

**BASIC
HEMODYNAMICS
and
Its Role in
Disease Processes**

by
Dali J. Patel
and
Ramesh N. Vaishnav

BASIC HEMODYNAMICS and Its Role in Disease Processes

by

Dali J. Patel, M.D., Ph.D.

Professor of Research in Physiology
Department of Physiology and Biophysics
College of Medicine
Howard University
Washington, D.C.

and

Ramesh N. Vaishnav, Ph.D.

Department of Civil Engineering
The Catholic University of America
Washington, D.C.



University Park Press
Baltimore

UNIVERSITY PARK PRESS
International Publishers in Science, Medicine, and Education
233 East Redwood Street
Baltimore, Maryland 21202

Copyright © 1980 by University Park Press

Typeset by York Graphic Services, Inc.
Manufactured in the United States of America by
Universal Lithographers, Inc.,
and The Maple Press Company

All rights, including that of translation into other languages, reserved. Photomechanical reproduction (photocopy, microcopy) of this book or parts thereof without special permission of the publisher is prohibited.

Library of Congress Cataloging in Publication Data

Patel, Dali J.
Basic hemodynamics and its role in disease processes.

Bibliography: p.
Includes index.

I. Hemodynamics. 2. Atherosclerosis.

I. Vaishnav, Ramesh N., joint author. II. Title.

[DNLM: 1. Hemodynamics. WG106 P295b]

QP105.P37 612'.1181 79-23980

ISBN 0-8391-1552-0

Preface

Hemodynamics may be defined as the study of the physical aspects of the circulatory system, and includes, among other things, the study of the blood vessel wall as well as blood flow. The purpose of this book on hemodynamics is threefold: to serve as a basic hemodynamics text for a variety of students; to cover in considerable detail those areas of hemodynamics that we believe will help in the study of disease processes; and to point out, wherever possible, the applications of basic hemodynamic principles to clinical cardiology. We differentiate, somewhat arbitrarily, between the application of basic hemodynamic principles to clinical cardiology and applied hemodynamics as used in cardiology. For instance, measuring a pressure difference across a heart valve at cardiac catheterization to estimate valvular stenosis is an example of the latter, whereas using Navier-Stokes equations to compute blood velocity profiles and wall shear stress from a measured pressure gradient in the aorta of cardiac patients is an example of the former.

Over the past twenty years, following the lead of McDonald and co-workers, researchers have studied in great detail the quantitative aspects of pulsatile phenomena in the arterial system. While these studies have furthered our understanding of the mechanics of the circulatory system, the more recent thrust of hemodynamic research is to deliberately shape the concepts and tools of basic hemodynamics to answer questions relative to the pathogenesis of disease. We now ask more difficult questions of hemodynamics; for example, what is the role of hemodynamics in disease processes like atherosclerosis? Before such questions can be answered, we need detailed information on vascular geometry and rheology, local flow fields, and the blood-wall interface. Such information is not presently available, but study of such questions is underway in various laboratories, including our own. Because arteries are the dominant sites of many serious diseases, such as atherosclerosis, and because the arterial system is particularly suitable to illustrate many concepts of hemodynamics, our emphasis in this work has been on the arterial system.

The book is an outgrowth of many years of research in basic hemodynamics at the National Institutes of Health and The Catholic University of America and several years of teaching a course to a mixed audience of advanced graduate students in physiology and bioengineering, and to technicians and physicians engaged in basic and clinical cardiovascular research. The book has therefore been designed not only as a text for a two-semester course in basic hemodynamics for seniors and first-year graduate students from a variety of backgrounds but also to be of use to researchers in the field. The tutorial aspects are emphasized throughout the book and an effort has been made to deal in concepts rather than to review all the work done in the field. Wherever possible, we have tried to illustrate these concepts from our own work, and refer to others via review articles or books. Unfortunately, this scheme has precluded direct citation of many excellent articles in the field. As far as possible, a consistent set of symbols has been used throughout the book. Occasionally, however, we have departed from this in a few situations where it was considered more desirable to do so and where there was no likelihood of ambiguities. In any case, a selected list of symbols has been provided for each chapter. In order to keep the size of the book within reasonable limits, techniques of measurements as well as historical and descriptive aspects of the subject have been minimized. Moreover, for the same reason, some important topics, such as myocardial mechanics and microcirculation, have not been included.

