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Homogeneous, Isotropic Turbulence

Phenomenology, Renormalization, and Statistical Closures

W. DAVID McCOMB

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Phenomenology, Renormalization, and Statistical Closures

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Preface

In the preface to *The Physics of Fluid Turbulence,* which was published in 1990, I suggested that turbulence research had become ever more scientific and sophisticated over the previous three decades—to the point where it could be argued (in effect) that a monograph concentrating on the *physics* of turbulence was justified. Given that the subject was then, and remains now, dominated by engineers and applied mathematicians, this might have been seen as a rash step. However, in practice, that proved not to be the case. And what turned out to be true then is even more so some two decades on. In particular, the growth of numerical simulation as a discipline has had a positively transformative effect on the subject. For instance, Fourier methods, which once tended to arouse suspicion or even hostility, are now widely used. Indeed, the study of homogeneous, isotropic turbulence, which used to be regarded as a minority cult, with no relevance to applications, is now very much a major field of research. It even has its own three-letter acronym (HIT): a sure sign of success!

The principal justification for studying HIT is that it tends to focus our attention on the fundamental problem of turbulence. As we all know, turbulence presents a very difficult fundamental problem. This is because the equations that govern *all* fluid motion are nonlinear and hence, in general, insoluble. Naturally, that goes for turbulent motion as well. But it is made worse by the chaotic nature of that motion. This forces us to attempt a statistical treatment; and a statistical description of turbulence runs into the unsolved moment-closure problem, as formulated more than one hundred years ago by Osborne Reynolds.

In more recent years, the development of numerical simulation has also led to the concept of *large-eddy simulation* and its associated subgrid modelling problem. This is a 'reduced' form of the closure problem, but is still in principle insoluble. The result is that we are thrown back on a mixture of phenomenology, mathematics (which can be formidably complicated and sometimes rather exotic), and a variety of ad hoc approximations. In this respect, turbulence is just like all the other bedrock problems in theoretical physics.

Given that the turbulence problem is inherently difficult, the starting point of this book is a recognition of the general unease felt by some members of the turbulence community, myself included, about certain basic aspects of the present state of the subject—about, in fact, aspects that should be among the more tractable of our difficulties and that involve no more than clarification of basic phenomenology. In this connection, well-established researchers have written of the tentative nature of much of the work in turbulence, unlike in other subjects. Or of the need to resolve long-standing issues, which stand in the way of successful applications of the subject. Or on the difficulty of publishing a 'different' or 'unconventional' opinion. Recently, we have had a very remarkable example of this, when a theoretical paper rejected ten years

Now we come to Part III, which deals with the statistical theory. This brings us to an example of what, in the UK, is often referred to as 'the elephant in the room'—In other words, some large awkward fact, of which everyone must be aware, but which nobody discusses. Here, our 'elephant' is the fact that there are two rival approaches to a theory of turbulence. That is, to use a rough categorization, there are approaches in the spirit of *statistical physics* versus approaches based on *dynamical systems theory.* Both of these approaches have been part of turbulence research for much the same length of time, but have traditionally addressed different aspects of the problem. The statistical approach was initiated by Reynolds in 1895, when he averaged the Navier–Stokes equations. Thereafter, the main emphasis in turbulence research has been on the use of statistical methods. In contrast, the treatment of fluid motion as a dynamical system was limited to the study of hydrodynamic stability, and originally to the problem of the laminar–turbulent transition, beginning in 1907/08 with the work of Orr and Sommerfeld.

The statistical treatment of developed turbulence was dominant throughout most of the last century, but two factors led to increased interest in dynamical systems theory. First, there was the discovery of coherent structures in turbulence, which suggested that the view of the laminar–turbulent transition as a single, or once and for all, catastrophe was over-simplified. For instance, the bursting process in turbulent pipe flow could be interpreted as repeated laminar–turbulent transitions. Second, the development of personal computers gave a stimulus to the study of dynamical systems with a few degrees of freedom, with the onset of chaotic behaviour being of particular interest. Moreover, the study of atmospheric turbulence, in the context of weather forecasting, led quite naturally (and independently) to an interest in predictability, and also in reducing the number of modes necessary to describe turbulence.

All this is well known. But in HIT there are no really interesting coherent structures and, with a formulation appropriate to statistical physics, there is no obvious laminar–turbulent transition. As a result, some researchers tend to dismiss the dynamical systems approach as 'dealing with only a few degrees of freedom'. However, bridging the gap between small and large numbers of degrees of freedom is a matter of *renormalization,* the seminal technique that has dominated statistical closure approximations since the 1960s. Indeed, in microscopic physics, the successful use of the renormalization group involves *both* statistical physics and dynamical systems theory. So there seems to be no reason why these two approaches should be hermetically sealed from each other.

