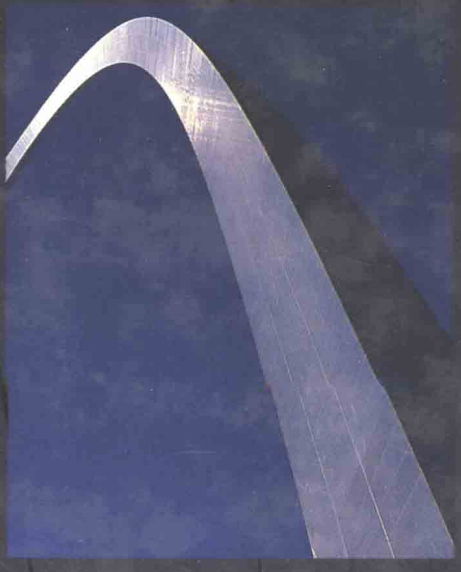
CALCULUS
WITH APPLICATIONS

SECOND EDITION



HARSHBARGER
REYNOLDS

CALCULUS

WITH APPLICATIONS

SECOND EDITION

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A Note to Instructors and Students About Custom Publishing

Applied calculus courses are offered nationally in a variety of formats. Courses can be one or two semesters (or two to four quarters) in length and cover a wide range of topics. To meet the diverse requirements of these courses, publishers have traditionally offered two versions of their applied calculus textbooks: one short version containing just those core topics covered in virtually all courses, and another, long version containing this same core subject matter plus all the additional topics covered in the lengthiest courses. The unfortunate, but frequent result: students must purchase a book containing chapters that will not be assigned.

D. C. Heath's solution is to offer *Calculus with Applications*, Second Edition, in a custom-published format, allowing instructors to build the texts most appropriate for their courses and helping students to save money. With this format, an instructor can request that we combine any or all of the last four chapters (on differential equations, trigonometric functions, probability, and infinite series and Taylor polynomials) with the beginning eight core chapters (which include an algebra review and the topics most often covered in all courses) to create the text best suited for his or her course. And pricing is structured so that students are charged only for those chapters included in their particular books. The result: instructors select—and students pay for—only those chapters they will use.

We at D. C. Heath are working hard to provide innovative, quality products for your courses. We value your feedback.

The Publisher

PREFACE

Calculus with Applications, Second Edition, presents mathematical skills and concepts for a brief calculus course and applies them to areas that are important to students in the management, life, and social sciences. The emphasis on applications allows students to view mathematics in a practical setting relevant to their intended careers. Almost every chapter of this book includes a section or two devoted to the applications of mathematical topics. An index of these applications on the inside covers demonstrates the wide variety used in examples and exercises. Although intended for students who have completed two years of high-school algebra or the equivalent, this text begins with a brief review of algebra, which, if covered, will aid in preparing students for the work ahead.

NEW FEATURES OF THE SECOND EDITION

Technology Corners Most sections of the text contain Technology Corners for which students can use a graphics calculator and/or a computer to solve more advanced applied problems, to study the concepts of the section, or to discover new relationships.

Consultant's Corners At the end of each chapter, Consultant's Corners pose real or realistic business problems; and, like real problems, they require students, as consultants, to provide several answers based on different conditions. Students will give advice based on a mathematical analysis of a problem and make decisions about the best way to solve it. Consultant's Corners can be assigned as projects requiring a written report or as discussion items for class. The questions are not connected to a specific section of the text, but are intended to have the consultant draw different mathematical skills and concepts together to solve a problem.

Checkpoints Each section of the text contains Checkpoint problems that permit students to check their understanding of the skills and concepts under discussion before proceeding farther in the section. Solutions to the Checkpoint problems are given at the end of the section.

Key Terms and Formulas Each chapter contains a summary of Key Terms and Formulas from that chapter. The terms and formulas are keyed to the sections in which they occur so that students can easily review any topic as needed.

Labeled Applications Each application problem in the exercise sets has a label identifying its subject matter. This permits the instructor to more easily assign application problems that are appropriate for the students in his or her class.