Although the book uses some sophisticated concepts of mathematics and physics, these are introduced in a gradual manner designed to provide an intuitive feel for the subject, even for readers unfamiliar with advanced physical and mathematical concepts. The first two chapters therefore cover some introductory mathematical, mechanical, and physiologic concepts of general use in hemodynamics.

Wherever possible, the mathematical concepts are illustrated with physiologic examples to serve an interdisciplinary audience. These two introductory chapters may be particularly helpful to physiologists and cardiologists and will also provide a physical basis for the subject matter of the later chapters. The rest of the text is modular in nature and chapters may be selected out of order without loss of continuity.

In Chapter 3, the structure of the normal and atherosclerotic vascular wall is discussed, with particular emphasis on the intimal region. This chapter will help provide a structural basis for the phenomenological studies of the arterial tissue to be discussed in later chapters.

Chapters 4, 5, and 6 deal in considerable detail with the rheologic properties of the vascular tissue. In Chapter 4 certain general aspects of vascular mechanics, such as geometry and motion of the vascular wall, longitudinal tethering of the vessel wall to the body cavity, and general material properties, such as incompressibility and elastic symmetry, are discussed. A knowledge of geometry, motion, and tethering is necessary to formulate the boundary conditions for modelling the circulatory system, and the general material properties are needed for formulating rheologic theories for the vessel wall, as is done in Chapter 5. In Chapter 5 we develop the theoretical and experimental basis for the linear incremental and nonlinear large deformation theories for the elastic and viscoelastic properties of the vessel wall. These formulations provide compact and quantitative rheologic descriptions of the arterial wall. Although such detailed formulations have not been used to date in circulatory mechanics (such as that described in Chapter 7), future, more detailed models of circulatory mechanics will undoubtedly require them. The discrete properties of the endothelial surface and intimal region, a knowledge of which is pertinent to the study of atherosclerosis, are discussed in Chapter 6. In particular, a microindentation method to study the compliance of the intimal region and a jet impingement method to study the yield strength of the endothelium are discussed. Taken together, the three chapters cover the state of the art in the field of blood vessel rheology.

Arterial blood flow fields and pulse propagation form the subject matter of Chapter 7. After introduction of a few fundamental concepts and the basic equations of fluid mechanics, two pertinent examples—steady and periodic flows in rigid tubes—are given. Then the linear and nonlinear theories of blood flow and pulse propagation are discussed. Although we expect the nonlinear theory to be used more often in the future, a detailed exposition of the linear theory is included in this chapter for tutorial purposes. Finally, the chapter ends with a concise discussion of a few special topics, such as entrance flow, flow through the aortic arch, branches, and stenoses.

Chapter 8 illustrates the clinical application of some basic hemodynamic concepts, such as hydraulic power and hydraulic input impedance. The impedance function characterizes the load the ventricle must face and the hydraulic power indicates the rate at which the ventricle is doing work against this load. The two together provide an opportunity to study the interaction between the heart and the rest of the circulatory system. Moreover, the data presented in this chapter are unique in that they were obtained from patients with cardiac pacemakers in whom the heart rate and the stroke volume could be varied independently over a wide range.

An understanding of the mechanical aspects of many physiologic flow problems, such as those in the heart, lung, kidney, and brain, requires a knowledge of the physics of flow in collapsible tubes. The flow in a collapsible tube becomes independent of the outlet pressure when the pressure on its outer wall exceeds the

intraluminal pressure. Although this phenomenon has been widely studied experimentally, an understanding of the associated physical mechanisms has emerged only recently. These mechanisms are discussed in terms of two inherently different models in Chapter 9.

Finally, in Chapter 10 transport of material into the arterial wall is discussed with special reference to the pathogenesis of atherosclerosis. The general problems of describing transport processes in arterial tissue systems are discussed with examples of various tentative approaches to specific mathematical descriptions. A number of simplified cases are then solved in detail with an eye toward providing insight into the mathematical methods of studying problems in this complex field.

It is important to point out that the book was designed as an *integrated textbook* rather than as an edited book. As a result, there was a great deal of interaction between us and the invited authors, particularly in Chapters 7, 8, and 10. By the same token, the guest authors, especially Drs. Fry and Atabek, provided significant input to the chapters written by us. In a work like this, it is inevitable that some errors remain in spite of meticulous care. We would be grateful to the readers if they would kindly draw our attention to any errors they may find.

