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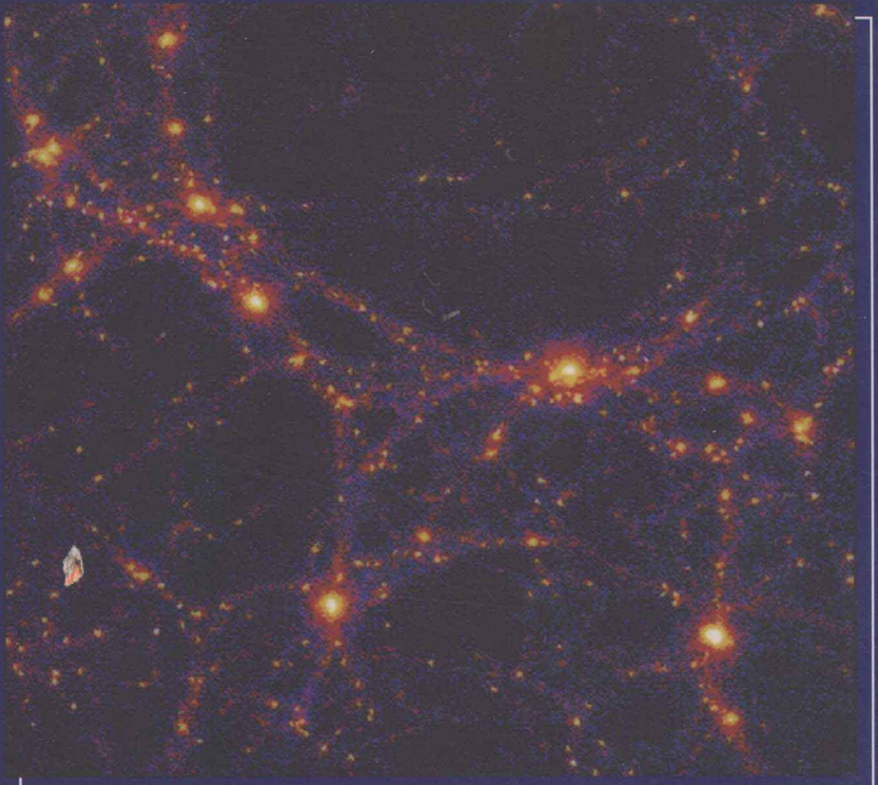
Cosmological Inflation and Large-Scale Structure

宇宙膨胀和大尺度结构



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图书在版编目（CIP）数据

宇宙膨胀和大尺度结构=Cosmological Inflation and Large-Scale Structure：英/（英）安德鲁（Andrew，R.L.）著.—北京：世界图书出版公司北京公司，2008.12
ISBN 978-7-5062-9278-8

Ⅰ.宇… Ⅱ.安… Ⅲ.宇宙学—英文 Ⅳ.P159

中国版本图书馆CIP数据核字（2008）第195301号

书　　名：	Cosmological Inflation and Large-Scale Structure
作　　者：	Andrew R. Liddle & David H. Lyth
中 译 名：	宇宙膨胀和大尺度结构
责任编辑：	高蓉 刘慧
出 版 者：	世界图书出版公司北京公司
印 刷 者：	三河国英印务有限公司
发　　行：	世界图书出版公司北京公司（北京朝内大街137号　100010）
联系电话：	010-64015659
电子信箱：	kjsk@vip.sina.com
开　　本：	16开
印　　张：	26
版　　次：	2009年01月第1次印刷
版权登记：	图字:01-2008-4938
书　　号：	978-7-5062-9278-8 / O・641　　定　价：86.00元

Preface

The 1990s have seen substantial consolidation of theoretical cosmology, coupled with dramatic observational advances, including the emergence of an entirely new field of observational astronomy – the study of irregularities in the cosmic microwave background radiation. A key idea of modern cosmology is *cosmological inflation*, which is a possible theory for the origin of all structures in the Universe, including ourselves! The time is ripe for a new book describing this field of research.

This book is based loosely on our 1993 *Physics Reports* article. We have widened the range of discussion and have made much of the material more pedagogical. We believe that this book will prove useful to starting graduate students in cosmology, to active researchers specializing in the field, and to all levels in between.

