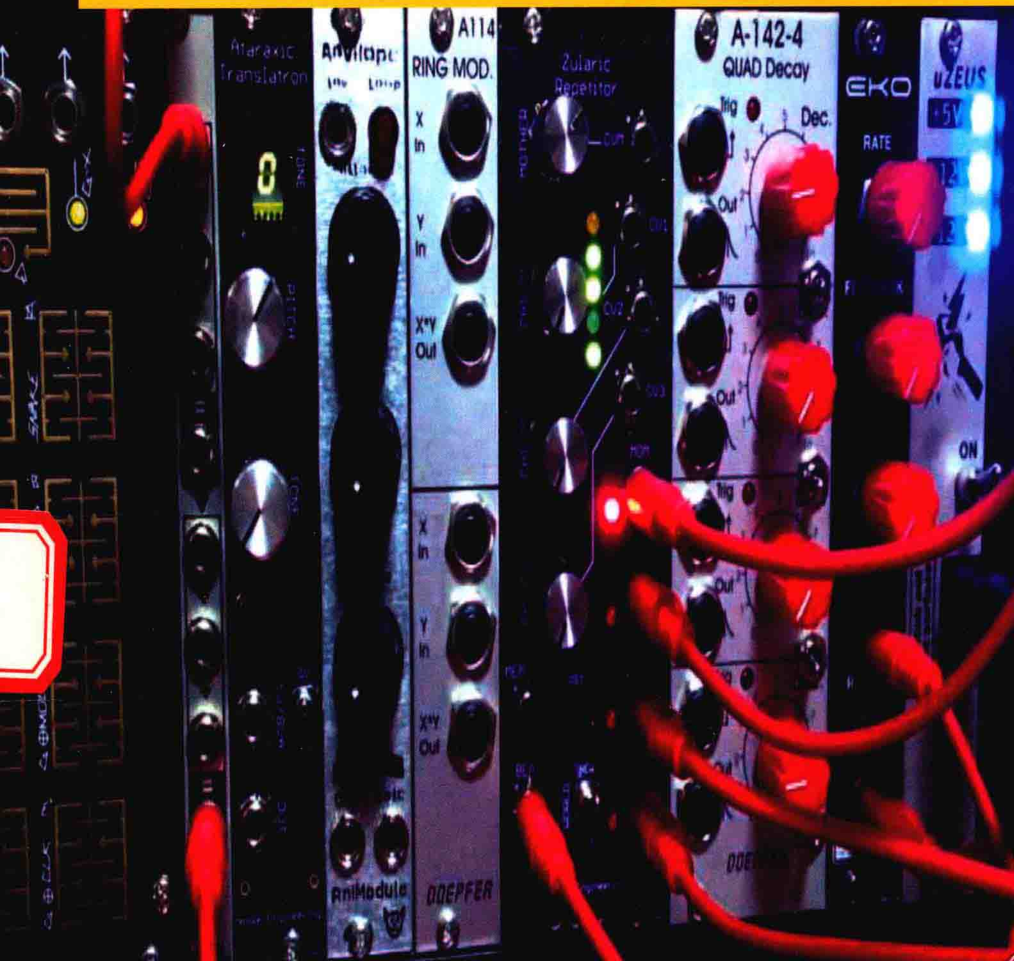
Handbook of
Mechanical Vibrations and Noise Engineering

Contributors | **Quansheng Ji, Xiaomei Ji et al.**



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List of Abbreviations

CFT	Complex Fourier transform
CLF	Coupling Loss Factor
CVD	Chemical Vapor Deposition
DAQ	Data Acquisition
DLF	Damping Loss Factor
EDX	X-Ray Spectrometry
EMFRF	Electro Mechanical Frequency Response Function
HVDC	High Voltage Direct Current
IRFT	Inverse Real Fourier transform
LECD	Localized Electrochemical Deposition
PD	Proportional Derivative
PDV	Portable Digital Vibrometer
SEA	Statistical Energy Analysis
SIL	Sound Intensity
SPL	Sound Pressure Level
SWL	Sound Power Level
TAF	Torque Amplification Factor
XRD	X-Ray Diffraction