It is our hope that *Basic Hemodynamics* will provide readers with at least a minimal background in the mechanics of the circulatory system. An effort has been made to do so in a clear, intuitive manner, to enable the readers to understand better the pertinent literature in the field, and to pursue independent study.

D.J.P.
R.N.V.

Acknowledgments

We extend our sincere thanks to Dr. Donald L. Fry, in whose laboratory most of the original work cited in this book was performed. His encouragement, support, and advice through the years are deeply appreciated. More than a few times we have benefited from Dr. Fry's unique insight into biophysical phenomena. We also thank Drs. Fry, Atabek, Ferrans, Greenfield, and their collaborators (Drs. McHale, Rember, Thomas, and Deshpande) for taking time out from their busy schedules to contribute their chapters.

We are grateful to many individuals who were instrumental in various ways in shaping this book into its present form. First and foremost we thank the late Dr. Alan C. Burton,* Professor Emeritus at University of Ontario, London, Ontario, Canada, for helpful suggestions in the initial planning stages of the book. We also thank Dr. Frederick J. Cornhill of the Ohio State University and Dr. Byron D. McLees of the National Institutes of Health, who critically read the entire manuscript. Mr. Jafar Vossoughi of The Catholic University of America not only read the whole manuscript, but also helped us with innumerable organizational details. Drs. Robert W. Mahley, John T. Young, John J. McCoy, and David M. Wetzel read specific chapters and gave us their valuable comments. Comments of these scholars were very useful to us and we extend to them our sincere gratitude.

We thank the graduate students, research colleagues, and the staff at the Laboratory of Experimental Atherosclerosis at The National Institutes of Health for their contributions over the years to the research upon which this book is based. In particular, we thank Mr. Fred Plowman for design and fabrication of some important instrumentation, Messrs. Joseph M. Pearce, Leander Brown, George Johnson, and Rufus Seabron for technical assistance in animal experiments, and Ms. Kathleen Holcombe for editorial assistance. We also thank various authors and institutions (Academic Press, American Heart Association, American Physiological Society, Edward Arnold Publ. Ltd., Elsevier/North Holland Biomedical Press, Grune and Stratton, McGraw Hill Book Co., C. V. Mosby Co., The New York Academy of Sciences, Pergamon Press, Raven Press, Rockefeller University Press, and Year Book Medical Publishers) for granting permission to reproduce various figures and tables as identified in the text.

In addition, Dr. Patel wishes to thank Dr. Fry for his support and encouragement during the years he worked under Dr. Fry at the National Institutes of Health. Dr. Vaishnav takes this opportunity to express his deep gratitude to The Catholic University of America for granting him a sabbatical leave during which the bulk of the work was performed, as well as for a generous grant in the form of computer time toward preparation of this book. He also acknowledges the support over the years of various grants from the National Science Foundation and The Public Health Service, on which part of the research work described in Chapters 4, 5, and 6 was based. He is particularly thankful to his able secretary, Ms. Elizabeth Gurubatham, who computerized the manuscript and was in charge of its general organization. She took pride and interest in pushing the manuscript to completion while suffering through the constant changes we made in our struggle to achieve clarity and perfection. He also thanks his student assistant, Ms. Mary Jane Flynn, who checked all the equations in Chapter 10 and critically read the page proofs of all the chapters. Above all, he thanks his wife Marianne, who helped us in many ways

*Dr. Burton died on June 27, 1979. Physiologists all over the world will remember him as the father of modern circulatory physics, but his students, including the senior author, will remember him as the kind, unselfish man they all loved.

during the preparation of this work. She also undertook the arduous task of preparing the index.

Finally, both of us extend our sincere thanks to Ms. Maria Coughlin, Mrs. Joan Sanow, and Mr. Albert A. Belskie of University Park Press. We owe a great deal to the efficiency and patience of all these fine people.

D.J.P.
R.N.V.