Our view of the inflationary cosmology and its consequences has been influenced by many people over the years. ARL especially thanks Alfredo Henriques and Gordon Moorhouse for showing the way into this research area. DHL would like particularly to acknowledge a long-term collaboration with Ewan Stewart. Much thanks is due to all our collaborators on the topics within this book, namely Mark Abney, Domingos Barbosa, Tiago Barreiro, John Barrow, Marco Bruni, Ted Bunn, Ed Copeland, Laura Covi, George Ellis, Mary Gaillard, Juan Garcìa-Bellido, Anne Green, Louise Griffiths, Ian Grivell, Rocky Kolb, Andrew Laycock, Jim Lidsey, Andrei Linde, Anupam Mazumdar, Milan Mijič, Manash Mukherjee, Hitoshi Murayama, Paul Parsons, Antonio Riotto, Dave Roberts, Leszek Roszkowski, Bob Schaefer, Franz Schunck, Douglas Scott, Qaisar Shafi, Ewan Stewart, Will Sutherland, Michael Turner, Pedro Viana, David Wands, Martin White, and Andrzej Woszczyna. Apart from our collaborators, we have had useful conversations with many others, far too many to mention. We hope they know who they are!

We are extremely grateful to Andrei Linde, Martin White, and especially Gordon Moorhouse for their careful reading of the manuscript. The figures for Chapter 12 were made by Pedro Viana, and the compilation of cosmic microwave background anisotropy data shown in Figures 5.9 and 9.2 was kindly provided by Martin White. Many figures were made using the superb publically available CMBFAST code (Seljak and Zaldarriaga 1996), which we strongly recommend everyone to get.

Although we wrote most of the book at our home institutes, occasionally we were somewhere more glamorous. ARL would like to thank the Università di Padova, the University of New South Wales, and the Aspen Center for Physics, and DHL the University of California at Berkeley. ARL acknowledges the generous support of the Royal Society throughout this endeavour.

Of course, we have done our best to ensure that the contents of this book are accurate; however, some errors may have slipped through. We would be very grateful if readers would inform us of any they spot. We plan to keep an up-to-date record of any errors, accessible at the book's World Wide Web Home Page at

`http://star-www.cpes.sussex.ac.uk/~andrewl/infbook.html`

which can be used to check for errors we already know about.

Andrew R. Liddle and David H. Lyth
October 1998

Contents

Frequently used symbols

Frequently used symbols and their place of definition

Symbol	*Definition*	*Page*
$M_{\rm Pl}$	Reduced Planck mass ($=2.436 \times 10^{18}$ GeV)	13
G	Gravitational constant ($=1/8\pi M_{\rm Pl}^2$)	13
a	Scale factor of the Universe	13
t	Time	13, 23
τ	Conformal time	14
H	Hubble parameter	14
H_0, h	Present Hubble parameter	15
z	Redshift	15
ρ, P	Density and pressure	15
Λ	Cosmological constant	15
K	Spatial curvature	16
ρ_c	Critical density	17
Ω	Density parameter	17
T	Temperature	18
n_g	Number density of any species g	18, 351
g_*	Effective number of particle species	18
w	Pressure-to-density ratio ($w = P/\rho$)	21
$r_{\rm hor}$	Particle horizon distance	24
ϕ	Scalar field	41
$V(\phi)$	Scalar field potential	41
ϵ, η	Slow-roll parameters	42
N	Number of e-foldings	43
$\epsilon_{\rm H}, \eta_{\rm H}$	Slow-roll parameters	51
k	Comoving wavenumber	59
δ	Density contrast	61
$T_g(k)$	Transfer function of any quantity g	62
$\mathcal{P}_g(k)$	Spectrum of any quantity g	65
$\xi(R)$	Correlation function	66
ℓ, m	Spherical expansion variables	68
$Y_{\ell m}$	Spherical harmonics	68
$W(kR)$	Window function	71

Continued ...