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Preface

The study of vibrations is concerned with the oscillating motion of elastic bodies and the force associated with them. Mechanical vibration is defined as the measurement of a periodic process of oscillations with respect to an equilibrium point. Handbook of Mechanical Vibrations and Noise Engineering provides essential concepts involving vibrational analysis, uncertainty modeling, and vibration control. It should also give a good fundamental basis in computational results, mathematical modeling and assessment in performance of different systems and system components. In first chapter, we propose a unified differential operator method to study mechanical vibrations, solving inhomogeneous linear ordinary differential equations with constant coefficients. In second chapter, an extensive analysis of the mechanical noise due to the building vibration has been analyzed and possible solutions to the problem discussed. Third chapter presents an exploratory study of the slotted stand-off layer damping treatment for rail vibration and noise control using both theoretical analysis and laboratory tests. A theoretical model for predicting noise reduction in coupled workshops is presented in fourth chapter by using statistical energy analysis (SEA) method. An opening between the coupled workshops is considered into the theoretical model properly. In fifth chapter, an attempt is made to investigate the effect of sound vibration, in particular, the frequency of vibration on the deposition rate. In addition to deposition rate, the quality of deposited coating is also investigated. In sixth chapter, a new noise level prediction method is proposed based on a frequency response function considering both electrical and mechanical characteristics of capacitors. In seventh chapter, several techniques have been proposed to reduce gear noise and vibrations in recent years. Noise source identification of a ring-plate cycloid reducer based on coherence analysis has been presented in eighth chapter. Ninth chapter emphasizes on noise of induction machines and automotive applications of active vibration control are presented in tenth chapter. Eleventh chapter mainly refers to the literatures on torsional vibration issue published in recent years, summarizes on the modeling of torsional vibration, corresponding analysis methods, appropriate measures and torsional vibration control, and points out the problems to be solved in the study and some new research directions. Last chapter presents an application of zero-power controlled magnetic levitation for active vibration control. Vibration isolation are strongly required in the field of high-resolution measurement and micromanufacturing, for instance, in the submicron semiconductor chip manufacturing, scanning probe microscopy, holographic interferometry, cofocal optical imaging, etc. to obtain precise and repeatable results.

Contents

Chapter 1

A NEW DIFFERENTIAL OPERATOR METHOD TO STUDY THE MECHANICAL VIBRATION

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ABSTRACT

In this paper, we propose a unified differential operator method to study mechanical vibrations, solving inhomogeneous linear ordinary differential equations with constant coefficients. The main advantage of this new method is that the differential operator D in the numerator of the fraction has no effect on input functions (i.e., the derivative operation is removed) because we take the fraction as a whole part in the partial fraction expansion. The method in various variants is widely implemented in related fields in mechanics and engineering. We also point out that the same mistakes in the differential operator method are found in the related references [1-4].

INTRODUCTION

A very serious equipment accident occurred in 1982 in a certain factory in the iron and steel industry in Jinan, China, where the safety pin of the middle plate of a three-roll mill for mechanical protection was not broken, but the main reducer (2800 kw, center distance 1900 mm) was so damaged that the whole gear produced crack with six connection bolts whose diameter is 64 mm of highspeed side pulled off, resulting in that the production had been halted for more than 20 days, and leading to huge economic losses. In the analysis of such serious equipment accident, we employed differential operator method in [3], Laplace transform and modal analysis method to calculate the natural frequency of torsional vibration and torque amplification factor for the main transmission system in medium plate rolling-mill. It was surprising that some mistakes were found in the solutions to the dynamic resonance of the above system through differential operator method, which was inconsistent with the correct results by Laplace transform and modal analysis method. Having compared with Laplace transform, we proposed a new differential operator method and solved the above same problem, obtaining the correct results completely consistent with modal analysis method and Laplace transform. We thus solved the disastrous equipment accident with little time. Here we illustrate the new differential operator method as follows: that differential operator D of numerator part in the fraction has no effect on input function because we do not need derivative operation working on input function. However, the fraction is taken as a whole part, using partial fraction expansion. Here we apply the related property of D operator in [5,6]. In [5], there is strict mathematical foundation for the partial fraction expansion of D operator. Finally, we obtain particular solution and general solution based on Cramer's rule as well as initial conditions. This method is suitable and powerful for solutions to different governing equations in many related vibration problems in the mechanics and engineering fields, showing more flexibility and superiority to other methods. It has not only enriched analytic methods in mechanical dynamics but also made contributions to the dynamic analysis in metallurgical equipments, where it was originally reported in [7]. Currently, in general teaching references in [1,2], for constant coefficients differential equations, the differential operator D in numerator part in its fraction affects input function through derivative operation and gives rise to errors herein.

