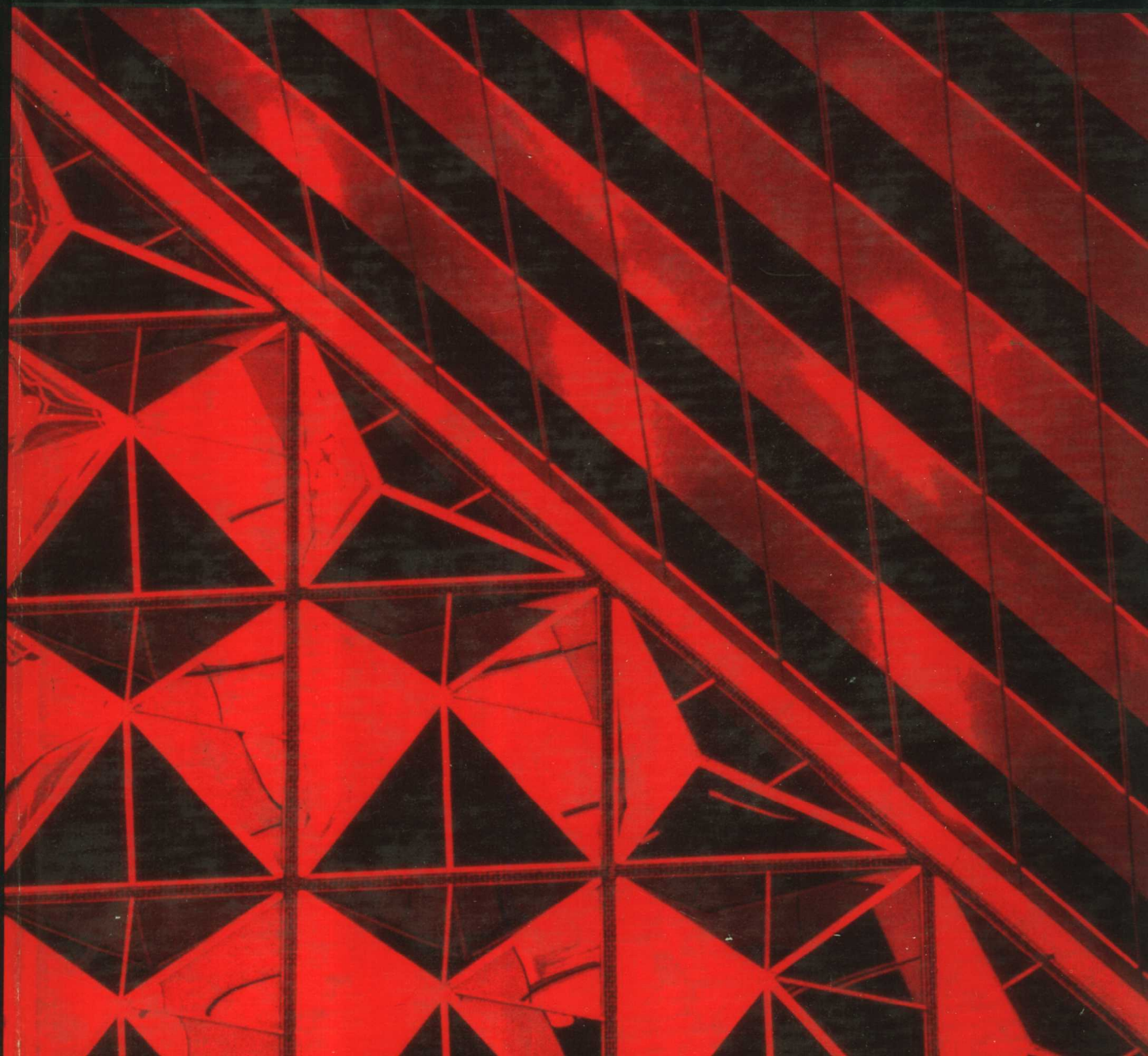
PRECALCULUS MATHEMATICS

SECOND EDITION

Hungerford & Mercer



Second Edition

Precalculus Mathematics

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Preface

This book is intended for students who have had two to three years of high school mathematics. It is designed to provide the essential mathematical background needed in calculus and is suitable for use in courses such as college algebra, trigonometry, algebra and trigonometry, elementary functions, and precalculus mathematics.