Contents

	Preface	xi
	Acknowledgments	xv
CHAPTER 1	MATHEMATICAL PRELIMINARIES	1
	FUNCTIONS	3
	DIFFERENTIATION	5
	INTEGRATION	5
	SOME EXAMPLES OF DIFFERENTIATION AND INTEGRATION	6
	PARTIAL DERIVATIVES	10
	DIFFERENTIAL EQUATIONS	11
	The Rate Problem	12
	Steady Flow Through a Rigid Tube	13
	Theory of Pressure-Recording Systems as an Example of a Linear Second-Order Differential Equation	19
	ELEMENTS OF VECTOR ANALYSIS	28
	THE CONCEPT OF LINEARITY	35
	COMPLEX NUMBERS, SINUSOIDAL WAVES, AND FOURIER ANALYSIS	37
	Complex Numbers	37
	Sinusoidal Waves	39
	Fourier Analysis	41
	Use of Fourier Analysis in Hemodynamics	43
	LINEAR MODELS	46
	General Considerations	46
	Lumped-Parameter Models	52
	Distributed Parameter Models	54
	Experimental Measurements of Characteristic Impedance, Propagation Constant, and Wave Velocities	60
	Propagation of a Pressure Wave	62
CHAPTER 2	SOME ELEMENTARY HEMODYNAMIC CONCEPTS	65
	MECHANICS OF SOLIDS	66
	Elasticity	66
	Viscoelasticity	68
	Some Basic Concepts of Stress Analysis	70
	FLUID MECHANICS	81
	Hydrostatics	81
	Surface Tension	83
	Potential and Kinetic Energies in a Moving Fluid	85
	Blood Viscosity	87
	Laminar and Turbulent Flows	90
	Boundary Layer and Flow Separation	92
	Local Flow Patterns in Arteries	94
	BLOOD-WALL INTERACTIONS	97
CHAPTER 3	VASCULAR STRUCTURE	105
	<i>Victor J. Ferrans</i>	
	GENERAL STRUCTURE OF THE NORMAL VASCULAR SYSTEM	106

Elastic Arteries	106
Muscular Arteries	107
Arterioles	110
Capillaries and Veins	112
STRUCTURE OF CELLULAR COMPONENTS OF THE	
VASCULAR WALL	112
Endothelium	112
Smooth Muscle Cells	122
STRUCTURE OF EXTRACELLULAR COMPONENTS OF	
THE VASCULAR WALL	126
Collagen	126
Elastic Fibers	130
Proteoglycans	133
STRUCTURE OF FIBROMUSCULAR INTIMAL	
THICKENINGS	134
Intimal Pads	134
Diffuse Intimal Thickenings	138
GENERAL FEATURES OF THE ARTERIAL WALL IN	
ATHEROSCLEROSIS	138
PATHOLOGY OF ATHEROSCLEROTIC LESIONS	139
Fatty Streaks	139
Fibrous Plaques	140
Complicated Plaques	141
TOPOGRAPHY OF ATHEROSCLEROTIC LESIONS	147
CONCEPTS OF ATHEROGENESIS	151

CHAPTER 4	GENERAL MECHANICAL PROPERTIES OF THE	
	VASCULAR WALL	155
	GEOMETRY AND MOTION OF THE ARTERIAL	
	WALL	157
	Geometry of the Arterial System	157
	Motion of the Arterial System	163
	VASCULAR TETHERING	172
	Theoretical Concepts	172
	Experimental Determination of Vascular Tethering	176
	GENERAL PROPERTIES OF THE VASCULAR TISSUE	180
	Homogeneity	182
	Incompressibility	183
	Orthotropy	186
	Nonlinearity	193
	Incremental Linearity	194
	Cylindricity	195
	Thinness of the Vascular Wall	195

CHAPTER 5	RHEOLOGY OF THE VESSEL WALL—THEORETICAL	
	AND EXPERIMENTAL CONSIDERATIONS	199
	INCREMENTAL ELASTIC PROPERTIES	201
	Theoretical Considerations	201
	Experimental Evaluation of Static Elastic Incremental	
	Moduli	207
	INCREMENTAL VISCOELASTIC PROPERTIES	209
	Theoretical Considerations	209