THE NEW DIFFERENTIAL OPERATOR METHOD

Without loss of generality, we introduce the new differential operator method via the rectal serial five masses torsional vibration system, see **Figure 1**, which is simplified in **Figure 2** in the transmission system in the above original accident in 1982 we mentioned, assuming initial conditions are zero (i.e., we suppose initial displacement and initial velocity as zero, and as a general description for initial conditions, see [8]). We state the related parameters as follows:

1) Motor (2800 kW); 2) Gear coupling; 3) Flywheel; 4) Main reducer(center distance 1900 mm); 5) Safety pin coupling; 6) Herringbone gear stand; 7) Universal joint; 8) Top roller; 9) Medium roller; 10) Steel plate; 11) Lower roller.

j_i(i = 1, 2, 3, 4, 5): Moment of inertia;

k_{ij}(i,= 1, 2, 3, 4;j=2,3,4,5): Torsional elastic coefficient of every axial segment (rigidity);

φ_i (i = 1, 2, 3, 4, 5): Angular position;

$\ddot{\varphi}_i$ (i = 1, 2, 3, 4, 5): Angular acceleration;

M_{ij}(i,= 1, 2, 3, 4; j = 2, 3, 4, 5): Torque of every axial segment;

M: Excitation torque;

D: Differential operator on behalf of d/dt;

Without the viscosity and damping, we set up the governing equations in the system:

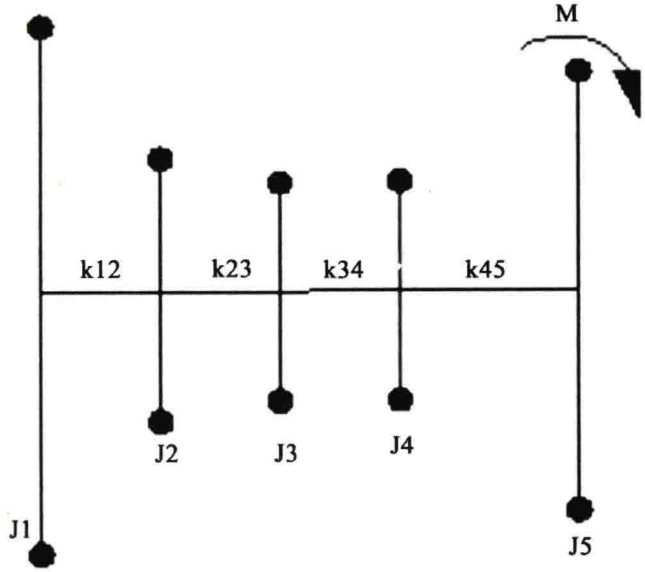

Figure 1. Diagram of five straight string masses system.