Our goal in this second edition of *Precalculus Mathematics* is the same as in the first: to produce a text that can be understood by an average student with a minimal amount of outside assistance, and to do so without any sacrifice in rigor. We have done our best to present sound mathematics in an informal manner that stresses detailed explanations, extensive use of pictures and diagrams, numerous examples, and emphasis on the real world origins of basic concepts (especially functions).

There is, alas, no way to do this in a short space. A given topic may well occupy more space here than in some other texts, but this length is misleading. We have found that a student can usually read the discussion here more easily and with greater comprehension than a terse, compact presentation elsewhere. So in the long run the extra length should cause no serious scheduling problems. Most important, it provides the added benefit of greater understanding.

There are several significant changes in the second edition:

New Material The coverage of conic sections has been greatly expanded. An entire chapter is now devoted to systems of equations. Determinants are discussed in greater detail and an optional section on nonlinear systems has been included. Two sections on sequences and sums have been added.

Improved Presentations Every section of the book was carefully examined and changed where necessary to improve clarity. The material on inverse functions has been completely rewritten and now appears in Chapter 2 instead of Chapter 7. An easier method of graphing polynomial functions is presented. The section on trigonometric equations has been redone, and the solution algorithm for such equations more clearly explained.

Exercises Many new exercises have been added. The routine calculation and drill problems are now paired and the answer to *every* odd-numbered problem is given at the end of the book. The exercise numbering scheme of the first edition has been replaced by conventional numbering.

Chapter Reviews Each chapter now concludes with a list of important concepts (referenced by section), a summary of important facts and formulas, and a list of sample examination questions. These questions are not just a duplication of exercises in earlier sections, but are questions used by the authors and others on actual exams. They include many questions that tie together material from several sections in the chapter, or from other chapters.

Order of Topics The order of presentation has been changed extensively to provide greater flexibility and to make it possible to omit less important topics more easily. See the note To the Instructor on page xiii for complete details.

At various stages in the writing of both editions of this book the authors received crucial assistance and advice from a number of people, for which we are profoundly grateful: Sybil Barrier (co-author of the first edition), Caspar Curjel (University of Washington), Steve Monk (University of Washington), Vivian Klein Lawrence (Wellesley College), and Jim Hassel (Cleveland State University and Berea High School). We also want to thank the following reviewers for many constructive comments: Robert Arnold (California State University, Fresno); Rick Billstein (University of Montana); Milton Cox (Miami University, Oxford, OH); Albert Fadell (State University of New York, Buffalo); Gerald Gannon (California State University, Fullerton); Joan M. Grahlfs (University of Wisconsin, Marathon Center); Jerrold W. Grossman (Oakland University); Thomas E. Hale (California Polytechnic State University); Douglas Hall (Michigan State University); James E. Hall (University of Wisconsin, Madison); Adam J. Hulin (University of New Orleans); Robert Lohman (Kent State University); Curtis McKnight (University of Oklahoma); Mike Morse (East Los Angeles Community College); James Mueller (California Polytechnic State University); Joan Mundy (University of New Hampshire); Anne Pfenning (Rose State College); Antoinette Trembinska (Northwestern University); Loyd Wilcox (Golden West College); and Edward Zeidman (Essex Community College).

We are happy to acknowledge the expert work of our typists Ann Melville and Joyce Pluth. Finally we want to express our appreciation to the staff of Saunders College Publishing for their assistance, with special thanks to Jay Freedman, Leslie Hawke, Sally Kusch, and Kate Pachuta.

T. W. H.
R. E. M.

Cleveland, Ohio
Dayton, Ohio
September, 1984

To The Instructor

Every effort has been made to make this text as flexible as possible, so that it may be effectively used in a wide variety of courses. You can easily adapt it to the needs of your own class by consulting the chart on the following page and the information below.