	Experimental Evaluation of Dynamic Incremental Viscoelastic Moduli	212
	COMPARISON OF INCREMENTAL MODULI FROM VARIOUS STUDIES	215
	NONLINEAR ELASTIC PROPERTIES	216
	Theoretical Considerations	216
	Interrelation Between the Incremental Elastic Theory and the Overall Nonlinear Elastic Theory	223
	Experimental Determination of Nonlinear Elastic Constitutive Constants Using the Thin-Wall Theory	224
	Distribution of Stresses and Strain Energy Density Through the Wall Thickness	227
	NONLINEAR VISCOELASTIC PROPERTIES OF ARTERIES	230
	Theoretical Considerations	230
	Experimental Determination of Nonlinear Viscoelastic Constitutive Functions	233
	OTHER FORMULATIONS TO CHARACTERIZE THE NONLINEAR RESPONSE OF THE VASCULAR TISSUE	235
	THE ROLE OF VASCULAR SMOOTH MUSCLE IN DETERMINATION OF MECHANICAL PROPERTIES	236
CHAPTER 6	LOCAL RHEOLOGY OF THE VASCULAR INTIMA	243
	TOPOGRAPHY OF ATHEROSCLEROTIC LESIONS	244
	METHODS FOR STUDYING SURFACE RHEOLOGY	247
	The Microindentation Technique	247
	The Saline Jet Technique	252
CHAPTER 7	BLOOD FLOW AND PULSE PROPAGATION IN ARTERIES	255
	<i>H. Bülent Atabek</i>	
	FUNDAMENTAL CONCEPTS AND EQUATIONS	258
	Kinematics of Flow	258
	Basic Equations of Flow	265
	Boundary Conditions	272
	Examples	273
	LINEAR THEORY OF FLOW AND PULSE PROPAGATION	286
	A General Discussion of Progressive Waves	286
	A Simplified Linear Theory	289
	Wave Reflection	297
	Physiologic Factors Affecting Pulse Propagation	310
	Experimental Results and Discussion	312
	NONLINEAR THEORY OF FLOW AND PULSE PROPAGATION	314
	Theory of Local Flow	315
	Experiments and Results	324
	Comparison with Linearized Theories	332
	Pulse Propagation	336
	SPECIAL TOPICS (with Mohan D. Deshpande)	345
	Flow in the Entrance Region	345
	Flow Through the Aortic Arch	349

	Flow Through Branches	351
	Flow Through Arterial Stenosis	354
CHAPTER 8	MEASUREMENT AND SIGNIFICANCE OF PRESSURE-FLOW RELATIONSHIPS IN MAN	363
	<i>*Philip A. McHale, Judith C. Rembert, and Joseph C. Greenfield, Jr.</i>	
	METHODS	366
	Measurement of Phasic Aortic Blood Flow	366
	The Pressure Gradient Technique	366
	Patient Material	367
	Computer Techniques	368
	Data Analysis	369
	Calculation of Pressure and Flow Parameters	370
	Calculation of Aortic Hydraulic Power	373
	Calculation of Aortic Input Impedance Data	375
	RESULTS	376
	Analysis of Hemodynamic Data	376
	Analysis of Hydraulic Power Data	385
	Analysis of Aortic Input Impedance Data	396
	DISCUSSION	399
	CONCLUSION	404
CHAPTER 9	FLOW IN COLLAPSIBLE TUBES	407
	<i>Donald L. Fry, Lewis J. Thomas, and Joseph C. Greenfield</i>	
	EXPERIMENTAL OBSERVATIONS ON A STARLING RESISTOR	409
	PHYSICS OF FLOW IN COLLAPSIBLE TUBES	412
	Inertial Mode of Flow Limitation	413
	Frictional Mode of Flow Limitation	414
	Simplified Frictional Mode Model	415
	BIOLOGIC EXAMPLES	418
	Simple Lung Model	418
	Simple Cerebral Vessel Model	419
	CONCLUDING REMARKS	423
CHAPTER 10	MASS TRANSPORT IN THE ARTERIAL WALL	425
	<i>Donald L. Fry and Ramesh N. Vaishnav</i>	
	COORDINATE SYSTEM: SPATIAL	429
	COORDINATE SYSTEM: TEMPORAL	429
	CONCENTRATION: Definition	431
	Activity	433
	Definitions of Fractional Volumes, Partition Coefficient, Porosity, and Solubility	434
	TRANSPORT PROCESSES	440
	Diffusion	442
	Convection	444
	Fick's Second Law for Combined Diffusion and Convection	448
	CHEMICAL REACTIONS	451
	EQUATION OF MASS BALANCE	452

SOLUTION TO SOME SIMPLE EQUATIONS OF MASS	
BALANCE	455
STEADY-STATE SOLUTIONS	455
Case 1: Simple Slab	455
Case 2: Slab Comprised of Two Zones	458
Case 3: Slab With Surface Barrier	459
Case 4: Slab With Surface Barrier and Chemical Reaction	461
TRANSIENT SOLUTIONS	463
Case 5: Simple Slab with Flux from One Boundary	465
Case 6: Simple Slab with Flux from Both Boundaries	467
Case 7: Slab with Surface Barrier	469
Case 8: Simple Slab with Chemical Reaction	470
Case 9: Slab with Surface Barrier and Chemical Reaction	471
ACCUMULATION OR UPTAKE	472
CONCLUDING REMARKS	476
APPENDIX: Units of Measurement	483
Index	485