In Part III, we begin with a consideration of renormalized perturbation theories that lead to two-point closures. Our intention here is to draw a clear distinction between the uncontroversial aspects of the subject and those issues that require some resolution. Accordingly, in Chapter 9, we present the Kraichnan–Wyld–Edwards covariance equations, which essentially reformulate the closure problem as a search for an appropriate renormalized response function. Despite the large number of acronyms (DIA, SCF, LET, EDQNM, and so on) that are associated with turbulence theory (and which are confusing to the non-specialist), there really is just one covariance equation (that is, if one considers single-time covariances—there are two such equations in the two-time case). In Chapter 10, we discuss the more controversial aspects, and show that in recent years some of these have been resolved. Chapter 11 deals with the renormalization group, and our emphasis here is on distinguishing the inappropriate use of the field-theoretic methods of microscopic physics from the relatively small body of work that involves a genuine attempt to directly apply the basic RG algorithm to turbulence. Chapter 12 is a postscript dealing, as its title suggests, with some approaches that are still being developed, and attempts to indicate some promising lines of research.

In all, the aim of this book is to simplify, and at every stage to draw clear distinctions between what one can believe and what still requires caution or further clarification. To this end, Chapter 1 provides a particularly simple overview of the material of the rest of the book. It has been done in this way in order to allow the main ideas to stand out.

One thing that I should emphasize is that this book is in an entirely different category to my 1990 book, most of which is still as relevant today as when it was published. *The Physics of Fluid Turbulence* aimed to formalize turbulence theory as part of statistical physics and to demystify renormalization methods by providing very detailed mathematical expositions, and also presented accounts of work in drag reduction by additives, and in turbulent diffusion, both of which required a consideration of various shear flows. In contrast, the present book is restricted to homogeneous, isotropic turbulence, in order to consider only the core fundamental problems. At various points, it refers to the previous book for the detailed mathematics, and concentrates on physical interpretation of renormalization theories in order to make it clear what can be believed and what requires caution. It also presents recently published research aimed at clearing up long-held misconceptions and unresolved issues in the phenomenology of turbulence.

Lastly, it is a pleasure to acknowledge the help received from my students. Sam Yoffe (who has now completed his PhD) helped with the preparation of some figures and some parts of the text (in particular, Section 12.2 and Appendix B), and also read some chapters in draft. Moritz Linkmann has also helped with the preparation of some figures, and has read the entire book, pointing out errors and raising queries as appropriate. At various points in the book, I have drawn on the PhD theses of Mark Filipiak, Adrian Hunter, Craig Johnston, Khurom Kiyani, Bill Roberts, David Storkey, Alastair Young, and Taek-Jin Yang. This is in addition to the citation of their published work, and is acknowledged as appropriate in the text. However, it seemed a good idea to collect them all together here and offer my retrospective thanks. During the course of writing the book, my re-reading of these theses took on an aspect of time travel, and it was a great pleasure to be reminded of so much fine work on their part. This made me think of all my other students (and post-docs, too) who over the years contributed so much. In some cases, their published work has been cited here, but in other cases it would not have been relevant. For this reason, I regard the present book as being dedicated to them all.

David McComb
Edinburgh
February 2014

Acknowledgements

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Notation

General remarks

The list of notation given here is not intended to be complete. It is provided to help the reader identify symbols that are encountered at various points throughout the book. Those symbols that only occur near to where they are defined are not included. The use of the subscript 'zero' on the kinematic fluid viscosity is to draw attention to the fact that it can be renormalized. This avoids having to rename it ν_0 when beginning renormalization methods. I hope that readers can live with it in the vast majority of cases where there is no renormalization involved!

Lastly, to avoid any possible confusion, I should mention some changes of notation. Until recent years, I followed the example of Edwards and used Q for correlations, G for response (i.e. Green) functions, and D for projection operators. There is no universal convention. Some people follow Kraichnan and use U for correlations and G for response, while others use a variety of symbols. The use of P for projectors is, however, near universal, and belatedly I have adopted it, although I still use M for the inertial transfer operator. As regards correlation and response functions, when studying the application of fluctuation–response theory to macroscopic systems in recent years, I noticed that dynamical systems theorists seemed to be standardizing on C for correlations, covariances, etc., while R was used for response functions. This seemed eminently logical to me, so I adopted it and have used it in this book.