Symbol	*Definition*	*Page*
$\sigma_g(R)$	Dispersion of any quantity g	72
n	Spectral index (density perturbations)	75, 187
$\mathbf{v}$	Peculiar velocity	76
Φ, Ψ	Gravitational potentials	79, 101, 342
V	Peculiar velocity (irrotational part)	83, 338
c_s	Sound speed	84, 349
t_{pr}	Proper time	93, 318
$\mathcal{R}$	Curvature perturbation	98, 341
Σ_{ij}, Π_{ij}	Anisotropic stress	101, 322, 337
$\delta_H(k)$	Density perturbation amplitude	106
$T(k)$	Cold dark matter transfer function	107
Γ	Shape parameter	108
C_ℓ	Spectrum of the cosmic microwave background anisotropy	116
Θ	Photon brightness function	123, 359
Q, U	Stokes' parameters	126
E, B	Polarization components	128
κ	Optical depth	131
$g(\Omega)$	Growth suppression factor	144, 148
n_{grav}	Spectral index (gravitational waves)	154, 193
n_{iso}	Spectral index (isocurvature)	159
S	Action	164
$\mathcal{L}$	Lagrangian density	165
r	Tensor-to-scalar ratio	193
b, b_I	Optical and infrared bias parameters	264, 265
$T_{\mu\nu}$	Stress–energy tensor	320, 331

1 Introduction

1.1 This book

The study of the early Universe came into its own as a research field during the 1980s. Though there had been occasional forays during the seventies and even before that, it was during the 1980s that a wide range of topics, united by the adoption of modern particle physics ideas in a cosmological context, were investigated in detail. This era of study culminated with the publication in 1990 of the classic book *The Early Universe* by Kolb and Turner, in which the authors described ideas across the whole range of what had become known as **particle cosmology** or **particle astrophysics**, including such topics as topological defects, inflationary cosmology, dark matter, axions, and even quantum cosmology.

Although all these topics matured during the 1980s, if we look back at the papers of that era, we are struck by the rarity with which any detailed comparison with observations could be made. In that regard, particle cosmology in the nineties and onward has become a very different subject from what it was during the eighties because, for the first time, there are observations of a quality that seriously constrains some of the possible physics of the early Universe. Those observations are of structure in the Universe, and a starring role among them is played by the first detection of microwave background anisotropies by the Cosmic Background Explorer (COBE) satellite, announced in 1992. These were the first observations that could be more or less directly interpreted as constraints on early Universe physics. As we will see, by the middle of the first decade of the twenty-first century, we should have a wealth of data constraining our conceptions of what may have occurred during the Universe's earliest stages, and, most likely, several of the ideas described in Kolb and Turner's book will have been banished from serious discussion.

This book is not about particle cosmology as a whole, but rather is about a single topic, **inflationary cosmology**, introduced in a seminal paper by Guth (1981). This has been a research field of lasting popularity; more papers have been written about inflation than any other area of early Universe cosmology, and one of Guth's favourite transparencies in review talks charts the rise of the publication count. Although introduced to resolve problems associated with the initial conditions needed for the Big Bang cosmology, its lasting prominence is owed to a property discovered soon after its introduction: It provides a possible explanation for the initial inhomogeneities in the Universe that are believed to have led to all the structures we see, from the earliest objects formed to the clustering of galaxies to the observed irregularities in the microwave background.

Our aim is twofold. First, we wish to give a unified view of the entire process of modelling the inflationary epoch, predicting the small irregularities that it generates, and evolving these irregularities using linear equations that are valid as long as the irregularities remain small. The resulting theoretical structure, starting with the quantum fluctuations of a free field, continuing with general-relativistic gas dynamics, and ending with the free fall of photons and matter, is perhaps one of the most beautiful and complete in the entire field of physics. Certainly, it lies at the opposite extreme from ad hoc models, not of course confined to physics, whose only merit is sometimes to make the author feel better than if the desired result had been written down immediately. Let us hope that the theory is true as well as beautiful!

Second, we wish to describe the state of the art, with respect to both inflation model-building and the confrontation of theory with observation. In the former area, Kolb and Turner's above mentioned book and Linde's *Particle Physics and Inflationary Cosmology* were both written in 1990, and since then, there have been many developments in the theoretical modelling of inflation, including major shifts in the perception of which ideas are the most relevant. Techniques for generating predictions for generic inflationary models also have come some way during that period. The latter area is in the process of being revolutionized by observations of the cosmic microwave background (cmb) anisotropy; the approval of two separate satellite experiments, Microwave Anisotropy Probe (**MAP**) by the National Aeronautics and Space Administration and **Planck** by the European Space Agency, to explore anisotropies down to angular scales of a few arc-minutes promises data of a quality that will be hard to surpass when it comes to constraining or excluding the inflationary cosmology. The rapid progress in both areas means that we are providing something resembling a snapshot of the current situation, though we believe that it will provide a useful orientation for at least some years to come.