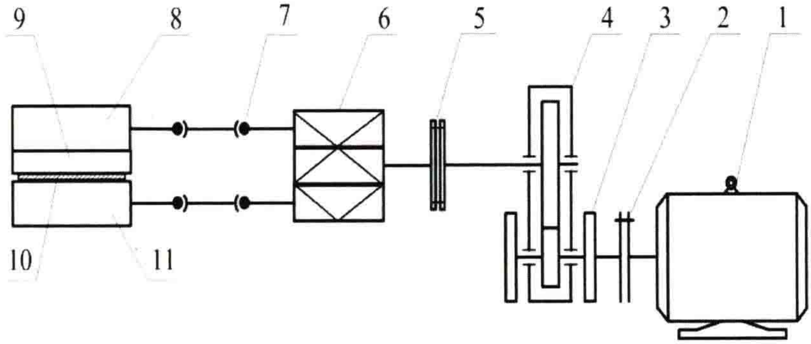

Figure 2. Diagram of transmission system.

$$\begin{cases} J_1\ddot{\phi}_1 + K_{12}(\phi_1 - \phi_2) = 0 \\ J_2\ddot{\phi}_2 + K_{23}(\phi_2 - \phi_3) - K_{12}(\phi_1 - \phi_2) = 0 \\ J_3\ddot{\phi}_3 + K_{34}(\phi_3 - \phi_4) - K_{23}(\phi_2 - \phi_3) = 0 \\ J_4\ddot{\phi}_4 + K_{45}(\phi_4 - \phi_5) - K_{34}(\phi_3 - \phi_4) = 0 \\ J_5\ddot{\phi}_5 - K_{45}(\phi_4 - \phi_5) = -M \end{cases} \tag{2.1}$$

Let $P_{ij}^2 = K_{ij}\dfrac{J_i + J_j}{J_i J_j}, i = 1,2,3,4; j = 2,3,4,5,$

and

$$\begin{cases} M_{12} = K_{12}(\phi_1 - \phi_2) \\ M_{23} = K_{23}(\phi_2 - \phi_3) \\ M_{34} = K_{34}(\phi_3 - \phi_4) \\ M_{45} = K_{45}(\phi_4 - \phi_5) \end{cases} \tag{2.2}$$

Then

$$\begin{cases} \ddot{M}_{12} + P_{12}^2 M_{12} - \dfrac{K_{12}}{J_2} M_{23} = 0 \\ \ddot{M}_{23} + P_{23}^2 M_{23} - \dfrac{K_{23}}{J_2} M_{12} - \dfrac{K_{23}}{J_3} M_{34} = 0 \\ \ddot{M}_{34} + P_{34}^2 M_{34} - \dfrac{K_{34}}{J_3} M_{23} - \dfrac{K_{34}}{J_4} M_{45} = 0 \\ \ddot{M}_{45} + P_{45}^2 M_{45} - \dfrac{K_{45}}{J_4} M_{34} = \dfrac{K_{45}}{J_5} M \end{cases} \tag{2.3}$$

And

$$\begin{cases} (D^2 + P_{12}^2) M_{12} - \dfrac{K_{12}}{J_2} M_{23} = 0 \\ (D^2 + P_{23}^2) M_{23} - \dfrac{K_{23}}{J_2} M_{12} - \dfrac{K_{23}}{J_3} M_{34} = 0 \\ (D^2 + P_{34}^2) M_{34} - \dfrac{K_{34}}{J_3} M_{23} - \dfrac{K_{34}}{J_4} M_{45} = 0 \\ (D^2 + P_{45}^2) M_{45} - \dfrac{K_{45}}{J_4} M_{34} = \dfrac{K_{45}}{J_5} M \end{cases} \tag{2.4}$$

Let the determinant of the coefficient of (2.4) be Δ. Then $\Delta = (D^2 + P_1^2)(D^2 + P_2^2)(D^2 + P_3^2)(D^2 + P_4^2)$ where P_1, P_2, P_3, P_4 are from the first order to the fourth order natural frequencies in the main transmission system.

By Cramer's rule, we calculate $\Delta_{12}, \Delta_{23}, \Delta_{34}, \Delta_{45}$, where

$$\Delta_{12} = \frac{K_{12}K_{23}K_{34}K_{45}}{J_1J_2J_3J_4J_5} J_1 M,$$

$$\Delta_{23} = \frac{K_{12}K_{23}K_{34}K_{45}}{J_1J_2J_3J_4J_5}(J_1 + J_2)M + \frac{K_{23}K_{34}K_{45}}{J_3J_4J_5} D^2 M,$$

$$\Delta_{34} = \frac{K_{12}K_{23}K_{34}K_{45}}{J_1J_2J_3J_4J_5}(J_1 + J_2 + J_3)M$$

$$+\frac{K_{34}K_{45}}{J_4J_5}\left(P_{12}^2 + P_{23}^2\right)D^2 M + \frac{K_{34}K_{45}}{J_4J_5} D^4 M.$$

and similarly, we obtain Δ_{45}.

