devices.

In Chapter 8, we introduce the finite element method as a general engineering design tool in preparation for specific applications in the following chapters. The concepts associated with the finite element method are discussed in depth and are accompanied by simple examples. A small finite element program for second order elements is included to show how the concepts are linked to practical implementation.

In Chapter 9, we present the variational method. This theoretical formulation, associated with finite elements, constitutes a well-adapted method to solve static problems. The axi-symmetric formulation, the Newton-Raphson method (for nonlinear problems) as well as several examples are described in this chapter.

Chapter 10 introduces Galerkin's residual method as it relates to time dependent applications. Two-dimensional (2D) and three-dimensional (3D) finite elements and their application are shown.

Chapter 11 introduces the concept of edge elements. The hexahedral edge element shape functions are presented in a didactic manner and subroutines showing how they are constructed are listed. The application of edge elements to static 3D and eddy current problems is explained and graphical results are provided to show their utility.

In Chapter 12, we describe several computational aspects associated with the implementation of Finite Element programs, with special attention to the matrix system, to memory use, and to the solution methods needed to solve the system of equations obtained. Iterative, direct, and eigenvalue problems are discussed.

In the thirteenth and last chapter, many aspects of the general organization of field calculation software are discussed. Pre- and post-processing methods (such as mesh generation and equipotential lines drawing) are described. The more modern systems (extension of traditional software packages) and trends in this research field are also discussed in the final sections of this chapter.

Finally, we would like to thank R. Carlson, B. Davat, R. Mesquita, G. Meunier, A. Raizer, C. Rioux, and N. Sadowski who have, in different ways, contributed to the realization of this work. Special thanks goes to many others, and in particular students, who pointed out errors, misspellings, and possible ways of improving the presentation.

N. Ida
J. P. A. Bastos
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1

Mathematical Preliminaries

1.1. Introduction

In this chapter we review a few ideas from vector algebra and calculus, which are used extensively in future chapters. We assume that operations like integration and differentiation as well as elementary vector calculus operations are known.

This chapter is written in a concise fashion, and therefore, only those subjects directly applicable to this work are included. Readers wishing to expand on material introduced here can do so by consulting specialized books on the subject. It should be emphasized that we favor the geometrical interpretation rather than complete, rigorous mathematical exposition.

We look with particular interest at the ideas of gradient, divergence, and curl as well as at the divergence and Stokes' theorems. These notions are of fundamental importance for the understanding of electromagnetic fields in terms of Maxwell's equations. The latter are presented in local or point form in this work.

1.2. The Vector Notation

Many physical quantities possess an intrinsic vector character. Examples are velocity, acceleration, and force, with which we associate a direction in space. Other quantities, like mass and time, lack this quality. These are scalar quantities. Another important concept is the vector field. A force, applied at a point on a body is a vector; however, the velocity of a gas inside a tube is a vector defined throughout a region (i.e., the cross-section of the tube, or a volume), not only at one point. In the latter case, we have a vector field. We use this concept extensively since many of the electromagnetic quantities (electric and magnetic fields, for example) are vector fields.