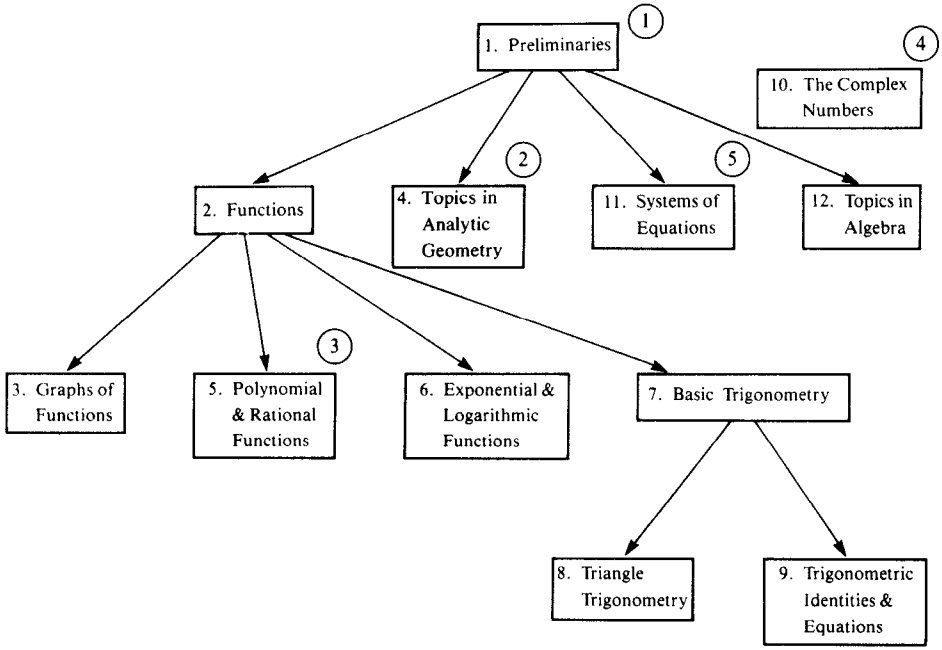
Among other possibilities, it should be noted that the order of topics used in the first edition (where analytic geometry preceded the introduction of functions) can still be used here: simply cover Chapter 4 after Chapter 1 and before Chapter 2. At least three of the possible permutations of Chapters 5, 6, 7 have been successfully used by the authors when teaching a course in elementary functions.

The *Algebra Review Appendix* contains all the intermediate algebra needed to read the text. Well prepared students should not need it (except perhaps as an occasional reference). With weaker students, however, you may want to cover this material in a formal way by treating the *Algebra Review* as a regular chapter. In this case it may be covered either before Chapter 1 or immediately after Section 1.1.

The *Geometry Review Appendix* is intended as a convenient reference for individual students. It summarizes those facts from plane geometry that are frequently used in the text. When such a fact is used in the text, there is usually a reference to the *Geometry Review*. So it should not be necessary to cover this material in class.

Interdependence of Chapters

Circled superscripts refer to the Notes below.



1 The Algebra Review Appendix is a prerequisite for Chapter 1; it is discussed on page xiii.

2 Section 4.6 (parabolas) depends in part on Section 3.3 (quadratic functions).

3 Graphs of first degree polynomial functions are treated in Section 4.2.

4 Section 10.1 (definition and arithmetic of complex numbers; the complex plane) may be covered at any time after Chapter 1. Section 10.2 (equations and complex numbers) may be covered at any time after Section 5.3. Section 10.3 (polar form, powers, and roots) may be covered at any time after Section 7.4.

5 Excursion 1.A (nonlinear systems of equations) depends on Section 5.2 (solutions of polynomial equations).

Interdependence of Sections Within Each Chapter

The list below may be used in conjunction with the Table of Contents to design a course that exactly fits the needs of your students.

Chapter 1 All of this material is essential for the rest of the book and should be covered in order.

Chapter 2 Section 3 (graphing) and Section 4 (operations on functions) depend on Sections 1 and 2, but are independent of each other and may be read in either order. Sections 1–4 are essential for Chapters 3–9. Section 5 (inverse functions) depends on Section 4; it is needed only in Section 6.3 (logarithmic functions) and Section 9.5 (inverse trigonometric functions) and may be omitted until then if desired.