CHAPTER 1

MATHEMATICAL PRELIMINARIES

FUNCTIONS
DIFFERENTIATION
INTEGRATION
SOME EXAMPLES OF DIFFERENTIATION AND INTEGRATION
PARTIAL DERIVATIVES
DIFFERENTIAL EQUATIONS
The Rate Problem
Steady Flow Through a Rigid Tube
Theory of Pressure-Recording Systems as an Example of a Linear Second-Order Differential Equation
<i>The Complementary Solution</i>
<i>Particular Solution</i>
<i>Testing the Manometer System</i>
<i>The Direct Method</i>
<i>The Indirect Method</i>
ELEMENTS OF VECTOR ANALYSIS
THE CONCEPT OF LINEARITY
COMPLEX NUMBERS, SINUSOIDAL WAVES, AND FOURIER ANALYSIS
Complex Numbers
Sinusoidal Waves
Fourier Analysis
Use of Fourier Analysis in Hemodynamics
LINEAR MODELS
General Considerations
Lumped-Parameter Models
Distributed Parameter Models
Experimental Measurements of Characteristic Impedance, Propagation Constant, and Wave Velocities
Propagation of a Pressure Wave

LIST OF SYMBOLS

A	area
a	acceleration; real part of a complex number z ; also used as a constant
α	attenuation constant
b	imaginary part of a complex number z ; also used as a constant
β	phase constant
C	capacitance
C'	capacitance per unit length
c	damping coefficient; also used as a constant
C_c	critical damping coefficient
F	F_{acc} = acceleration force; F_{app} = applied force; F_s = spring force; F_d = force in dashpot; F_{visc} = viscous force
f	frequency
G	conductance
G'	conductance per unit length
h	thickness of blood vessel wall
i	current

sm	denotes imaginary part of a complex variable
j	$= \sqrt{-1}$
K	spring constant
\mathcal{L}	inductance
\mathcal{L}'	inductance per unit length
L	length
\lim	limit
l_1, l_2, l_3	inductances
M	mass; amount of substance
NE	norepinephrine
p	pressure
p_i	inside pressure
p_o	outside pressure
$ p_n $	magnitude of n^{th} harmonic of pressure
Q	charge
$ Q_n $	magnitude of n^{th} harmonic of flow
Q	cardiac output; volume flow rate
\mathcal{R}	resistance; hydraulic resistance
\mathcal{R}'	resistance per unit length
R	radius; also reflection coefficient
r	radial variable
r_1, r_2, r_3	resistances
\vec{r}	radius vector
\Re	denotes real part of a complex variable
Re	Reynold's number
S_θ	circumferential stress
T	period
TMP	transmural pressure
t	time
u, v, w	components of \vec{V}
V	volume
\mathcal{V}	voltage
\vec{V}	vector; velocity vector
V_x, V_y, V_z	cartesian components of \vec{V}
$ \vec{V} $	magnitude of \vec{V}
v	velocity
v_p	phase velocity
W	work
X_c	complementary solution
X_p	particular solution
x, y, z	Cartesian coordinates
x_s	extension of spring
x_d	extension in dashpot
z	complex variable; coordinate variable in axial direction
Z	impedance
Z_o	characteristic impedance
β_o	damping factor
Δc	concentration difference
γ	propagation constant
ζ	damping coefficient
η	viscosity coefficient
θ	angle
θ_n	phase angle of n^{th} harmonic
λ	wavelength
ρ	density
τ	shear stress

ϕ	a scalar; phase angle
ϕ_n	phase angle of n^{th} harmonic
ϕ_f	starting angle for flow vector
ϕ_p	starting angle for pressure vector
ω	angular frequency
ω_0	undamped natural frequency
ω_d	damped natural frequency
∇	vector operator "del"

The subject of hemodynamics is vast in scope, and the cardiovascular system, which it addresses, is highly complex. Not only are the components of this system complex in nature, but also the interactions of these components in the living system are complicated and varied. Characterization of such a system necessarily requires sophisticated mathematical and mechanical concepts and tools with which the average physiologist may not be familiar. As a matter of fact, some concepts of mechanics used in modern hemodynamics are too specialized even for an average mechanic. The first two chapters of this book have therefore been set out solely to explain some fundamental concepts of mathematics, solid mechanics, and fluid mechanics. Biologic examples are used whenever possible. Once these concepts have been introduced, we can continue the development of the subject with minimal interruption. To facilitate understanding of some of the complicated concepts of hemodynamics to be discussed later on, the subject is developed from an elementary to a more sophisticated level. The tutorial aspects of the subject are emphasized throughout.