Computer-Generated Art To ensure the accuracy of the figures and graphs, the art has been computer-generated. The graphs and axes appear in the same color, eliminating the possibility of inaccurate graphs caused by color registration problems during printing.

Precise Mathematical Language The Second Edition continues its emphasis on concepts, problem solving, and applications rather than on formal proofs. Although the text retains its clarity and readability, the mathematical language has been honed for more precision in the Second Edition, with definitions and theorems stated accurately but in language that students can understand.

Pedagogical Features Retained in the Second Edition

Separate Application Sections Offering applied topics such as cost, revenue, and profit functions in separate sections brings the preceding mathematical discussion into clear and concise focus. There are eight such sections in the book. In all, there are more than 1260 application problems and hundreds of applied examples.

Exercise Sets The quantity of the exercise sets has been increased by more than 17% in the Second Edition. The new problems are graded better, are more challenging, and are of a wider variety. A number of problems are drawn from real-life data and are documented by references. As in the previous edition, those exercises that are best worked with a calculator are highlighted with a ▦ symbol.

Warmups The Warmup at the beginning of each chapter invites students to test themselves on the skills needed for that chapter. They present several prerequisite problem types that are taken from parts of upcoming problems. Each prerequisite problem type is keyed to the upcoming section where that skill is needed, and students who have difficulty with any particular skill are directed to specific sections of the text for review. Instructors may find the Warmups useful in creating a syllabus.

Objectives Every section begins with a brief list of objectives that outlines the goals of that section for the student.

Procedure/Example and **Property/Example Tables** Placed where helpful, these tables aid student understanding by giving step-by-step descriptions of important procedures and properties with illustrative examples worked out beside them.

Boxed Information All definitions and other important information are boxed for easy reference, and key terms are highlighted in boldface.

Review Exercises At the end of each chapter, a set of Review Exercises offers students extra practice on the topics in that chapter. These exercises are annotated with section numbers so that students having difficulty can turn to the appropriate section for review.

Content Changes in the Second Edition

Significant changes within the chapters of the Second Edition include the following.

In Chapter 0, Sections 0.1 and 0.2 from the First Edition have been combined, and a discussion of intervals as special subsets of the real numbers, as well as interval notation has been added. In Section 0.6, Algebraic Fractions, we discuss complex fractions. Also, in Section 0.7, we have expanded our treatment of the solution of linear equations to include the solution of linear inequalities.

In Chapter 1, we rewrote our discussion of functions and graphs and expanded it to two sections. To the original section, we added operations with functions, including function composition. In the new Section 1.5, we look at simple polynomial, rational, and piecewise defined functions and at the special features of their graphs.

We have written a more precise presentation of limits, continuity, and the derivative in the first three sections of Chapter 2. We have moved limits involving piecewise defined functions out of Section 2.1 and have used graphs to a greater extent in the remaining material, all in an effort to make the introduction to limits more easily understood and to join the geometric and algebraic concepts of limits. There is also a greater emphasis on graphs in our discussion of continuity in Section 2.2, and this section now contains the material on limits of piecewise defined functions. Our discussion of asymptotes has been moved to a new section in Chapter 3 that covers both asymptotes and graphing. In Section 2.3, we introduce instantaneous rates of change with an example involving velocity and follow this with the discussion of marginal revenue, so that students first encounter the idea of derivative through a concept from their personal experience. This section also includes a new Procedure/Example table that clearly outlines how to use the definition of derivative to find the derivative of a function. Finally, throughout the chapter, we have increased our mathematical precision with carefully stated theorems, more proofs of the derivative formulas, and more justification in our discussions.

We have completely reorganized Chapter 3 to cover curve sketching using the first and second derivatives in Sections 3.1 and 3.2. These discussions include more justification of theory and make a stronger connection between information from the derivatives and the shape of the graph. In particular, we have introduced sign diagrams for the derivatives and mainstreamed our discussion of critical points where $f'(x)$ does not exist. Section 3.3 is a new section that covers asymptotes and curve sketching and acts as a summary section for Sections 3.1 and 3.2.