Chapter 3 Section 3 (quadratic functions) depends on Section 2 (new graphs from old); Sections 1, 2, and 4 are independent and may be read in any order. Sections 2 and 3 provide the complete algebraic treatment of graphs of quadratic functions that some instructors prefer. However, this material may be safely omitted by instructors who would rather leave the finding of vertices (i.e., maxima and minima of quadratic functions) to calculus. A student need only know that the graph of a quadratic function is a parabola in order to deal with quadratic inequalities in Section 5.6 (the location of the vertex is irrelevant). For other purposes, the general graphing techniques for polynomials in Section 5.4 should be sufficient.

Chapter 4 Section 2 (equations of lines) depends on Section 1 (slopes of lines); Section 6 (parabolas) depends in part on Section 3.3 (quadratic functions). Otherwise the sections in this chapter are independent and may be read in any order. A briefer treatment of conic sections is possible by covering the standard forms of equations of conics centered at the origin, and omitting the remaining material in Sections 3–5.

Chapter 5 Section 1 (polynomials) and Section 2 (linear and quadratic equations) are essential for the rest of the chapter. Section 3 (roots of higher degree polynomials) is used in occasional exercises and examples in later sections, but may be omitted without causing difficulty for students. Section 4 (graphs of polynomial functions) and Section 5 (graphs of rational functions) are independent of each other. So either one of them may be omitted if desired. A brief, simplified treatment of rational functions is possible if one covers only the first part of Section 5 (through the subsection on linear rational functions).

Section 6 (inequalities) is divided into four essentially independent parts: linear inequalities; quadratic inequalities (which uses only the fact that the graph of a quadratic polynomial is a parabola); higher degree polynomial inequalities (which depends on Section 4); and rational inequalities (which depends on Section 5). The first part of Section 7 (absolute value equations) depends on Sections 1 and 2. The second part (absolute value inequalities) has examples corresponding to the four types of inequalities discussed in Section 6.

Chapter 6 Each of Sections 1–5 depends on the preceding section. Section 6 (logarithmic growth) is independent of Section 5 (exponential growth), but does depend on Section 4.

Chapter 7 With two exceptions each section in this chapter depends on the preceding one. However, Section 4 (evaluation of sine and cosine) may be covered immediately after the definition of sine and cosine in Section 2 if desired. Section 7 (periodic graphs) is independent of Sections 5 and 6, but does depend on Section 4. The last half of Section 7 (graph of $f(t) = A \sin(bt + c)$) also depends on Section 3.2 (new graphs from old). Section 7 is not needed in Chapters 8 and 9, but the rest of Chapter 7 is used there.

Chapter 8 Section 2 (law of cosines) and Section 3 (law of sines) depend in part on Section 1 (right triangle trigonometry), but are independent of each other.

Chapter 9 Sections 1–3 (trigonometric identities) form a unit in which each section depends on the preceding one. However, Section 4 (trigonometric equations) and Section 5 (inverse trigonometric functions) are independent of Sections 1–3 and of each other; they may be covered first if desired.

Chapter 10 Each section in this chapter depends on the preceding one.

Chapter 11 Each section in this chapter depends on the preceding one.

Chapter 12 Section 2 (arithmetic and geometric sequences) depends on Section 1 (sequences and sums). Section 3 (binomial theorem) and Section 4 (induction) are independent of Sections 1–2 and of each other.

Excursions

Certain sections of the text are labeled "Excursion" instead of "Section." Each excursion is closely related to the section that precedes it and usually has that section as a prerequisite. Some excursions include topics that an instructor may want to include as a regular part of the course. Others provide interesting mathematical background or useful applications of topics discussed in the text. Still others are of use to some students but not to others.

Exercises

Each exercise set begins with routine calculation and drill problems (the ones that were labeled as A problems in the first edition); these are followed by exercises that are somewhat less mechanical and require some thought, but are able to be done by almost all students who have read the text carefully (the B problems in the first edition). Some exercise sets also include a section labeled "Unusual Problems" (the C problems in the first edition). It should be emphasized that these "unusual problems" are often not difficult, but are simply different from the sort of thing most students have seen before. Some of the "unusual problems" are mathematically difficult, but most of them should be well within reach of the majority of students.

Answers for all odd-numbered exercises and sample exam questions are given at the end of the book. Answers for even-numbered exercises are available in the Instructor's Manual for this text.