We must pay attention to that we just take differential operator D as an algebra symbol, so it has no effect on input function, that is to say, we need not derivative operation, and we take the fraction with differential operator D in numerator and denominator as a whole part in partial fraction expansion. If we assume initial conditions of the system are zero and M is a step function, we can obtain analytic formulae of each axial segment torque individually below:

$$M_{12} = \frac{\Delta_{12}}{\Delta} = \frac{1}{\left(D^2 + P_1^2\right)\left(D^2 + P_2^2\right)\left(D^2 + P_3^2\right)\left(D^2 + P_4^2\right)} \frac{K_{12}K_{23}K_{34}K_{45}}{J_1J_2J_3J_4J_5} J_1 M$$

$$= \frac{J_1 M}{J_1 + J_2 + J_3 + J_4 + J_5} \times \left[\frac{P_2^2P_3^2P_4^2\left(1-\cos P_1 t\right)}{\left(P_2^2 - P_1^2\right)\left(P_3^2 - P_1^2\right)\left(P_4^2 - P_1^2\right)} + \frac{P_1^2P_3^2P_4^2\left(1-\cos P_2 t\right)}{\left(P_1^2 - P_2^2\right)\left(P_3^2 - P_2^2\right)\left(P_4^2 - P_2^2\right)}\right.$$

$$\left. + \frac{P_1^2P_2^2P_4^2\left(1-\cos P_3 t\right)}{\left(P_1^2 - P_3^2\right)\left(P_2^2 - P_3^2\right)\left(P_4^2 - P_3^2\right)} + \frac{P_1^2P_2^2P_3^2\left(1-\cos P_4 t\right)}{\left(P_1^2 - P_4^2\right)\left(P_3^2 - P_4^2\right)\left(P_2^2 - P_4^2\right)}\right];$$

$$M_{23} = \frac{\left(J_1 + J_2\right)}{J_1 + J_2 + J_3 + J_4 + J_5} M \left[\frac{P_2^2P_3^2P_4^2\left(1-\cos P_1 t\right)}{\left(P_2^2 - P_1^2\right)\left(P_3^2 - P_1^2\right)\left(P_4^2 - P_1^2\right)} + \frac{P_1^2P_3^2P_4^2\left(1-\cos P_2 t\right)}{\left(P_1^2 - P_2^2\right)\left(P_3^2 - P_2^2\right)\left(P_4^2 - P_2^2\right)}\right.$$

$$\left. + \frac{P_1^2P_2^2P_4^2\left(1-\cos P_3 t\right)}{\left(P_1^2 - P_3^2\right)\left(P_2^2 - P_3^2\right)\left(P_4^2 - P_3^2\right)} + \frac{P_1^2P_2^2P_3^2\left(1-\cos P_4 t\right)}{\left(P_1^2 - P_4^2\right)\left(P_3^2 - P_4^2\right)\left(P_2^2 - P_4^2\right)}\right]$$

$$-\frac{K_{23}K_{34}K_{45}}{J_3J_4J_5} M \left[\frac{1-\cos P_1 t}{\left(P_2^2 - P_1^2\right)\left(P_3^2 - P_1^2\right)\left(P_4^2 - P_1^2\right)} + \frac{1-\cos P_2 t}{\left(P_1^2 - P_2^2\right)\left(P_3^2 - P_2^2\right)\left(P_4^2 - P_2^2\right)}\right.$$

$$\left. + \frac{1-\cos P_3 t}{\left(P_1^2 - P_3^2\right)\left(P_2^2 - P_3^2\right)\left(P_4^2 - P_3^2\right)} + \frac{1-\cos P_4 t}{\left(P_1^2 - P_4^2\right)\left(P_3^2 - P_4^2\right)\left(P_2^2 - P_4^2\right)}\right];$$