Also in Chapter 3, inventory-cost models are now discussed in the text of Section 3.5. Section 3.6 is a new section devoted to differentials. Section 3.7 now covers implicit differentiation and elasticity, followed by related rates in Section 3.8.

In Chapter 5, we have added the change of base formula to our development of logarithmic functions. In Sections 5.3 and 5.4, we have focused our discussion and theoretical development on the natural logarithm and the natural exponential functions, although derivatives of logarithmic and exponential functions with other bases are still included.

In Chapter 6, Section 6.1, we have expanded our work with a fixed number of subdivisions to approximate the area under a curve. Also, in Section 6.3, the exercises on finding areas between two curves have been reworked so that they are more interesting and will help students to conceptualize the topic better.

Chapter 7 now includes a new discussion of linear regression. This discussion introduces a technique important for many business applications, and the formula development provides an application of the max-min techniques for functions of two variables.

Chapter IS has a new section on sequences. We have also expanded our discussion of partial sums, with special emphasis on geometric series.

SUPPLEMENTS

Instructor's Guide This guide contains four forms of a test for each chapter of the text with answers provided, solutions to the even-numbered text exercises, and transparency masters. In addition, a section containing the answers to the even-numbered text exercises is included.

Study and Solutions Guide by Gordon Shilling. In addition to the solutions to all the odd-numbered exercises in the text, this guide contains supplementary exercises that reinforce the concepts and techniques presented in the text. Answers to these problems are also provided.

Computerized Testing Computerized testing for the IBM PC and the Apple Macintosh is available to instructors free of charge. This test bank contains more than 1600 test items.

Test Item File This is a printed file of all the test items and answers appearing in the computerized testing program.

Interactive Applied Calculus by The Math Lab. For use with the IBM PC or Apple II, this software program provides labwork for business, life, and social science majors. A total of 40 labs are provided, allowing instructors to select those labs appropriate for their use. Eight of the labs are related to business and economic applications, while many others model real-life business problems and situations.

BestGrapher by George Best. Available for the IBM PC and the Apple Macintosh, this highly accessible program and its accompanying workbook emphasize the concepts of calculus. The software can be used for graphing, evaluating functions, and symbolic differentiation among other tasks. The workbook provides activities that encourage experimentation and discovery about functions, derivatives, areas, and other topics.

RELATED TEXTS

This book is one of three covering finite mathematics and applied calculus. All three of our texts heavily emphasize real-world applications of the mathematics featured as the students in these courses are typically majoring in management or the life or social sciences. The other texts in this series are:

Finite Mathematics for Management, Life, and Social Sciences, Third Edition.
This text is intended for a one-term course covering sets, matrices, inequalities and linear programming, mathematics of finance, probability, statistics, and game theory.

Mathematical Applications for Management, Life, and Social Sciences, Fourth Edition.
This text is designed for a one- or two-term course in finite math and calculus. It contains topics from *Finite Mathematics* as well as most of the calculus topics discussed in this text.

Acknowledgments

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Ronald J. Harshbarger
James J. Reynolds

CONTENTS

0 | ALGEBRAIC CONCEPTS

This chapter provides a brief review of the algebraic concepts that will be used throughout the text.

The review begins with sets and the real numbers, the number system used in the remainder of the text. Special subsets of the real numbers, including intervals, are also covered.

Exponents, rules of exponents, and radicals are then introduced. Expressions involving powers and radicals are the building blocks for algebraic expressions. Operations with algebraic expressions and factoring are reviewed and then followed by operations with and simplification of algebraic fractions.

The chapter concludes with a study of the solution of linear equations and inequalities and a look at some of their applications.

You should already be familiar with the topics covered in this chapter, but it may be helpful to spend some time reviewing them. In addition, each chapter after this one opens with a warm-up page that identifies prerequisite skills needed for that chapter. If algebraic skills are required, the warm-up cites their coverage in this chapter. Thus, you will find that the following sections are a useful reference as you study later chapters.