As mentioned already, our discussion focuses on the evolution of *small* irregularities. Because inflation ultimately is supposed to provide the origin of all structure, potentially any measure of that structure can, in principle, be used to constrain inflation. This provides a connection to a research area known variously as large-scale structure or physical cosmology, which on its own is a much vaster research area than all of particle cosmology put together. Much has been written on this topic; for example, four books produced after the crucial COBE observations are those of Padmanabhan (1993), Peebles (1993), Coles and Lucchin (1995), and Peacock (1999). By restricting ourselves almost entirely to the linear regime, our focus is both narrower and deeper.

Incidentally, we use the phase "large-scale structure" to refer only to irregularities in matter density, such as the galaxy distribution and motions. The term usually does not include microwave background anisotropies, the exception being the title of this book!

1.2 The Universe we see

Extensive discussion of the nature of the observed Universe has been given in the recent textbooks just mentioned, and so, we will be brief in this introduction.

A description of our observed Universe can be broken into two parts: the global description of the Universe, which is given in terms of a set of parameters that we call the **cosmological parameters**, and the irregularities observed in the Universe.

The cosmological parameters tell us about the geometry of the Universe, and about the material contained within it. These parameters are defined in Chapter 2. The dynamics of an expanding Universe are characterized by two quantities: the expansion rate, given by the Hubble parameter, and the spatial curvature. The latter, in fact, is determined by the amounts of different types of material in the Universe. Direct observation shows that the Universe contains quite a significant amount of baryonic matter, of which we are made, and also contains quite a bit of radiation in the form of the cmb, which can be characterized by a thermal distribution at a temperature $T_0 = 2.728$ K. These are the only two forms of matter that are observed directly. However, on the basis of standard particle physics, it is assumed that there is also a cosmic neutrino background, contributing about the same energy density as the radiation. Beyond that, there is substantial circumstantial evidence (though, as we write, no direct detection of it) that the Universe contains a large (and probably dominant) amount of nonbaryonic dark matter, of some as yet unknown form. The details of how the Universe, and particularly any irregularities within it, will evolve depends on the nature of this dark matter. To get structure formation models to work, it normally is assumed that there must be at least some so-called **cold dark matter**, comprising particles with negligible velocity. However, there also may be a component of **hot dark matter** (particles whose velocities are relativistic for at least some of their evolution) or something more exotic yet. Another possibility, for which there is increasing observational support, is that the Universe might possess a nonzero cosmological constant.

Determination of the various cosmological parameters is a key goal in cosmology, but one in which much progress remains to be made. Of all those just listed, only the present microwave background temperature is known to a satisfying level of accuracy. Other parameters, such as the Hubble constant or the density parameter, remain the subjects of much controversy. In Chapter 2, we briefly review the current observational status. We hope that, in the near future (and for you the reader maybe even the recent past), the situation will become much more definite; in particular, satellite measurements of cmb anisotropies promise to pin down many of the cosmological parameters to a high degree of accuracy.

The second aspect of the observed Universe is the long-established realization that material within it is distributed irregularly. Such irregularities are known as **density perturbations**. An understanding of the origin and evolution of structure in the Universe is the outstanding problem in cosmology at the moment, and this book is primarily about this topic in the context of the inflationary cosmology. Measures of structure in the Universe now come from a variety of sources. Historically, the distribution of galaxies was the most studied, popularized though large galaxy redshift surveys such as the CfA survey in the mid-1980s. Nowadays, we have access to a much more diverse range of measures. The many observations of anisotropies in the cmb, across a range of angular scales, tell us about structure in the Universe long ago when the microwave background was created. The velocities of galaxies can be determined quite accurately, telling us about the gravitational attraction they experience. The abundance of different types of object probes the size of the irregularities in the density of the Universe – at the present epoch, clusters of galaxies are a useful probe, and the study of very distant objects such as quasars can tell us about structure when the Universe was younger, as can observations of distant galaxies with technology such as the Hubble Space Telescope and the Keck telescope on Hawaii. All of these are discussed in Chapters 9 through 12.

