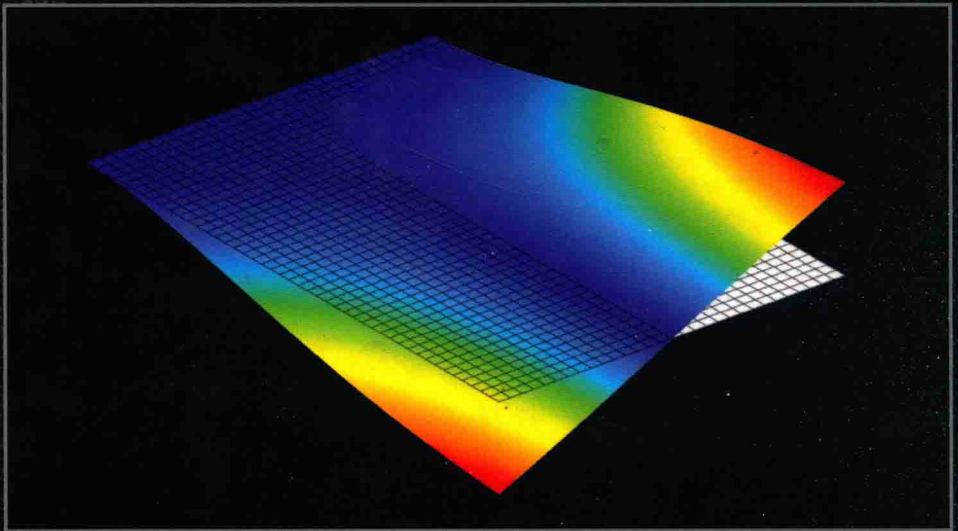


Mechanical Vibrations

Theory and Application
to Structural Dynamics

Third Edition



Michel Géradin
Daniel J. Rixen

WILEY

MECHANICAL VIBRATIONS

THEORY AND APPLICATION TO STRUCTURAL DYNAMICS

Third Edition

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WILEY

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Foreword

The first two editions of this book had seven skillfully written chapters, organized in my mind in three parts. Collectively, they aimed at giving the reader a coherent presentation of the theory of vibrations and associated computational methods, in the context of structural analysis. The first part covered the analytical dynamics of discrete systems, and both undamped and damped vibrations of multiple-degree-of-freedom systems. It also served as a good introduction to the second part, which consisted of two chapters. The first one focused on the dynamics of continuous systems and covered the subject of wave propagation in elastic media. It was followed by a chapter which bridged this topic with the first part of the book, by introducing the novice to the concept of displacement methods for semi-discretizing continuous systems. It also culminated with a brief and yet well-executed initiation to the finite element method. All this led to the third part of the book, which indulged into a concise and effective treatment of classical numerical methods for the solution of vibration problems in both frequency and time domains. Covering all of these topics in a unified approach, making them interesting to both students and practitioners, including occasional references to experimental settings wherever appropriate, and delivering all this in less than 400 pages, was a daunting challenge that the authors had brilliantly met. For this reason, the previous editions of this book have been my favourite educational publication on this subject matter. I have used them to teach this topic at the MS level, first at the University of Colorado at Boulder, then at Stanford University.

So what can one expect from a third edition of this book?

In its third edition, the overall organization of this book and that of its chapters has remained mostly unchanged. However, several enhancements have been made to its technical content. The notion of the response of a system to a given input has been refined throughout the text, and its connections to the concepts of dynamic reduction and substructuring (which remain timely) have been made easier to observe, follow, and understand. Chapter 3 has gained a new section on experimental methods for modal analysis and some associated essentials in signal processing and system identification. The mathematical content of Chapter 6 has been somehow refreshed, and its scope has been enhanced by two welcome enrichments. The first one is a new section on linear equation solvers with particular emphasis on singular systems. Such systems arise not only in many mechanical and aerospace engineering problems where the structure of interest is only partially restrained or even unrestrained, but also as artifacts of many modern computational methods for structural analysis and structural dynamics. The second enrichment

brought to Chapter 6 is an updated section on the analysis of the sensitivity of frequencies and mode shapes to parameters of interest, and its association with model updating. Most importantly, the third edition comes now with carefully designed problem sets (and occasionally some solutions) that will certainly enhance both processes of teaching and learning. Overall, the third edition has added about 150 pages of technical content that make it a better textbook for students and teachers, a useful reference for practitioners, and a source of inspiration for researchers.