$$M_{34}=\frac{(J_1+J_2+J_3)}{J_1+J_2+J_3+J_4+J_5}M\left[\frac{P_2^2P_3^2P_4^2(1-\cos P_1t)}{(P_2^2-P_1^2)(P_3^2-P_1^2)(P_4^2-P_1^2)}+\frac{P_1^2P_3^2P_4^2(1-\cos P_2t)}{(P_1^2-P_2^2)(P_3^2-P_2^2)(P_4^2-P_2^2)}\right.$$
$$\left.+\frac{P_1^2P_2^2P_4^2(1-\cos P_3t)}{(P_1^2-P_3^2)(P_2^2-P_3^2)(P_4^2-P_3^2)}+\frac{P_1^2P_2^2P_3^2(1-\cos P_4t)}{(P_1^2-P_4^2)(P_3^2-P_4^2)(P_2^2-P_4^2)}\right]$$
$$+\frac{K_{34}K_{45}}{J_4J_5}M\left[\frac{P_1^2(1-\cos P_1t)}{(P_2^2-P_1^2)(P_3^2-P_1^2)(P_4^2-P_1^2)}+\frac{P_2^2(1-\cos P_2t)}{(P_1^2-P_2^2)(P_3^2-P_2^2)(P_4^2-P_2^2)}\right.$$
$$\left.+\frac{P_3^2(1-\cos P_3t)}{(P_1^2-P_3^2)(P_2^2-P_3^2)(P_4^2-P_3^2)}+\frac{P_4^2(1-\cos P_4t)}{(P_1^2-P_4^2)(P_3^2-P_4^2)(P_2^2-P_4^2)}\right]$$
$$-\frac{K_{34}K_{45}}{J_4J_5}(P_{12}^2+P_{23}^2)M\left[\frac{1-\cos P_1t}{(P_2^2-P_1^2)(P_3^2-P_1^2)(P_4^2-P_1^2)}+\frac{1-\cos P_2t}{(P_1^2-P_2^2)(P_3^2-P_2^2)(P_4^2-P_2^2)}\right.$$
$$\left.+\frac{1-\cos P_3t}{(P_1^2-P_3^2)(P_2^2-P_3^2)(P_4^2-P_3^2)}+\frac{1-\cos P_4t}{(P_1^2-P_4^2)(P_3^2-P_4^2)(P_2^2-P_4^2)}\right];$$