Calculators

In our experience most students entering a course in which this book is used already have calculators. The text is written with this in mind. In particular, the text takes full advantage of calculators when evaluating logarithmic, exponential, and trigonometric functions. Tables are dealt with in appendices at the end of the book, but play no role in the body of the text. Instructors who want to present such tables will have no difficulty doing so, but they should be aware that in Chapters 6–9 it is tacitly assumed that calculators will be used for the exercises

whenever needed. If tables are used instead, some exercises may be significantly longer or more difficult.

One area in which the presence of calculators has significantly affected the mathematical presentation is in the solution of trigonometric equations. In Section 9.4 we present a calculator-based algorithm for solving equations such as $\sin x = .473$ or $\tan x = 7$ that are rarely if ever discussed in traditional texts. It also works readily without calculators when the solution is a special angle such as $\pi/6$ or $3\pi/4$. If you have not thought about this question before, the algorithm may first appear to be a bit cumbersome. But it has been successfully classroom-tested: students find that they can deal with sine, cosine, and tangent in a uniform manner that minimizes the possibility of wrong or incomplete solutions.

In solving trigonometric equations and dealing with triangles one must answer questions such as: "if the sine of an acute angle is .642787, then what is the angle?" In the text we routinely answer such questions by using the INV SIN key on a calculator. Some instructors, however, are uneasy about this approach. They seem to feel that it is bad pedagogy or poor policy to use the INV SIN key without prior mention of the inverse sine *function*.

Our position is that the SIN and INV SIN keys on a calculator have two distinct roles. They are, of course, evaluation keys for the sine and arcsine functions. But they also can be thought of as part of a push-button trig table. Instead of looking through the columns of a table, you enter a number and push the button. We have never heard anyone argue that using a trig table to determine which angle has sine .642787 requires one to study inverse trigonometric functions first. But if there is no reason not to use tables, then there is no reason not to use a calculator as a push-button table. This is what is done in the text when appropriate (see for example, page 322).

Test Bank

A computerized test bank (suitable for use on an Apple II) is available for this book. It is accompanied by a printed copy of the test questions, which can be used even if a suitable computer is not available. Instructors may obtain it from the publisher.

To The Student

Read this—or you will turn into a toad!

If you want to succeed in this course, remember that *mathematics is not a spectator sport.* You can't learn math simply by listening to your instructor lecture or work problems. *You* have to take an active role, making wise use of your chief resources: your instructor and this book.

When it comes to math textbooks, many students use their books only for finding out what the homework problems are. If they get stuck on a problem, they page back through the text until they find a similar example. If the example doesn't clarify things, they may try reading part of the text (as little as possible). Rarely, if ever, do such students read through an entire section (or subsection) from beginning to end.

If this description fits you, don't feel guilty. Some mathematics texts are unreadable. But don't use your bad past experiences as an excuse for not reading this book. It has been classroom-tested for years by students like yourself. It is understandable by an average student, with a minimal amount of outside assistance. So if you want to get the most out of this course, we strongly suggest that you follow these guidelines:

1. Read the pages assigned by your instructor from beginning to end before starting the homework problems. If you find calculations you don't understand, take pencil and paper and try to work them out. If you don't understand a particular statement, reread the preceding material to see if you missed something.

2. If you have spent a reasonable amount of time trying to figure something out, mark the place with a question mark and continue reading. Plan to ask your instructor about the material you have marked.
3. Now do the homework problems. You should be able to do all, or almost all, of the assigned problems. After you've worked at the homework for a reasonable amount of time and answered as many problems as you can, mark the exercises that are still causing trouble. Plan to ask your instructor about them.

If you follow these guidelines, you will get the most out of this book. But it won't be enough unless you actually *ask* your instructor about the things you don't understand. Unfortunately, many students are afraid to ask questions in class for fear that the questions will seem "dumb." Such students should remember this:

> **If you have honestly followed the guidelines above and still have unanswered questions, then there are at least six other students in your class who have the same questions.**

So it's not a dumb question. Furthermore, your instructor will welcome questions that arise from a serious effort on your part. In any case, your instructor is being paid (with your tuition money) to answer questions. So do yourself a favor and get your money's worth—*ask questions.*

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