Within the context of inflation, all of these structures can be quantified by a small number of parameters describing the initial perturbations, whose subsequent evolution is determined by the cosmological parameters. Such parameters could be called the **inflationary parameters.** They include as a minimum the overall amplitude and scale dependence of the density perturbations; this might be, for example, a power law requiring two parameters that we aim to fix through observation. In fact, the amplitude is already rather well determined by the COBE satellite. In the simplest inflation models, this is all we need, but in more complicated versions, some additional parameters might be necessary. If so, they too in principle can be determined from observations.

1.3 Overview: From cosmological inflation to large-scale structure

This book can be divided loosely into four parts. In Chapters 2 and 3, we introduce the homogeneous Universe and the role that inflation plays in setting its initial conditions. The second part, from Chapters 4 through 8, concerns the development of inhomogeneities in the Universe, from their inception during inflation up to the present. The third part, from Chapters 9 through 13, concerns observations and the way in which they constrain the theoretical development in the first eight chapters. This part ends with an overview. Finally, the last two chapters, separated from the main flow of the book, give a more advanced treatment of inhomogeneities in the Universe, including complete derivations of some results that were assumed for the simpler treatment in the main body of the book.

1.3.1 Hot Big Bang cosmology

We begin our discussion proper in Chapter 2 with a rather rapid summary of the Hot Big Bang theory. This sets down some of our notation and allows us quickly to summarize the results that we use later. Anyone desiring a more leisurely account will find one in any of the books mentioned in Section 1.1. We collect quite a range of different results; in particular, we analyze low-density Universes, both with and without a cosmological constant, as well as the case of a spatially flat Universe with a critical matter density. The last case is the simplest but is disfavoured by observation; we show that there are both theoretical and observational reasons for also considering the low-density cases. By contrast, there is little motivation from either theory or observation to consider closed Universes, where there is greater than a critical density of matter, and we do not concern ourselves with that situation.

1.3.2 Inflation

In Chapter 3, we move on to a discussion of inflationary cosmology (Guth 1981), looking at the general properties rather than at specific models. The definition of inflation is extraordinarily simple: it is any period of the Universe's evolution during which the scale factor, describing the size of the Universe, is accelerating. This leads to a very rapid expansion of the Universe, though perhaps a better way of thinking of this is that the characteristic scale of the Universe, given by the Hubble length, is *shrinking* relative to any fixed scale caught up in the rapid expansion. In that sense, inflation is actually akin to zooming in on a small part of the initial Universe.

Inflation does not in any way replace the Hot Big Bang theory, but rather is an accessory attached during its earliest stages. Inflation certainly cannot proceed forever; the great successes of the Big Bang theory, such as nucleosynthesis (the formation of light elements) and the origin of the thermal microwave background radiation, require the standard evolutionary progression from radiation domination to matter domination, and it is assumed that inflation must end some considerable time before that to allow generation of observed properties such as the baryon–antibaryon asymmetry of the Universe.

As we see later, a sufficiently long period of inflation can resolve certain concerns about the initial conditions necessary for the Big Bang cosmology to lead to a Universe such as our own. In particular, it can explain why the Universe should be close to spatial flatness and why it should appear homogeneous, at least on large scales. It was these problems that motivated the original introduction of inflation by Guth (1981); although accelerated expansion, in fact, already had been considered, most notably by Starobinsky (1980) but also much earlier, it was the strong connection Guth made between rapid expansion and these problems that was the true beginning of inflationary cosmology.

Nevertheless, these problems can no longer be regarded as the strongest motivation for inflationary cosmology because it is not at all clear that they could ever be used to falsify inflation. In fact, they have even been eroded to some extent; for example, it formerly was thought that inflation necessarily gave a spatially flat Universe if it gave homogeneity, but there now exist inflationary models that can give a homogeneous open Universe as well (see Chapter 8). Linde in particular has been vocal (e.g., Linde 1997) in suggesting that the idea of inflation as a theory of initial conditions may be very hard to exclude, and indeed only a few possible observational signals, such as a global rotation of the observable Universe, would be in conflict with inflation in this context (Albrecht 1997; Barrow and Liddle 1997).