Italic symbols

$\mathbf{c}$	shift velocity of the Galilean transformation
$C_{\alpha\beta}(\mathbf{x}, \mathbf{x}'; t, t')$	covariance or correlation of fluctuating velocities
$C(r)$; $C(r,t)$	correlation function of fluctuating velocities in isotropic turbulence
$C_{\alpha\beta}(\mathbf{k}; t, t')$	covariance of fluctuating velocities in wavenumber space
$C(k)$; $C(k,t)$	energy spectral density function
$C_{LL}(r)$	longitudinal correlation function
$C_{NN}(r)$	transverse correlation function
C_ε	coefficient for Taylor dissipation surrogate, dimensionless dissipation rate
$D(k)$; $D(k,t)$	energy dissipation spectrum
$\partial_t \equiv \partial/\partial t$	contracted notation for the partial derivative with respect to time
$E(k)$; $E(k,t)$	energy spectrum
$E = \int_0^\infty E(k)\, dk$	total kinetic energy of turbulence
$f_\alpha(\mathbf{x}, t)$	random stirring force with zero mean
$f_\alpha(\mathbf{k}, t)$	random stirring force with zero mean in wavenumber space

$F(k)$	prescribed covariance of stirring forces
$f(r)$	longitudinal correlation coefficient
$g(r)$	transverse correlation coefficient
k_{bot}, k_{top}	smallest and largest wavenumbers bounding the inertial range
k_C	cutoff wavenumber for large-eddy simulation
k_d	Kolmogorov dissipation wavenumber
k_{min}, k_{max}	smallest and largest resolved wavenumbers in direct numerical simulation
k_*	wavenumber where the transfer spectrum $T(k)$ crosses zero
l	general length scale, its meaning being defined locally where it is used
L	integral length scale
L_{box}	length of side of the cubical box containing the tubulence; other characteristic length of experimental apparatus
$L(\mathbf{k}, \mathbf{j})$	coefficient in the (k, j, μ) formulation of the turbulence problem
$M_{\alpha\beta\gamma}(\mathbf{k})$	inertial transfer operator in wavenumber space
$P_{\alpha\beta}(\mathbf{k})$	transverse projector in wavenumber space
$R(\mathbf{k}; t, t')$	response function
R_L	integral-scale Reynolds number
R_λ	Taylor–Reynolds number
S	skewness factor of the longitudinal velocity derivative
$S_n(r)$	longitudinal structure function of order n
$T(k)$; $T(k.t)$	energy transfer spectrum
U	root-mean-square turbulent velocity
$\mathbf{u}(\mathbf{x}, t)$; $u_\alpha(\mathbf{x}, t)$	fluctuating velocity field with zero mean
$\mathbf{u}(\mathbf{k}, t)$; $u_\alpha(\mathbf{k}, t)$	fluctuating velocity field with zero mean in wavenumber space
$W(k) = 4\pi k^2 F(k)$	energy injection spectrum due to stirring forces

Greek symbols

α	Kolmogorov spectral prefactor
$\widehat{\varepsilon}$	instantaneous dissipation rate
ε	mean dissipation rate
$\varepsilon_D = -\partial E/\partial t$	energy decay rate
$\varepsilon_T = \Pi_{\mathrm{max}}$	maximum rate of inertial transfer
$\varepsilon_W = \int_0^\infty W(k)\, dk$	rate at which stirring forces do work on the fluid
ζ_n	exponents for power-law region of $S_n(r)$
η	Kolmogorov dissipation length scale
λ	Taylor microscale
ν_0	kinematic fluid viscosity
$\Pi(k, t)$	transport power, or flux of energy into mode k due to inertial transfer
Π_{max}	maximum value of the transport power
Σ_n, σ_n	scaling exponents for $S_n(r)$ as determined by extended self-similarity

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Abbreviations

ALHDI	abridged Lagrangian-history direct-interaction
DIA	direct-interaction approximation
DNS	direct numerical simulation of the Navier–Stokes equations
EDI	Eulerian direct-interaction
EDQNM	eddy-damped quasi-normal Markovian
EFP	Edwards' (Fokker–Planck) theory
ESS	extended self-similarity
FNS	Forster, Nelson, and Stephen
FRN	finite-Reynolds-number (effects)
FRR	fluctuation–response relation
GI	Galilean invariance/invariant
HIT	homogeneous, isotropic turbulence
h.o.t.	higher-order terms
K41	Kolmogorov's (1941) theory
KHE	Kármán–Howarth equation
LES	large-eddy simulation
LET	local-energy-transfer theory
LRA	Lagrangian-renormalized approximation
MSR	Martin–Siggia–Rose
NSE	Navier–Stokes equation(s)
pdf	probability distribution function(al)
PFT	*The Physics of Fluid Turbulence*, by W.D. McComb (Oxford University Press, 1990)
QN	quasi-normality
RG	renormalization group
RGI	random Galilean invariance
RGT	random Galilean transformation
RPT	renormalized perturbation theory
SCF	self-consistent field theory
TFM	test-field model

Contents

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