*Charbel Farhat
Stanford University
1 January 2014*

Preface

This monograph results from a complete recasting of a book on Mechanical Vibrations, initially written in French and published by Masson Éditions in 1992 under the title *Théorie des vibrations, Application à la dynamique des structures*. The first edition in English was issued shortly after, thanks to the support of DIST (French Ministry of Scientific Research and Space) and published by John Wiley & Sons in 1994. The book was indubitably felt to fill a gap since both editions were a success in France as well as internationally, so that both versions were almost immediately followed by a second edition by the same publishers: in French in 1996, and in 1997 for the English version. Due to the short delay between editions, only minor changes – essentially corrections – took place between the first and second versions of the manuscript.

The numerous constructive comments received from readers – university colleagues, students and practising engineers – during the following decade convinced both of us that a deep revision of the original manuscript was definitely needed to meet their expectations. Of course there were still remaining errors to be corrected – and the very last one will never be discovered, error-making being a common trait of human beings – and more rigor and accuracy had to be brought here and there in the presentation and discussion of the concepts. But the subject of mechanical vibration has also rapidly evolved, rendering the necessity of the addition of some new important topics. Proposed exercises to help, on the one hand, teachers explain the quintessence of dynamics and, on the other hand, students to assimilate the concepts through examples were also missing.

We were already planning to produce this third edition in French in the early 2000s, but the project could never be achieved due to overwhelming professional duties for both of us. The necessary time could finally be secured from 2010 (partly due to the retirement of the first author). However, priority has now been given to the English language for the writing of this third, entirely new edition since our perception was that the demand for a new, enhanced version comes essentially from the international market. We are indebted to Éditions Dunod for having agreed to release the rights accordingly.

We are thus pleased to present to our former readers a new edition which we hope will meet most of their expectations, and to offer our new readers a book that allows them to discover or improve their knowledge of the fascinating world of mechanical vibration and structural dynamics.

Without naming them explicitly, we express our gratitude to all those who have helped us to make this book a reality. Indeed, we received from many colleagues, friends and relatives much support, which could take various forms, such as a careful and critical reading of some parts, the provision of some examples and figures, appropriate advice whenever needed, personal support and, not the least, the understanding of our loved ones when stealing from them precious time to lead such a project to its very end.

Michel Gérardin and Daniel J. Rixen
München
24 January 2014

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Introduction

We owe to Lord Rayleigh the formulation of the principles relative to the theory of vibration such as they are applied and taught nowadays. In his remarkable treatise entitled *Theory of Sound* and published in 1877 he introduced the fundamental concept of oscillation of a linear system about an equilibrium configuration and showed the existence of vibration eigenmodes and eigenfrequencies for discrete as well as for continuous systems. His work remains valuable in many ways, even though he was concerned with acoustics rather than with structural mechanics.

Because of their constant aim to minimize the weight of flying structures, the pioneers of aeronautics were the first structural designers who needed to get vibration and structural dynamic problems under control. From the twenties onwards, aeronautical engineers had to admit the importance of the mechanics of vibration for predicting the aeroelastic behaviour of aircraft. Since then, the theory of vibration has become a significant subject in aeronautical studies. During the next forty years, they had to limit the scope of their analysis and apply methods that could be handled by the available computational means: the structural models used were either analytical or resulted from a description of the structure in terms of a small number of degrees of freedom by application of transfer or Rayleigh-Ritz techniques.

The appearance and the progressive popularization of computing hardware since 1960 have led to a reconsideration of the entire field of analysis methods for structural dynamics: the traditional methods have been replaced by matrix ones arising from the discretization of variational expressions. In particular, the tremendous advances in the finite element method for setting up structural models gave rise to the development of new computational methods to allow design engineers to cope with always increasing problem sizes.

Today, the elaboration of efficient computational models for the analysis of the dynamic behaviour of structures has become a routine task. To give an example, Figure 1 illustrates the computational prediction of the vibration modes of a stator section of an aircraft engine. The fineness of the finite element model has been adapted in this case for the needs of the associated stress analysis, the latter requiring a level of detail that is not really needed for a modal analysis. The eigenmode represented is a 3-diameter mode exhibiting a global deformation of the structure. What makes the modal analysis of such a structure very difficult is the high level

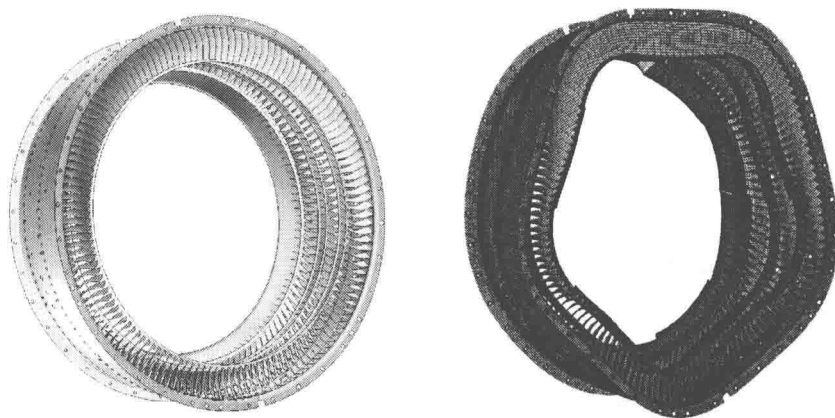


Figure 1 Finite element model of a stator section of aircraft engine. Source: Reproduced with permission from Techspace Aero – SAFRAN Group.

of cyclic symmetry (resulting from the number of stator blades) which is responsible for the appearance of a high number of nearly equal eigenvalues.