By contrast to inflation as a theory of initial conditions, the model of inflation as a possible origin of structure in the Universe is a powerfully predictive one. Different inflation models typically lead to different predictions for the observed structures, and observations can discriminate strongly between them. Future observations certainly will exclude most of the models currently under discussion, and they are also capable of ruling out all of them. Inflation as the origin of structure is therefore very much a proper science of prediction and observation, meriting detailed examination. It is true that even if inflation fails as a model for structure formation, one may be left with the possibility of inflation to fix the initial conditions and some other mechanism for the origin of structure [topological defects being the only known candidate, and a rather unpromising one at that; see Allen et al. (1997) and Pen et al. (1997)], but if we learn that much, we have already learned a lot.

All the standard models of inflation are based on a type of matter known as a scalar field; scalar fields are, among other things, thought to be responsible for the physics of symmetry breaking. Particle physics has yet to offer a definitive view on the detailed properties of such fields and, in particular, has not specified the potential energy, which, it turns out, is responsible for driving the inflationary expansion. The freedom exists to build a wide range of different inflationary models, based on different choices of the potential energy and perhaps different motivations for its particle physics origin. We reserve discussion of specific models until Chapter 8, to be able to discuss them in relation to their predictions for structure formation. In Chapter 3, we develop the machinery needed to deal with scalar fields in an expanding Universe, including extensive discussion of an analytical scheme known as the slow-roll approximation, which is used widely throughout the book. We also briefly discuss the end of inflation, an epoch known as reheating, though its details are not important when considering structure formation from inflation.

1.3.3 Simplest model of structure formation

The key idea in studying structure in the Universe is that of **gravitational instability**. Stated simply, this notes that if the material in the Universe is distributed irregularly, then the overdense regions provide extra gravitational attraction and draw material toward them, thus becoming more overdense. That is, under the action of gravity, irregularities become more pronounced as time passes. At the present epoch, we find that on moderate scales (e.g., less than 10 Mpc), the material in our Universe is very unevenly distributed, in the form of galaxies and clusters of galaxies. On larger scales, it begins to appear homogeneous. On the other hand, at very early times, as sampled by the cmb anisotropies, the Universe is distributed much more evenly. Gravitational instability provides a mechanism to get from a fairly smooth distribution at that time to the more irregular present Universe. It is a dramatic success that this simple picture goes a long way to explaining what is observed, and current attention is focused entirely on the details of the gravitational instability process. This depends on the nature of the Universe as a whole, for example, on how rapidly it is expanding and on how much material is in it to provide the gravitational attraction, and it also depends on the form of the initial irregularities. As we see, inflation provides the most promising theory for the origin of these initial irregularities.

The detailed study of cosmological perturbations is a highly technical topic, and we have chosen to give a simplified treatment within the main body of the book, in order to keep it at roughly the same technical level as the rest of the book. Ideas from general relativity are avoided as far as possible. From time to time, our simplified approach requires us to quote and use results without proper mathematical justification. Because an understanding of cosmological perturbations is so central to current developments in cosmology, we also provide two advanced chapters, 14 and 15, at the end of the book. For readers who are interested, these give a fully self-contained and mathematically rigorous general-relativistic treatment of cosmological perturbations, in which all the results quoted within the book are derived. These can be studied either in their own right or used to fill in the gaps of the earlier discussion.

We begin our discussion of structure formation in Chapters 4 and 5 by setting up some of the machinery for the description of perturbations in an expanding Universe. Our strategy is to keep the discussion as simple as possible, and so we focus on the simplest model, the

cold dark matter (CDM) model. In this model the Universe contains a critical density of material (making it spatially flat), all of the dark matter is cold, and the density perturbations are of a type known as adiabatic. In these chapters, we carry out an analysis with minimal reference to general relativity, really only needing the idea of a locally inertial frame in which the laws of special relativity apply. Further, in certain circumstances perturbations on small scales are amenable to a treatment using only Newtonian gravity. Here, small means relative to the characteristic length scale of an expanding Universe, the **Hubble length**.

In these chapters, we do not concern ourselves with the origin of the perturbations, deferring that until Chapter 7. We simply assume that there is an initial spectrum of perturbations that can be taken to have power-law form. We discuss the statistical nature of the perturbations, their description via their spectrum, and their evolution. This leads ultimately to predictions for the present form of the perturbations, and for the anisotropies in the microwave background. In addition to the temperature anisotropies, we discuss the polarization of the microwave background, which carries additional valuable information, as well as the effect on the microwave background if the atoms in the Universe are reionized at some epoch well before the present, enabling scattering of the microwave photons from the liberated electrons.