$$M_{45}=\frac{(J_1+J_2+J_3+J_4)}{J_1+J_2+J_3+J_4+J_5}M\left[\frac{P_2^2P_3^2P_4^2(1-\cos P_1t)}{(P_2^2-P_1^2)(P_3^2-P_1^2)(P_4^2-P_1^2)}+\frac{P_1^2P_3^2P_4^2(1-\cos P_2t)}{(P_1^2-P_2^2)(P_3^2-P_2^2)(P_4^2-P_2^2)}\right.$$
$$\left.+\frac{P_1^2P_2^2P_4^2(1-\cos P_3t)}{(P_1^2-P_3^2)(P_2^2-P_3^2)(P_4^2-P_3^2)}+\frac{P_1^2P_2^2P_3^2(1-\cos P_4t)}{(P_1^2-P_4^2)(P_3^2-P_4^2)(P_2^2-P_4^2)}\right]$$
$$-\frac{K_{45}}{J_5}M\left[\frac{P_1^4(1-\cos P_1t)}{(P_2^2-P_1^2)(P_3^2-P_1^2)(P_4^2-P_1^2)}+\frac{P_2^4(1-\cos P_2t)}{(P_1^2-P_2^2)(P_3^2-P_2^2)(P_4^2-P_2^2)}+\frac{P_3^4(1-\cos P_3t)}{(P_1^2-P_3^2)(P_2^2-P_3^2)(P_4^2-P_3^2)}\right.$$
$$\left.+\frac{P_4^4(1-\cos P_4t)}{(P_1^2-P_4^2)(P_3^2-P_4^2)(P_2^2-P_4^2)}\right]+\frac{K_{45}}{J_5}(P_{12}^2+P_{23}^2+P_{34}^2)M$$
$$\times\left[\frac{P_1^2(1-\cos P_1t)}{(P_2^2-P_1^2)(P_3^2-P_1^2)(P_4^2-P_1^2)}+\frac{P_2^2(1-\cos P_2t)}{(P_1^2-P_2^2)(P_3^2-P_2^2)(P_4^2-P_2^2)}+\frac{P_3^2(1-\cos P_3t)}{(P_1^2-P_3^2)(P_2^2-P_3^2)(P_4^2-P_3^2)}\right.$$
$$\left.+\frac{P_4^2(1-\cos P_4t)}{(P_1^2-P_4^2)(P_3^2-P_4^2)(P_2^2-P_4^2)}\right]-\frac{K_{45}}{J_5}\left(P_{12}^2P_{23}^2+P_{12}^2P_{34}^2+P_{23}^2P_{34}^2-\frac{K_{23}K_{34}}{J_3^2}-\frac{K_{23}K_{34}}{J_2^2}\right)$$
$$\times M\left[\frac{1-\cos P_1t}{(P_2^2-P_1^2)(P_3^2-P_1^2)(P_4^2-P_1^2)}+\frac{1-\cos P_2t}{(P_1^2-P_2^2)(P_3^2-P_2^2)(P_4^2-P_2^2)}\right.$$
$$\left.+\frac{1-\cos P_3t}{(P_1^2-P_3^2)(P_2^2-P_3^2)(P_4^2-P_3^2)}+\frac{1-\cos P_4t}{(P_1^2-P_4^2)(P_3^2-P_4^2)(P_2^2-P_4^2)}\right].$$

If we follow the differential operator D of numerator part in its faction has effect on input function in general mathematics handbook [4] and the teaching material of constant differential equations in textbook [1,2] (that is to say, we carry out derivative operation), thus we can acquire the following items if M is a step function and initial conditions are zero:

$$\overline{\Delta_{12}}=\frac{K_{12}K_{23}K_{34}K_{45}}{J_1J_2J_3J_4J_5}J_1M;$$

$$\overline{\Delta_{34}}=\frac{K_{12}K_{23}K_{34}K_{45}}{J_1J_2J_3J_4J_5}(J_1+J_2+J_3)M+\frac{K_{34}K_{45}}{J_4J_5}(P_{12}^2+P_{23}^2)D^2M+\frac{K_{34}K_{45}}{J_4J_5}D^4M=\frac{K_{12}K_{23}K_{34}K_{45}}{J_1J_2J_3J_4J_5}(J_1+J_2+J_3)M;$$

$$\overline{\Delta_{23}} = \frac{K_{12}K_{23}K_{34}K_{45}}{J_1J_2J_3J_4J_5}(J_1+J_2)M + \frac{K_{23}K_{34}K_{45}}{J_3J_4J_5}D^2M = \frac{K_{12}K_{23}K_{34}K_{45}}{J_1J_2J_3J_4J_5}(J_1+J_2)M;$$

$$\overline{\Delta_{45}} = \frac{K_{12}K_{23}K_{34}K_{45}}{J_1J_2J_3J_4J_5}(J_1+J_2+J_3+J_4)M - \frac{K_{45}}{J_5}D^6M - \frac{K_{45}}{J_5}\left(P_{12}^2+P_{23}^2+P_{34}^2\right)D^4M - \frac{K_{45}}{J_5}$$
$$\times\left[P_{12}^2P_{23}^2+P_{12}^2P_{34}^2+P_{23}^2P_{34}^2-\frac{K_{23}K_{34}}{J_3^2}-\frac{K_{23}K_{34}}{J_2^2}\right]D^2M = \frac{K_{12}K_{23}K_{34}K_{45}}{J_1J_2J_3J_4J_5}(J_1+J_2+J_3+J_4)M;$$