A rudimentary knowledge of calculus is essential to the understanding of the mathematical concepts presented in this book. If a brief review of the subject is desired, we recommend two sources, the books by Ayres (1964) and Richmond (1971), which contain many worked examples and can be adapted for self-study. In the following pages we state, explain, and illustrate a few useful concepts from calculus and other pertinent branches of mathematics.

FUNCTIONS

If, for each value of a variable x , there exists a corresponding value of y , then y is called a *function* of x . The usual notation for such a correspondence is $y = f(x)$. Simple examples of the general statement $y = f(x)$ are: $y = x^2$, $y = \sin x$, and $y = x^2 \log x$, with x as the *independent* variable and y , whose value depends on x , as the *dependent* variable. These are examples of an *explicit* function, as contrasted with an *implicit* function, such as $x^2y^2 + \sin xy = 2$, which is not expressed explicitly as $y = f(x)$.

If, for a function $y = f(x)$, y has a value y_0 when $x = x_0$, this can be expressed as

$$y|_{x=x_0} = f(x_0) = y_0$$

For example, if $y = x^2$, then

$$y|_{x=2} = f(2) = 4$$

If, as in Figure 1.1a, there is only one value of y for each value of x , then f is said to be *single-valued*. In Figure 1.1b, f is *multi-valued* at x_0 , because there are three values of y corresponding to $x = x_0$. Finally, f is said to be *discontinuous* at x_0 if different values of $f(x_0)$ are obtained, depending on whether we approach x_0 by gradually increasing x or by gradually decreasing x , as shown in Figure 1.1c.

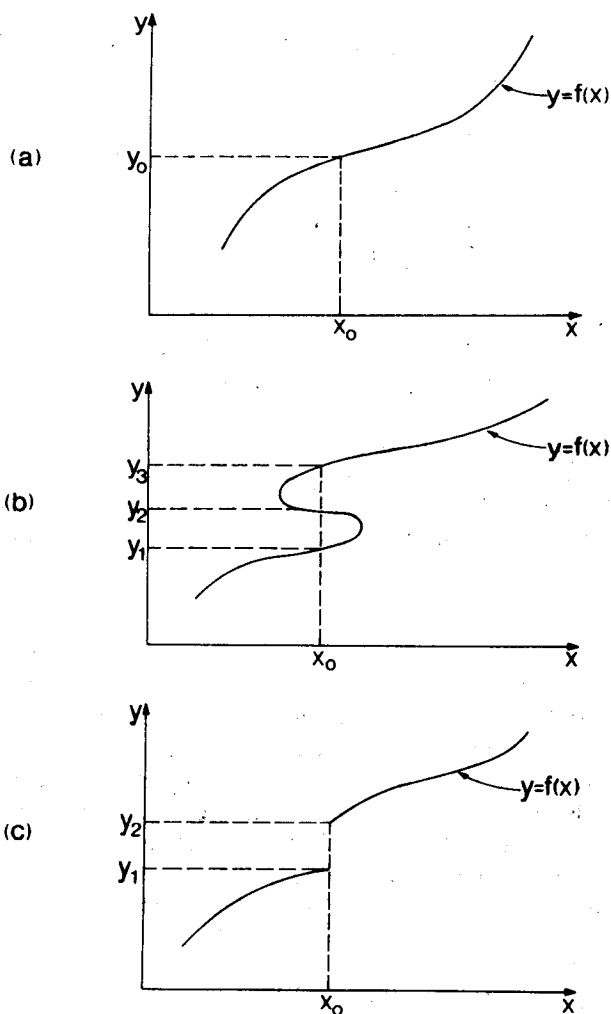


Figure 1.1. Graph of a function $y = f(x)$. (a) A single-valued continuous function. (b) A multi-valued continuous function. (c) A function with a discontinuity at $x = x_0$.