1.3.4 Extensions to the simplest model

Although theoretically the simplest scenario, a model in which the density is critical and all dark matter is cold is not the only possibility, and we study extensions in Chapter 6. There is considerable observational evidence that the density is less than critical, and the dark matter need not all be cold. In particular, moving to a low-density Universe, either with or without a cosmological constant, brings better concordance with large-scale structure observations.

Concerning the initial perturbations, an alternative to an adiabatic perturbation is an isocurvature perturbation, where the relative amounts of different materials are perturbed while leaving the total density constant. However, this gives much larger microwave anisotropies for a given size of density perturbation, and most likely cannot be the sole source of perturbations, though they may accompany the usual adiabatic perturbation.

In Chapter 6, we also discuss gravitational wave perturbations. These are inevitable at some level in all inflationary models. The amplitude of gravitational waves reduces rapidly once they come within the Hubble length, and they can be important only on large angular scales in the microwave background (those scales being larger than the Hubble radius at the time the background was formed). As we write there is no way of telling whether a significant fraction of the anisotropies that COBE sees are due to gravitational waves rather than density perturbations, a possibility that has been given substantial attention in the literature [see Lidsey et al. (1997) for a review]. However, in most models of inflation, and in particular within the context of a class of models known as hybrid inflation, the gravitational waves have a negligible effect.

We do not consider magnetic fields, either their possible generation during inflation or their possible effects on observations of structure such as early structure formation and the polarization of the microwave background radiation.

1.3.5 Scalar fields and the vacuum fluctuation

In Chapter 7 we carry out a detailed calculation of the density perturbations produced by inflation. Inflation gives rise to irregularities in the Universe because we live in a quantum world, not a classical one. Inflation is assumed to be driven by a scalar field and, classically, the result of the accelerated expansion is to drive the observable Universe toward a state of perfect homogeneity. However, in a quantum world, perfect homogeneity cannot be attained; we are always left with some residual fluctuations in the scalar field. The typical size of these fluctuations is a property of quantum mechanics, which means that they can be predicted using standard techniques, as we demonstrate in detail in Chapter 7. In particular, they do not depend on the initial conditions before inflation, and so, the theory is highly predictive.

It is an extravagant claim that quantum fluctuations, normally associated with microscopic phenomena, can lead to structures such as clusters of galaxies. The quantum fluctuations occurring in the space between you and this page certainly do not do anything of that sort. And, although it is true that in the early Universe the quantum fluctuations are much bigger than those we normally consider, because the timescales are so much shorter and the energy scales higher, they still will turn out to be small in the sense of being only a very minor perturbation on the classical behaviour. The crucial difference is rather that the Universe is accelerating, which means that the quantum fluctuations can be caught up in the rapid expansion and stretched to huge sizes, orders of magnitude larger than the Hubble scale, which sets the scale of causal physics. Once the fluctuation is taken to such a large scale, it is unable to evolve and becomes frozen-in; crucially, scales are pulled outside the horizon with such swiftness that the amplitude has no chance to tend to zero, but instead is frozen-in at a fixed nonzero value.

In early work on inflationary perturbations, they normally were described as giving a type of density perturbations known as a scale-invariant or Harrison–Zel'dovich spectrum (basically, because physical conditions do not change much as perturbations on all relevant scales are produced). Since then, observations have developed to such an extent that this approximation can no longer be used, and must be replaced by something more accurate. For the observations described in this book, it normally proves adequate to approximate the perturbations by a power-law spectrum, with different inflation models leading to different spectral indices. Future observations can measure the spectral index very precisely, and hence discriminate strongly between different models of inflation.

By the same mechanism, inflationary models inevitably produce gravitational waves at some level. In some models, these may be significant enough to be detectable.

Although the inflationary perturbations typically are calculated in terms of the scalar field perturbation, this leads to a perturbation in the total energy density of the Universe, and hence in the spatial curvature of the Universe. This last is the most useful when one comes to consider how the perturbations will evolve; the scalar field is not useful because it decays long before the present. In the standard scenario, where the perturbations are adiabatic, the perturbation in the spatial curvature remains constant as long as the scale is larger than the Hubble length. This allows us to evolve the perturbations forward in time until the Hubble length grows to encompass them in the postinflationary Universe; for the scales of interest for structure formation, this happens in the recent past, where the standard Big Bang model applies.