$$\overline{M_{12}} = \frac{J_1}{J_1+J_2+J_3+J_4+J_5}M\left[\frac{P_2^2P_3^2P_4^2(1-\cos P_1t)}{(P_2^2-P_1^2)(P_3^2-P_1^2)(P_4^2-P_1^2)}+\frac{P_1^2P_3^2P_4^2(1-\cos P_2t)}{(P_1^2-P_2^2)(P_3^2-P_2^2)(P_4^2-P_2^2)}\right.$$
$$\left.+\frac{P_1^2P_2^2P_4^2(1-\cos P_3t)}{(P_1^2-P_3^2)(P_2^2-P_3^2)(P_4^2-P_3^2)}+\frac{P_1^2P_2^2P_3^2(1-\cos P_4t)}{(P_1^2-P_4^2)(P_3^2-P_4^2)(P_2^2-P_4^2)}\right];$$

$$\overline{M_{23}} = \frac{(J_1+J_2)}{J_1+J_2+J_3+J_4+J_5}M\left[\frac{P_2^2P_3^2P_4^2(1-\cos P_1t)}{(P_2^2-P_1^2)(P_3^2-P_1^2)(P_4^2-P_1^2)}+\frac{P_1^2P_3^2P_4^2(1-\cos P_2t)}{(P_1^2-P_2^2)(P_3^2-P_2^2)(P_4^2-P_2^2)}\right.$$
$$\left.+\frac{P_1^2P_2^2P_4^2(1-\cos P_3t)}{(P_1^2-P_3^2)(P_2^2-P_3^2)(P_4^2-P_3^2)}+\frac{P_1^2P_2^2P_3^2(1-\cos P_4t)}{(P_1^2-P_4^2)(P_3^2-P_4^2)(P_2^2-P_4^2)}\right];$$

$$\overline{M_{34}} = \frac{(J_1+J_2+J_3)}{J_1+J_2+J_3+J_4+J_5}M\left[\frac{P_2^2P_3^2P_4^2(1-\cos P_1t)}{(P_2^2-P_1^2)(P_3^2-P_1^2)(P_4^2-P_1^2)}+\frac{P_1^2P_3^2P_4^2(1-\cos P_2t)}{(P_1^2-P_2^2)(P_3^2-P_2^2)(P_4^2-P_2^2)}\right.$$
$$\left.+\frac{P_1^2P_2^2P_4^2(1-\cos P_3t)}{(P_1^2-P_3^2)(P_2^2-P_3^2)(P_4^2-P_3^2)}+\frac{P_1^2P_2^2P_3^2(1-\cos P_4t)}{(P_1^2-P_4^2)(P_3^2-P_4^2)(P_2^2-P_4^2)}\right];$$

$$\overline{M_{45}} = \frac{(J_1+J_2+J_3+J_4)}{J_1+J_2+J_3+J_4+J_5}M\left[\frac{P_2^2P_3^2P_4^2(1-\cos P_1t)}{(P_2^2-P_1^2)(P_3^2-P_1^2)(P_4^2-P_1^2)}+\frac{P_1^2P_3^2P_4^2(1-\cos P_2t)}{(P_1^2-P_2^2)(P_3^2-P_2^2)(P_4^2-P_2^2)}\right.$$
$$\left.+\frac{P_1^2P_2^2P_4^2(1-\cos P_3t)}{(P_1^2-P_3^2)(P_2^2-P_3^2)(P_4^2-P_3^2)}+\frac{P_1^2P_2^2P_3^2(1-\cos P_4t)}{(P_1^2-P_4^2)(P_3^2-P_4^2)()}\right](P_2^2-P_4^2).$$

Obviously, the above result of torsional vibration dynamic resonance in every axial segment shows only related to the distribution coefficients of moment of inertia with the same coefficients of frequency difference and the same time of every axial segment attained by peak moment, but it is inappropriate in practice. Above result seems only like the ordinary dynamic questions (the starting of acceleration movement and the braking decaleration movement); however, it apparently does not belong to questions of torsional vibration. Consequently, the results via the new differential operator method we mentioned above are not only as same as that by Laplace transform and