Development of computing, acquisition and sensing hardware has led to a similar revolution in the field of experimental techniques for identification of vibrational characteristics of structures. For more than thirty years, experimental modal analysis techniques have been developed which are based either on force appropriation or on arbitrary excitation.

The methods for dynamic analysis, whether they are numerical or experimental, have now taken an important place everywhere in engineering. If they were rapidly accepted in disciplines such as civil engineering, mechanical design, nuclear engineering and automotive production where they are obviously needed, they have now become equally important in the design of any manufactured good, from the micro-electromechanical device to the large wind turbine.

From its origin in the early sixties, the aerospace department of the University of Liège (Belgium) has specialized mainly in structural mechanics in its education programme. This book results from more than twenty years of lecturing on the theory of vibration to the students of this branch. It is also based on experience gathered within the University of Liège's Laboratory for Aerospace Techniques in the development of computational algorithms designed for the dynamic analysis of structures by the finite element method and implemented in the structural analysis code the team of the laboratory has developed since 1965, the SAMCEF™ software.¹

The content of the book is based on the lecture notes developed over the years by the first author and later formatted and augmented by one of his former students (the second author). This work reflects the teaching and research experience of both authors. In addition to his academic activity at the University of Liège, the first author has also spent several years as head of the European Laboratory for Safety Assessment at the Joint Research Centre in Ispra (Italy). The second author has accumulated until 2012 lecturing and research experience at

¹ From 1986, SAMCEF™ has been industrialized, maintained and distributed by SAMTECH SA, a spin-off company of the University of Liège.

the Delft Technical University (The Netherlands) and is currently pursuing his career at the Technische Universität München (Germany). The book has been adopted internationally as course reference in several universities.

Due to its very objective, the book has a slightly hybrid character: the concepts of vibration theory are presented mainly with the intention of applying them to dynamic analysis of structures and significant attention is paid to the corresponding methods. Even though the foundations of analytical mechanics are reviewed, a preliminary acquaintance with this subject is necessary. A good knowledge of matrix algebra and theory of complex numbers, calculus, structural mechanics and numerical analysis for linear systems is required. It is also assumed that the reader is familiar with the theory of the single-degree-of-freedom oscillator. However, the presentation of the finite element method is deliberately made simple since its study requires a course of its own. Finally, the very important fields of nonlinear vibration and random vibration have been intentionally omitted in the present text since they are highly specialized subjects.

What is new in this third edition?

Although the overall structure of the book, its organization into individual chapters and the main topics addressed, remain unchanged, this new version is the result of important revision work to achieve major improvements.

Regarding the theoretical content itself, the main changes with respect to the previous edition are the following:

- The response of an either discrete or continuous system has been the object of deep rethinking and turned out to be a thread towards the important concepts of dynamic reduction and substructuring. The latter are explained and developed, starting from the observation that the response of a part from the overall system is the result either from an excitation of its support, or from the application of a set of loads at selected points of the structure. Such duality can be exploited in at least two ways. On the one hand, it allows a system description in terms of the classical concepts of mechanical impedance or admittance. On the other hand, it naturally leads to the concept of dynamic substructuring based on an expansion of the response in terms of the spectral content of the impedance and admittance relationships.
- Experimental modal analysis is an essential ingredient in structural dynamics since it allows to confirm by experiment the structural properties predicted through numerical modelling. Therefore it was felt necessary to include in this new version of the book the essentials of signal processing and identification techniques that allow us to extract the spectral properties of a linear structure from measured dynamic responses.
- In the same spirit, the concept of eigensolution sensitivity to physical parameters has been further detailed since it is the basis for the development of appropriate numerical tools for improving the numerical model of a real dynamic system.
- The considerable evolution of the size of the structural systems to be considered for eigenvalue extraction and transient dynamic analysis in the context of large engineering projects had to be reflected and addressed properly. Models reaching the size of several millions of degrees of freedom (such as the one displayed on Figure 1) are now common practice. The efficiency of the eigenvalue solvers (such as the Lanczos method) and implicit